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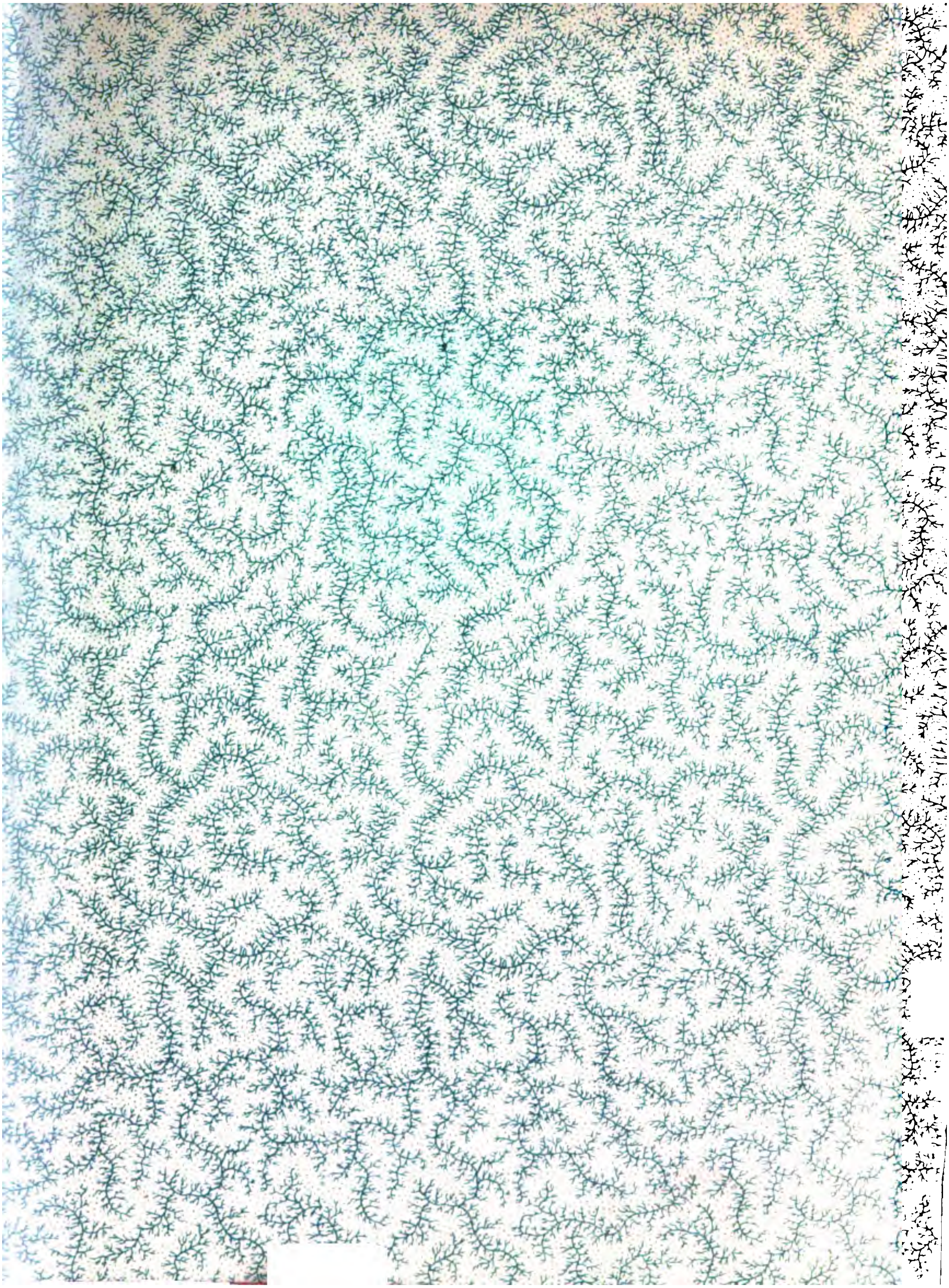

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THE

ESSENTIALS

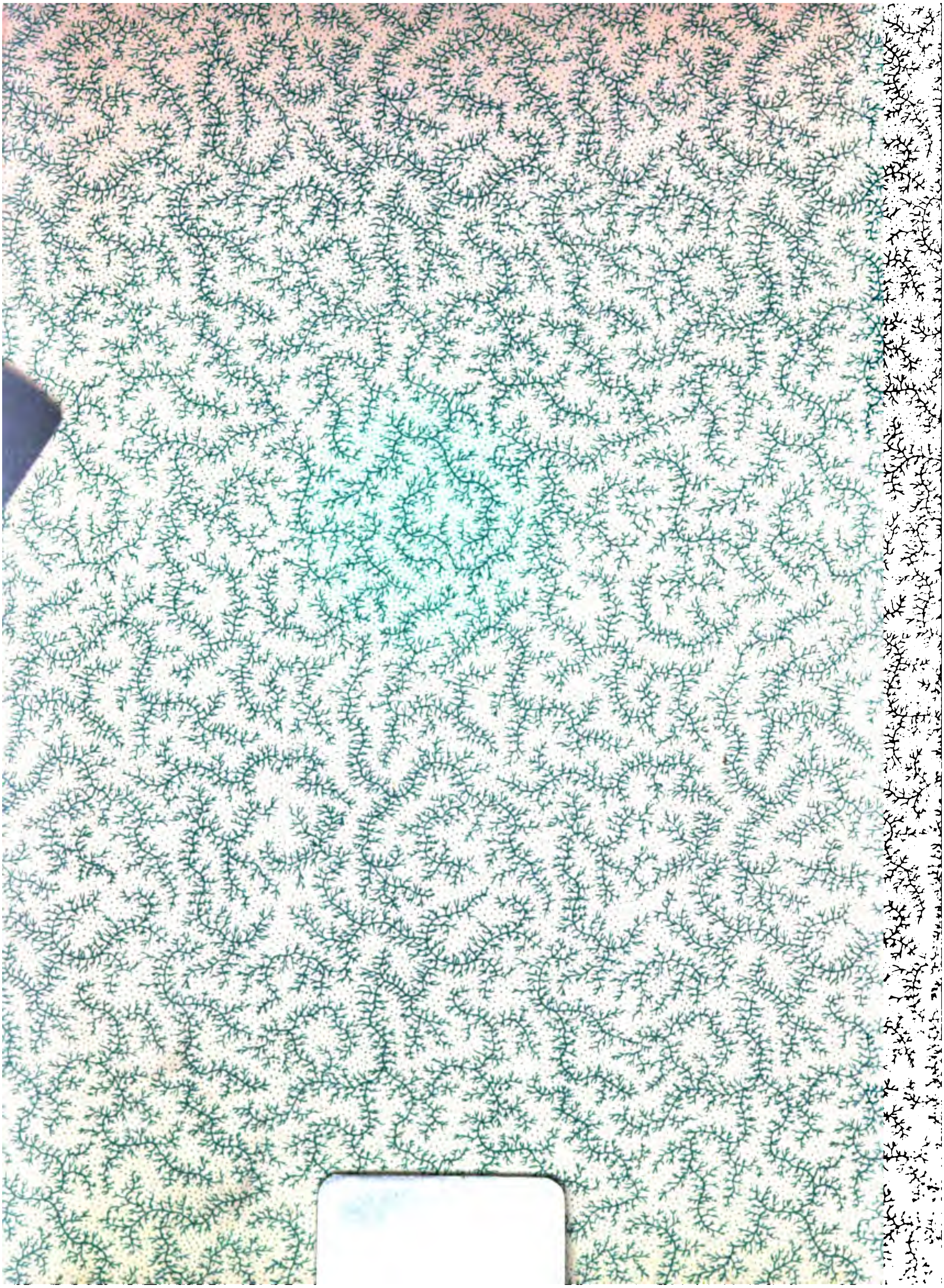
OF

BUSINESS ARITHMETIC

FOR USE IN BUSINESS COLLEGES

FIRST EDITION

BALTIMORE, MD.
SADLER PUBLISHING COMPANY
10 AND 12 N. CHARLES STREET
1889



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PREFACE.

THIS book embraces all the arithmetic necessary to success in business. It has been specially prepared for use in business colleges in which the average period of attendance is six months. Under the competent instruction to be found in those institutions, the essentials of arithmetic, as here presented, can be thoroughly mastered within this short period, if the student is not hampered by the study of useless theories or obsolete topics.

In its treatment of arithmetical subjects which every young business man must know, the book will be found unusually full, its special aim being to qualify young men in six months to handle quickly, accurately, and intelligently, the class of problems which are likely to arise in every-day business life.

How well the authors have succeeded in accomplishing their purpose in the present volume, they are content to leave to the enlightened judgment of business educators, for whose benefit it is especially designed.

THE AUTHORS.

TO TEACHERS.

TO secure an intelligent solution of arithmetical problems, students should become familiar with the generalizations of **52**, **53**, **54**, and **55**, relating to integers; of **113**, **114**, and **116**, relating to common and decimal fractions; and of **213** and **214**, relating to percentage and its applications. Students should be required to exhibit the application of these principles to particular problems as they are successively solved, until the general law of increase and decrease of numbers has been thoroughly mastered.

Topical rules and special analyses are soon forgotten or inextricably blended; but the constant iteration and reiteration of a few general principles, applicable to all classes of problems, are likely to leave a clear and permanent impression.

By thus generalizing many perplexing arbitrary rules into a few simple common-sense rules, many independent bases of reasoning into one common basis, it is possible for business colleges in six months to impart to their students a more thorough and enduring knowledge of arithmetic than can be taught in twice that time under the old system of presentation.

CONTENTS.

INTEGERS.

PAGE
Numeration and Notation . . 7
Addition 12
Subtraction 16
Multiplication 18
Division 23
United States Money . . . 29
PAGE
Relation of Integers 33
Review 35
Factoring 38
Common Dividends 40
Common Divisors 42
Cancellation 43

COMMON FRACTIONS.

Preliminary Definitions . . . 45
Reduction 47
Addition 51
Subtraction 53
Multiplication 55
Division 58
Relation of Common Fractions. 61
Review 65

DECIMAL FRACTIONS.

Preliminary Definitions . . . 68
Numeration and Notation . . 70
Reduction 72
Addition 74
Subtraction 75
Multiplication 76
Division 77
Review 79

COUNTING-HOUSE PRACTICE.

Commercial Calculations . . . 81
Bills and Accounts 87

DENOMINATE NUMBERS.

Measures 94
Reduction Descending . . . 98
“ Ascending 99
“ of Denom. Fractions 100
Addition of Comp. Numbers. . 102
Subtraction of Comp. Numbers. 103
Multiplication “ “ 104
Division “ “ 105
Review 107
Metric System 108

PRACTICAL MEASUREMENTS.

	PAGE		PAGE
Rectangular Surfaces	111	Stone Work	119
Carpeting, Paving, etc.	114	Brick Work	121
Rectangular Volumes	115	Bins, Granaries, etc.	122
Board Measure	118	Cisterns	124

PERCENTAGE.

Simple Percentage	125	Bank Discount	184
Profit and Loss	135	Partial Payments:	
Trade Discount	141	United States Rule	192
Commission	146	Mercantile Rule	195
Ordinary Interest	153	Stocks and Bonds	197
Accurate "	166	Exchange	206
Formulas of "	167	Domestic	207
Review	171	Foreign	210
Compound Interest	173	Custom-House Business	213
True Discount	177	Taxes	217
Commercial Paper	180	Fire and Marine Insurance	219

RATIO AND PROPORTION.

Ratio and Proportion	223	Averaging Accounts	244
Single Rule of Three	225	Averaging Accounts Sales	252
Double Rule of Three	228	Cash Balance	254
Bankruptcy	230	Partnership	259
Equation of Payments	233		

APPENDIX.

Addition	273	Marking Goods	286
Subtraction	275	Answers	289
Multiplication	276		

ARITHMETIC.

INTRODUCTION.

1. Arithmetic treats of numbers.

2. A **Number** is an expression of a quantity by means of characters or figures.

3. The **Unit** of a number is *one* of the things which it expresses. Thus, *one* inch is the unit of nine inches, *one* mile of sixty miles.

REMARK 1.—Numbers are classified as *concrete* when they name the kind of units which they represent, as five *gallons*, three *pounds*, six *books;* and as *abstract* when the kind of units is not named, as *five, three, six.*

REM. 2.—Numbers are said to be *like* when they express the same kind of units, as two *years* and nine *years;* and *unlike* when they express different kinds of units, as three *miles* and four *quarts.*

REM. 3.—Numbers are further distinguished as *integers* when they express *whole* units, and as *fractions* when they express *parts* of a whole unit.

NUMERATION AND NOTATION.

4. Numeration is the art of *reading* numbers when expressed by figures.

5. Notation is the art of *writing* numbers.

REM.—The method of expressing numbers by figures is called *Arabic Notation;* and by letters, *Roman Notation.*

6. The Arabic method embraces the following ten figures :

0	1	2	3	4	5	6	7	8	9
Naught,	*One,*	*Two,*	*Three,*	*Four,*	*Five,*	*Six,*	*Seven,*	*Eight,*	*Nine.*

Each of the above figures, when standing alone, represents the number written under it. Consequently, *nine* is the greatest number that can be expressed by one figure ; and numbers greater than nine are expressed by the artifice of increasing the local value of each figure *tenfold* by placing *one* figure to the right of it ; a *hundredfold* by placing *two* figures to the right of it ; a *thousandfold* by placing *three* figures to the right of it ; etc.

REM.—The distance of a figure from the right of a number is called its order ; thus, the *first* figure at the right is the *first*, or *units'* order ; the second figure, the *second*, or *tens'* order ; the *third* figure, the *third*, or *hundreds'* order ; the *fourth* figure, the *fourth*, or *thousands'* order, etc. ; *ten units of any order equaling one unit of the next order on the left.* Hence, the representative value of a figure increases tenfold for each order that it is moved to the left, and decreases tenfold for each order that it is moved to the right. For the names and position of the several orders, see DECIMAL NOTATION TABLE, **138.**

ILLUSTRATIVE EXERCISE.

7. Read 7862.

EXPLANATION.—7862 is read *seven thousand eight hundred and sixty-two ;* the local value of 7 being *thousand* because it has three figures at its right, the local value of 8 being *hundred* because it has *two* figures at its right, the local value of 6 being *tens* because it has *one* figure at its right, and the local value of 2 being its *simple value* (two) because it has *no* figure at its right.

Read, or express by written words, the following numbers :

1. 8.
2. 19.
3. 16.
4. 25.
5. 34.
6. 87.
7. 52.
8. 49.
9. 91.
10. 63.
11. 372.
12. 548.
13. 275.
14. 118.
15. 407.
16. 4195.
17. 3861.
18. 9138.
19. 7413.
20. 5279.
21. 5009.
22. 8074.
23. 6905.
24. 4260.
25. 3006.

8. With numbers greater than hundreds, the further artifice is employed of considering them in *periods* or sets of *three figures* each. The first set of three figures at the right is named the *units' period ;*

the second set, the *thousands' period;* the third, *millions;* the fourth, *billions;* the fifth, *trillions;* the sixth, *quadrillions;* etc.

ILLUSTRATIVE EXERCISE.

9. Read 51827168394.

Explanation.—Beginning at the right, separate the number into periods of three figures each, as follows, 51,827,168,394. As the right-hand period (394) is pointed off, think of its name (units); as the second period (168) is pointed off, think of its name (thousands); and so continue with the remaining periods until the left-hand period (51) is reached, which will thus prove to be billions.

Then commencing with this left-hand period, read it as if it stood alone, adding its name (51 billion); then read the following period as if it stood alone, adding its name (827 million); then the next period (168 thousand); and finally the last period (394), omitting its name (units). Hence,

Rule.—I. *Commence at the right and separate the number into periods of three figures each.*

II. *Then begin at the left and in regular order read each period as if it stood alone, adding the name of each period except that of units.*

Read, or express by written words, the following numbers:

1. 59285.
2. 1718219.
3. 745364.
4. 39219283.
5. 65175.
6. 8721293592.
7. 2195629837.
8. 9127438.
9. 5375816253.
10. 29285.
11. 92162715208508.
12. 4000273001.
13. 180005107.
14. 8000000026.
15. 1900107500.
16. 400034.
17. 2900107.
18. 9800001800070.
19. 31768290005.
20. 2831000.
21. 91000146.

ILLUSTRATIVE EXERCISE.

10. Express, by means of figures, nine trillion, twenty-seven billion, two hundred thirty-five million, five hundred and eighty-six.

Ans. 9,027,235,000,586.

Explanation.—Commence with the highest period of the given number (trillions), writing it with a comma (thus, 9,); then write the next lower period (billions), but as 27 contains only two of the *three* figures required for a period (**8**), write a cipher in the place of the omitted figure (thus, 9,027,); then the next lower period (millions), placing a comma after it (thus, 9,027,235,); then the next lower period

(thousands), and as there are no thousands given, indicate the omission of the three figures belonging to that period by writing three ciphers (thus, 9,027,235,000,); then conclude by writing the following and lowest period of the scale (units), obtaining, as the final result, 9,027,235,000,586.

RULE.—I. ***Commence with the highest period of the given number, writing it and each succeeding period in regular order, and filling vacant periods with three ciphers.***

II. ***If any period except the highest should contain only one or two figures, prefix ciphers thereto until it contains three figures.***

Express, by means of figures, the following numbers:

1. Six thousand seven hundred and thirty-six.
2. Ninety-three thousand four hundred and fifty-three.
3. Twelve thousand and twenty-nine.
4. Five thousand and two.
5. Three hundred and five thousand and sixty-eight.
6. Seventy thousand and sixty.
7. Forty-three thousand five hundred.
8. Nine hundred and eight thousand and five.
9. Two million, one hundred and seven thousand, and fifty-six.
10. Nine million, twenty-eight thousand, eight hundred and fifty.
11. Five million, three hundred and seventy-nine thousand, eight hundred and fifteen.
12. Six million, four hundred and thirty-two thousand, three hundred and forty-nine.
13. Five hundred and twenty million, two hundred and sixty-one thousand, nine hundred and seventy-four.
14. Three billion, one hundred and seventy-four million, eight hundred and thirteen thousand, two hundred and twenty.
15. Six billion, eighty-one thousand, eight hundred and fifty-three.
16. Seventeen trillion, eight hundred and forty-two billion, nine hundred and eight million, twenty-one thousand, eight hundred and forty-five.
17. Ninety-six trillion, twenty-nine thousand, and seven.
18. Seventy-two quadrillion, five hundred and thirty-six trillion, eighty-two billion, five million, six hundred and ninety-nine thousand, and thirty-one.

ROMAN NOTATION AND NUMERATION.

11. The **Roman Notation** employs the following seven letters:

I	V	X	L	C	D	M
One,	*Five,*	*Ten,*	*Fifty,*	*One Hundred,*	*Five Hundred,*	*One Thousand.*

12. The method of combining these seven characters to express numbers is shown in the following

ROMAN NOTATION TABLE.

I	One.	XIV	Fourteen.	XC	Ninety.
II	Two.	XV	Fifteen.	C	One hundred.
III	Three.	XVI	Sixteen.	CC	Two hundred.
IV	Four.	XVII	Seventeen.	CCC	Three hundred.
V	Five.	XVIII	Eighteen.	CCCC	Four hundred.
VI	Six.	XIX	Nineteen.	D	Five hundred.
VII	Seven.	XX	Twenty.	DC	Six hundred.
VIII	Eight.	XXX	Thirty.	DCC	Seven hundred.
IX	Nine.	XL	Forty.	DCCC	Eight hundred.
X	Ten.	L	Fifty.	DCCCC	Nine hundred.
XI	Eleven.	LX	Sixty.	M	One thousand.
XII	Twelve.	LXX	Seventy.	MM	Two thousand.
XIII	Thirteen.	LXXX	Eighty.	MDCCC	Eighteen hundred.

REM.—A horizontal line or bar placed over a letter multiplies its simple value by one thousand. Thus, $\overline{\text{V}}$ denotes five thousand; $\overline{\text{VII}}$, seven thousand; $\overline{\text{XIX}}$, nineteen thousand; etc.

ILLUSTRATIVE EXERCISE.

13. Write 2876 by the Roman method.

EXPLANATION.—First write its highest order (thousands), thus, MM; then its next highest order (hundreds), thus, MMDCCC; then its next highest order (tens), thus, MMDCCCLXX, and finally its lowest order (units), thus, MMDCCCLXXVI. Hence,

RULE.—*Write the highest order, then the next highest order, and so continue until the lowest order is written.*

Express the following numbers by the Roman notation:

1. 17.
2. 25.
3. 19.
4. 38.
5. 43.
6. 57.
7. 81.
8. 75.
9. 113.
10. 163.
11. 194.
12. 242.
13. 676.
14. 839.
15. 921.
16. 354.
17. 1846.
18. 1086.
19. 2739.
20. 9461.

Express the following numbers by the Arabic notation:

21. XXIV.
22. XLVI.
23. LXXIX.
24. XCVIII.
25. CXIX.
26. CCCCXLI.
27. DCCXCIX.
28. MCCXVI.
29. MMDCCIV.
30. DCCCCXIX.
31. MMMCXXXVI.
32. $\overline{\mathrm{V}}$DCCCXCVIII.
33. $\overline{\mathrm{LXV}}$DCXXIV.
34. $\overline{\mathrm{IX}}$CCXLVII.
35. $\overline{\mathrm{XV}}$CCCCLXXI.

ADDITION.

14. **Addition** is the process of finding *one number* which will express as many units as two or more other numbers.

Rem. 1.—The several numbers to be added are called *Addends;* the number which expresses as many units as its addends is called the *Sum* or *Amount.*

Rem. 2.—The *Sign of Addition* is an erect cross (+). It is read *plus,* and is placed between numbers to denote that they are to be added.

Rem. 3.—The *Sign of Equality* is two short, horizontal, and parallel lines (=). It is read *equals,* or *equal to,* and denotes that the quantity or quantities on its left are equal to the quantity or quantities on its right.

15. The **Principles of Addition** are:

1. *Only like numbers and like orders can be added.*
2. *The sum expresses units of the same name as its addends.*
3. *The sum of two or more numbers equals the sum of their respective similar orders, added separately.*

Note.—Orders are said to be *like* or *similar* when they are at the same distance from the right of their respective numbers.

ILLUSTRATIVE EXAMPLE.

16. What is the sum of $825, $367, $546, and $938?

SOLUTION.
$825
367
546
938
$2676
12

EXPLANATION.—Write the addends so that their right-hand figures shall fall in the same column. Add the figures of the right-hand column, obtaining $8+6+7+5 = 26$, or 2 tens and 6 units. Place 6 units under the units' column, and carry 2 tens to the tens' column.

Next add the tens' column, obtaining 2 (the carrying figure) $+ 3 + 4 + 6 + 2 = 17$ tens, or 1 hundred and 7 tens. Place 7 tens under the tens' column, and carry 1 hundred to the hundreds' column.

Then add the hundreds' column, obtaining 1 (the carrying figure) $+ 9 + 5 + 3 + 8 = 26$ hundreds.

RULE.—I. *Write the numbers so that units of the same order shall fall in the same column.*

II. *Beginning at the right, add each column separately; and write the sum underneath if it does not exceed nine.*

III. *If the sum of any column exceeds nine, set down its right-hand figure only; and add the number expressed by its remaining figures to the next column on the left.*

IV. *Set down the entire sum of the left-hand column.*

17. PROOF OF ADDITION.—*Add each column in the contrary direction, to form new combinations of figures. If the same results are obtained, the work is supposed to be correct.*

EXAMPLES FOR PRACTICE.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$5	$4	$9	$8	$53	$75	$526	$891
3	6	6	2	24	39	738	435
4	3	4	6	87	58	462	763
2	5	8	5	35	24	249	294
3	2	7	7	92	13	384	587
1	4	5	9	46	67	763	915

(9)	(10)	(11)	(12)	(13)
$8327	$6583	$2853	$52132	$91835284
5945	7354	6986	79825	24542525
6278	6782	3571	85274	86199746
4354	9483	4382	11618	52753672
7286	1629	3674	97583	17878391
5923	2807	5218	36929	65365435

(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
$5	$6	$8	$32	$51	$25	$519	$615
2	2	3	65	28	13	831	273
7	7	5	78	66	38	487	983
3	3	2	26	34	59	314	321
6	4	9	87	75	21	722	527
2	2	7	42	33	76	567	198
5	1	2	35	81	92	949	832
8	5	3	29	27	37	283	275
4	3	2	53	52	61	218	767
1	2	5	18	14	58	746	253
3	6	7	72	73	23	337	662
2	1	6	26	21	12	825	161
9	7	2	63	65	82	624	589
2	9	4	85	41	16	265	184
8	3	5	48	32	73	465	212

Find the sum of

22. $45 + $83 + $9187 + $725 + $68 + $9281 + $37 + $815 + $9.

23. $928 + $75 + $96 + $417 + $3263 + $2815 + $5 + $628 + $82715.

24. 575 lb. + 38 lb. + 7 lb. + 2196 lb. + 837 lb. + 6173 lb. + 25 lb.

25. 914 ft. + 475 ft. + 62 ft. + 427 ft. + 529 ft. + 6175 ft. + 8329 ft.

26. 8275 bbl. + 59 bbl. + 8173 bbl. + 493 bbl. + 8245 bbl. + 763 bbl.

27. 2173 bu. + 591 bu. + 6279 bu. + 87 bu. + 527 bu. + 6 bu.

28. 42 yd. + 324 yd. + 516 yd. + 7 yd. + 8219 yd. + 548 yd. + 413 yd.

29. $625 + $5218 + $62 + $54 + $839 + $6275 + $418 + $6128 + $95.

30. $23781 + $5183 + $192834 + $1625 + $93 + $82713 + $61825 + $41825 + $371862 + $68291 + $428 + $8291 + $67325 + $92 + $8293.

31. A merchant's sales were as follows: Monday, $392; Tuesday, $416; Wednesday, $285; Thursday, $347; Friday, $485; and Saturday, $1283. What were his total sales for the week?

32. A man bought a vessel for $9365, paid $673 for repairs, $218 for painting, and then sold it at a profit of $842. What did he receive for the vessel?

33. I sold a house for $5183 and lost $1570. How much would I have received if I had sold the house at a gain of $582?

34. In what year did a man die, if he was born in 1816 and died when he was 64 years old?

35. What is the sum of five thousand eight hundred and ninety-three miles, thirteen thousand two hundred and six miles, one thousand and seventy-eight miles, and twenty thousand and five miles?

Rem.—To rearrange addends in perpendicular columns involves unnecessary labor. Students should therefore be trained to add items horizontally, or in whatever other manner they may have been originally placed. The process is as follows: *First add all the right-hand orders of the several addends; then all the second orders from the right; then all the third orders, etc.; setting down and carrying as usual.*

Find the sum by horizontal addition:

36. \$513 + \$618 + \$475 + \$513.
37. \$918 + \$275 + \$292 + \$348.
38. \$418 + \$98 + \$6125 + \$417.
39. \$563 + \$175 + \$281 + \$9183.
40. \$2183 + \$79 + \$2938 + \$865.
41. \$592 + \$817 + \$284 + \$5137.
42. \$9 + \$218 + \$65 + \$3 + \$428.
43. \$51832 + \$61758 + \$419328.

In the following examples, add perpendicularly, then horizontally; and prove both by comparing the sums of their respective totals, which must agree.

44. POPULATION OF A CITY.

	FIRST WARD.	SECOND WARD.	THIRD WARD.	FOURTH WARD.	FIFTH WARD.	SIXTH WARD.	TOTALS.
Americans	2175	3251	2876	4531	3250	1760	*****
Germans	837	735	2531	1260	924	853	****
Irish	1235	1526	627	837	1537	526	****
French	125	271	182	206	193	175	****
Italians	376	582	163	293	185	116	****
Others	975	1236	927	2301	627	823	****
Totals	****	****	****	****	****	****	*****

45. GRAIN EXPORT OF A CITY FOR ONE WEEK (in bushels).

	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTALS.
Corn	28325	15236	35715	29128	75183	46217	******
Wheat	35719	41719	50108	32546	59275	81126	******
Oats	12136	9237	18265	7268	6950	17230	*****
Barley	18230	15738	21375	15928	19263	13637	******
Rye	5275	6829	7201	11325	7825	13261	*****
Totals ..	*****	*****	*****	*****	******	******	******

SUBTRACTION.

18. Subtraction is the process of taking one number from another number of the same kind.

REM. 1.—The number to be taken is called the *Subtrahend;* the number from which the subtrahend is taken is called the *Minuend;* and the number which is left after taking the subtrahend from the minuend is called the *Remainder* or *Difference.*

REM. 2.—The *Sign of Subtraction* is a short, horizontal line (—). It is read *minus,* and is placed between two numbers to indicate that the one on the right is to be subtracted from the one on the left.

19. The **Principles of Subtraction** are:

1. *Only like numbers and like orders can be subtracted, one from the other.*
2. *The remainder or difference expresses units of the same name as those expressed by the minuend and subtrahend.*
3. *The increase of any order in the subtrahend is equivalent to a corresponding decrease of the same order of the minuend.*
4. *The total difference of two numbers equals the differences of the several orders of the less number separately subtracted from similar orders of the greater number.*

ILLUSTRATIVE EXAMPLE.

20. Subtract $293 from $758.

SOLUTION.

$$\begin{array}{r} \$758 \\ 293 \\ \hline \$465 \end{array}$$

EXPLANATION.—Commencing at the right, subtract 3 units from 8 units, leaving 5 units to be placed below the line in the column of units.

Since 9 tens cannot be subtracted from 5 tens, borrow 1 hundred from 7 hundreds, and add the borrowed hundred (= 10 tens) to 5 tens, making 15 tens; then subtract 9 tens from 15 tens, leaving 6 tens to be written below the line.

Next subtract 2 hundred from 6 hundred (that is, from 7 hundred less the borrowed hundred), leaving 4 hundred.

REM.—The two preceding subtractions from 7 hundred, first of the borrowed hundred and then of the 2 hundred in the subtrahend, could have been consolidated into one subtraction of 2 plus the borrowed unit, or 3 hundred from 7 hundred, obtaining the same result, 4 hundred.

RULE.—I. *Place the subtrahend under the minuend so that units of the same order shall fall in the same column.*

II. *Begin at the right and subtract each order of the subtrahend from the like order of the minuend, writing the result under the order subtracted.*

III. *If any order of the subtrahend be greater than the like order of the minuend, add 10 to that order of the minuend and then subtract.*

IV. *If 10 are added to any order of the minuend, 1 should be mentally added to the next higher order of the subtrahend.*

21. PROOF OF SUBTRACTION.—*Add the remainder to the subtrahend; the result should equal the minuend. Or, subtract the remainder from the minuend; the result should equal the subtrahend.*

EXAMPLES FOR PRACTICE.

Find the difference between

1. $8271 and $5327.
2. $12587 and $6273.
3. $893528 and $691735.
4. 7241 bu. and 1589 bu.
5. 853912 ft. and 27289 ft.
6. $327832 and $98265.
7. $8129173 and $3281769.
8. 27183296 lb. and 18291563 lb.
9. $750000 and $293486.
10. $925876 and $260000.
11. $5003018 and $2910807.
12. $6750000 and $5002807.
13. $70200508 and $41083026.
14. $32807050 and $20608098.
15. $862700405 and $305086002.
16. $100100100 and $90090090.

22. Find the difference between the following terms as they stand. In business, this is frequently done to avoid a rearrangement of the numbers:

(17)	(18)	(19)	(20)	(21)
$28375	$52176	$8273625	$527183	$10908005
76257	91027	9859487	783065	71060280

23. Students should also be required to subtract horizontally, or in whatever other manner the subtrahend and minuend may have been originally written. First, subtract the right-hand figure of the subtrahend from the right-hand figure of the minuend, placing a check-mark over each at the time it is considered. Proceed in a similar manner with the second figures from the right, with the third figures, etc. The object of the check-mark is to show quickly the next figures to be considered. Employing this method, arrange the following numbers horizontally and find the difference between them:

22. $6825 — $3178.
23. $82917 — $69275.
24. $582938 — $168271.
25. $7208013 — $5060807.
26. $8713009 — $6090080.
27. $192083 — $5875.

28. A merchant's total deposits in a bank were $75813, and his withdrawals $63298. How much had he remaining in the bank?

29. A man bought a farm for $18275 and afterwards sold it for $29360. What was his gain?

30. I sold a steamboat for $43500, which was $2980 more than it cost. What did I pay for the steamboat?

31. If a man was born in 1827 and died in 1889, how old was he at the time of his death?

32. A man possesses property to the amount of $182716, and owes debts amounting to $95865. What is his true worth?

MULTIPLICATION.

24. Multiplication is a short process of finding the sum *when the several addends are equal.*

REM. 1.—The *Multiplicand* is the addend to be repeated; the *Multiplier* denotes how many repetitions of the addend are required; and the *Product* is the result obtained by repeating the multiplicand as many times as there are units in the multiplier.

REM. 2.—The multiplicand and multiplier are called *Factors* of the product.

REM. 3.—The *Sign of Multiplication* is an oblique cross (×). It is read *multiplied by*, or *times;* and is placed between numbers to indicate that they are to be multiplied.

25. The **Principles of Multiplication** are:

1. *The multiplicand in business practice is a concrete number.*

2. *The multiplier, though usually a concrete number, is always regarded abstractly as denoting* **times.**

3. *The product expresses units of the same name as the multiplicand.*

4. *The total product equals the sum of the products of the several orders of the multiplicand separately multiplied by each order of the multiplier.*

26. MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**

NOTE.—It will not be a waste of time to memorize the whole of the above table, especially if the learner is preparing himself for a mercantile life.

27. Multiply $483 by 7.

SOLUTION.
$483
7
$3381

EXPLANATION.—Separately take 7 times each of the orders of $483, commencing with the lowest. Thus, 7 times 3 units equal 21 units, or 2 tens and 1 unit. Write 1 and carry 2 tens to 7 times 8 tens, obtaining 58 tens, or 5 hundreds and 8 tens. Write 8 tens, and carry 5 hundreds to 7 times 4 hundreds, obtaining 33 hundreds.

EXAMPLES FOR PRACTICE.

Multiply
1. 41875 lb. by 6.
2. 37284 bu. by 4.
3. 9182765 yr. by 2.
4. 25318276 yd. by 5.
5. 627983624 gal. by 3.
6. 578263413 lb. by 7.

Multiply
7. 3291827168 gal. by 9.
8. $401783028102 by 11.
9. $270800630582 by 8.
10. $607008300209 by 12.
11. $281070068035 by 10.
12. $5107000908302 by 9.

ILLUSTRATIVE EXAMPLE.

28. Multiply $4827 by 73.

SOLUTION.

$$\begin{array}{r} \$4827 \\ 73 \\ \hline 14481 \\ 33789 \\ \hline \$352371 \end{array}$$

EXPLANATION.—First multiply $4827 by the right-hand order of the multiplier (3 units) as shown in **27**, obtaining $14481 as a partial product.

Then similarly multiply $4827 by the second order of the multiplier (7 tens), obtaining 33789 tens of dollars. Arrange the second partial product (33789 tens) so that its right-hand figure (9 tens) shall fall under the same order (8 tens) of the first partial product (Prin. 1, **15**).

Add the partial products, obtaining $352371 as the complete product.

When one of the factors consists of only one figure.

RULE.—I. *Place the factor having one figure under the right-hand order of the remaining factor.*

II. *Commence with the lowest order of the upper factor, and multiply it and each of its successive higher orders by the lower factor, setting down and carrying as in addition.*

When both factors consist of more than one figure.

RULE.—I. *Under the factor which has the greater number of figures, write the remaining factor so that units shall fall under units, tens under tens, etc.*

II. *Applying the preceding Rule, successively multiply the upper factor by each order of the lower factor, as if that order stood alone.*

III. *Place the first figure of each partial product under that order of the lower factor which was used to produce it.*

IV. *Add the several partial products to find the complete product.*

29. PROOF OF MULTIPLICATION.—*Reverse the position of the factors, and multiply. If the same product is obtained, the result may be presumed to be correct.*

EXAMPLES FOR PRACTICE.

Multiply
1. 82719 yd. by 42.
2. 571832 yr. by 68.
3. 19287536 lb. by 73.
4. 29827826 ft. by 59.
5. 425163827 oz. by 326.
6. 98731428 gal. by 587.
7. 659284372 yd. by 649.
8. 152783265 ft. by 813.

Multiply
9. $78359281 by 8273.
10. $42678928 by 5496.
11. $7812634 by 82732.
12. $9281763 by 50803.
13. $6829178 by 90027.
14. $5207008 by 604005.
15. $700208009 by 50304.
16. $407006004 by 70008.

17. What is the total weight of 5378 bales of cotton, which average 397 pounds per bale?

18. How many pounds in 472 barrels of flour, allowing 196 pounds to each barrel?

19. What is the cost of 293 barrels of apples at $2 per barrel?

20. If a man buy 285 acres of land at $34 per acre, and sell his purchase for $12732, what will be his gain?

21. An orchard contains 38 rows and each row contains 53 peach trees. How many peach trees are in the orchard?

30. **A Continued Product** is one which is produced by the multiplication of *more than two factors.*

ILLUSTRATIVE EXAMPLE.

31. What is the cost of 56 horses at $235 each?

SOLUTION.

56 = 8 × 7.

$235
8
―――
$1880
7
―――
$13160

EXPLANATION.—One of the factors (56) is separable into two other factors (8 × 7). Multiplying $235 by one of these factors (8) will produce $1880 as the cost of 8 horses, and multiplying this result ($1880) by the remaining factor (7) will produce $13160 as the cost of 7 times 8, or 56 horses.

EXAMPLES FOR PRACTICE.

In like manner, multiply

1. 7389 men by 72.
2. 6537 gal. by 48.
3. 59832 yr. by 96.
4. 283572 yd. by 54.
5. 4789623 lb. by 63.
6. $92783165 by 84.
7. $26832791 by 144.
8. $583329726 by 49.
9. $80170038 by 32.
10. $62918 by 315 (5 × 7 × 9).

ILLUSTRATIVE EXAMPLES.

32. Multiply $418 by 1000.

SOLUTION.

$418 × 1000 = $418000.

EXPLANATION.—Simply place at the right of $418 as many ciphers as are found in 1000.

33. Multiply $42800 by 53000.

SOLUTION.

$$\begin{array}{r} \$42800 \\ 53000 \\ \hline 1284 \\ 2140 \\ \hline \$2268400000 \end{array}$$

EXPLANATION.—$42800 = $428 × 100 (**32**); and 53000 = 53 × 1000. Applying **31**, first multiply $428 by 53, obtaining $22684; then multiply this result by 100 × 1000, by annexing *two* plus *three*, or *five* ciphers.

When the multiplier is 10, 100, 1000, 10000, etc.

RULE.—*At the right of the multiplicand place as many ciphers as are found in the multiplier.*

When either or both factors terminate in ciphers.

RULE.—I. *Arrange the factors so that the right-hand significant figure of the lower factor shall fall under the right-hand significant figure of the upper factor.*

II. *Multiply as if there were no ciphers at the right of either factor, and to the product thus obtained annex as many ciphers as are found at the right of both factors.*

EXAMPLES FOR PRACTICE.

1. 897 yr. by 1000.
2. 6285 gal. by 100.
3. 13721 bu. by 10000.
4. 7983 ft. by 800.
5. 25984 yd. by 7000.
6. 483927 lb. by 3200.
7. $6273849 by 875000.
8. $6250000 by 2900.
9. $183600 by 4270000.
10. $630000 by 380.

DIVISION.

34. Division is the process of finding one factor (Rem. 2, **24**) when the product and the remaining factor are given.

REM. 1.—The given product is called the *dividend;* the given factor, the *divisor;* and the required factor, the *quotient.*

REM. 2.—If the divisor or given factor is abstract, the quotient or required factor will be concrete, and its units will be of the same name as those of the dividend. If, however, the divisor is concrete, it should express units of the same name as the dividend; and the quotient will be abstract, simply denoting *times.*

REM. 3.—After the integral part of the required factor has been found, whatever is left of the dividend is called the *remainder.*

REM. 4.—Division is usually indicated by the sign (÷). It is read *divided by,* and is written after the dividend and before the divisor.

35. GENERAL PRINCIPLES GOVERNING DIVISION.

1. *Any change of the dividend, by multiplication or division, produces a like change of the quotient.*

2. *Any change of the divisor, by multiplication or division, produces an opposite change of the quotient.*

3. *A like change of both divisor and dividend, by multiplication or division, produces no change of the quotient.*

36. To divide when the divisor is 12 or less.

ILLUSTRATIVE EXAMPLE.

Divide $973 by 7.

SOLUTION.
Divisor. Dividend.
7) $973
$139 Quotient.

EXPLANATION.—9 hundreds ÷ 7 = 1 hundred and 2 hundreds over. Write this partial quotient (1) under the order divided, carry the remainder (2 hundreds or 20 tens) to the next order of the dividend (7 tens), and divide the result (27 tens) by 7, obtaining 3 tens as the next quotient figure and 6 tens as the next remainder. Carry the last remainder (6 tens or 60 units) to the next order of the dividend (3 units) and divide the result (63 units) by 7, obtaining 9 units as the last quotient figure.

RULE.—I. *From the left of the dividend, select the least number of orders that will contain the divisor. Find how many times the divisor is contained in these orders and write the result as the first quotient figure.*

II. *If the division is not exact, conceive the remainder to be placed before the next right-hand order of the dividend, find how many times the divisor is contained in the number thus formed, and write the result as the second quotient figure. Proceed in a similar manner until the entire dividend has been divided.*

37. PROOF OF DIVISION.—*Multiply the quotient by the divisor, and to the product add the remainder if any. The result should equal the dividend.*

EXAMPLES FOR PRACTICE.

Divide	Divide
1. 6783219 gal. by 3.	9. $62000150800 by 10.
2. 58297385 lb. by 5.	10. $4182700836 by 12.
3. 71829534 yd. by 2.	11. $6958293727 by 11.
4. 49273868 bu. by 4.	12. $80739100025 by 5.
5. 86291436 yr. by 7.	13. $42600308040 by 8.
6. 13729581 gal. by 9.	14. $1070820048 by 12.
7. 46273824 oz. by 6.	15. $5200000002 by 9.
8. 318293712 ft. by 8.	16. $620800100501 by 7.

17. A manufacturer sold a quantity of broadcloth at $5 per yard, and obtained $7160. How many yards did he sell ?

18. At $3 each, how many books can be bought for $1254 ?

19. A speculator invested $73096 in Western land at $8 per acre. How many acres did he purchase ?

38. Short Division is the method of obtaining the successive quotient figures by *mental processes,* as in the Ill. Ex., **36**. It should always be preferred when the divisor is not greater than 12.

39. Long Division is the method of obtaining the successive quotient figures by written processes. It is usually employed when the divisor exceeds 12.

40. To divide when the divisor exceeds 12.

ILLUSTRATIVE EXAMPLE.

Divide \$62938 by 73.

SOLUTION.

Divisor. Dividend. Quotient.

73) \$62938 (\$862$\frac{12}{73}$

$$\begin{array}{r} 584 \\ \hline 453 \\ 438 \\ \hline 158 \\ 146 \\ \hline 12 \end{array}$$ Remainder.

EXPLANATION. — 629 hundreds divided by 73 equals 8 hundreds. Write 8 as the first quotient figure; multiply the divisor (73) by this quotient figure; place the product (584 hundreds) under the orders divided; and subtract, obtaining 45 hundreds as a remainder.

To this remainder annex the succeeding order of the dividend (3 tens), and divide the result (453 tens) by the divisor (73), obtaining 6 tens as the second quotient figure. Multiply the divisor (73) by the second quotient figure (6 tens); write the result (438 tens) under 453 tens, and subtract, obtaining 15 tens as a remainder.

To this remainder bring down the next order of the dividend (8 units) and divide the result (158 units) by the divisor (73), obtaining 2 units as the next quotient figure. Multiply and subtract as before; under the final remainder (12) write the divisor (73), and annex the result ($\frac{12}{73}$) to the quotient.

RULE.—I. *Find the first quotient figure as directed in Rule for short division,* **36.**

II. *Multiply the divisor by the first quotient figure; write the product under the first partial dividend; subtract; and to the right of the remainder place the next undivided order of the dividend for a second partial dividend.*

III. *Divide the second partial dividend by the divisor, and write the result as the second quotient figure. Multiply, subtract, and bring down, as with the first partial dividend; and continue in this manner until all the orders of the dividend have been divided.*

IV. *If any partial dividend is less than the divisor, place a cipher in the quotient, and annex the next undivided order of the dividend for a new partial dividend.*

NOTE 1.—If any product be greater than the partial dividend, the quotient figure is too large.

NOTE 2.—If any remainder be equal to or greater than the divisor, the quotient figure is too small.

NOTE 3.—A convenient method of finding the quotient figure is to point off from the right of any partial dividend as many figures lacking 1 as there are

40

figures in the divisor, and divide the remaining figures of the partial dividend by the left-hand figure of the divisor (if the following figure of the divisor is less than 5) or by the left-hand figure of the divisor increased by 1 (if the following figure is 5 or more).

EXAMPLES FOR PRACTICE.

Divide

1. 75829134 yd. by 52 yd.
2. 981736289 bu. by 31.
3. 527821965 bbl. by 93 bbl.
4. 718312593 yr. by 42.
5. 19278193 yr. by 94 yr.
6. 57389247 gal. by 28.
7. 61827534 bu. by 69.
8. 28173081 lb. by 86 lb.

Divide

9. 872916432 miles by 528.
10. 521071835 gal. by 385.
11. 2753862148 oz. by 732 oz.
12. $807535148 by 3152.
13. $5718230248 by 4926.
14. $645342319 by $8276.
15. $2159382473 by 58234.
16. $415273684 by $276325.

17. An orchard contains 22144 trees planted in 128 equal rows. How many trees are in each row?

18. A plantation of 946 acres was sold for $54868. At what price per acre was it valued?

19. A drover invested $19454 in buying 137 horses. What was the average price per horse?

20. If the product of two factors is 143186076 inches, and one of the factors is 32871 inches, what is the other factor?

41. To divide by the factors of a divisor.

ILLUSTRATIVE EXAMPLE.

Divide $7695 by 168.

SOLUTION.

$$168 = 8\times 7\times 3$$

$$\begin{array}{lr} 8\,)\,\underline{\$7695} & \\ \quad 7\,)\,\underline{961} + 7 = & 7 \\ \quad\quad 3\,)\,\underline{137} + 2\times 8 = & 16 \\ \quad\quad\quad \$45 + 2\times 8\times 7 = & \underline{112} \\ & 135 \end{array}$$

Quotient, $\$45\frac{135}{168}$.

EXPLANATION.—Dividing $7695 by 8 (or one twenty-first of the true divisor, 168) produces $961 (or 21 times the true quotient, Prin. 2, **35**) and a remainder of $7. Since this remainder is left from the true dividend, it must be a part of the true remainder.

Dividing $961 (one twenty-first of the true quotient) by the second factor (7) produces $137, which must be one-seventh of 21, or 3 times the true quotient, and a remainder of $2. Since $2 is left from dividing one-eighth of the true dividend, this remainder must be one-eighth of the true remainder; hence, $2 × 8 = $16, the second part of the true remainder.

Dividing $137 (one-third of the true quotient) by the remaining factor (3) produces $45, the true quotient, and a remainder of $2. Since $2 is the remainder from dividing $137 (one-eighth of one-seventh of the true dividend), this remainder must be one-eighth of one-seventh of the true remainder; hence, $2 × 8 × 7 = $112, the third part of the true remainder.

Add the several parts of the true remainder, obtaining $135 as the total true remainder.

RULE.—I. *Separate the divisor into convenient factors.*

II. *Divide the dividend by one of these factors; divide the resulting quotient by one of the remaining factors; and so continue until each factor has been used as a divisor. The final quotient will be the required quotient.*

III. *Multiply each remainder by all the divisors that precede its own divisor; and add these products to the remainder, if any, of the first division, to find the true remainder.*

EXAMPLES FOR PRACTICE.

Find the quotient and remainder by the preceding rule.

1. 7128934 lb. by 56.
2. 582917831 ft. by 81 ft.
3. 6271382937 yd. by 35.
4. 817981362 bu. by 63 bu.
5. $9821763405 by $72.
6. $219534876 by 36.
7. $5827139826 by 48.
8. $829173 by $125 ($5 × 5 × 5).
9. $658397 by $336 ($8 × 6 × 7).
10. $968574 by 224 (8 × 7 × 4).

42. To divide by 10, 100, 1000, 10000, etc.

ILLUSTRATIVE EXAMPLE.

Divide 41673 gallons by 100.

SOLUTION.

1 | 00) 416 | 73

416 quo., 73 rem.

$416\frac{73}{100}$ gallons, *Ans.*

EXPLANATION.—Cut off the two right-hand figures (73) of the dividend, which is equivalent to moving the remaining figures of the dividend two places to the right, thus decreasing the dividend one hundredfold (Rem., **6**).

The figures cut off (73), being the undivided part of the true dividend, must be the true remainder.

RULE.—*Cut off from the right of the dividend as many figures as there are ciphers in the divisor. The remaining* *figures will express the quotient, and the figures cut off will express the remainder.*

EXAMPLES FOR PRACTICE.

Divide

1. 91872683 yd. by 1000.
2. 51791836 lb. by 100 lb.
3. 82967283 bu. by 10000.

Divide

4. 43579386 yr. by 100000.
5. $95843765 by $1000000.
6. 27836291 ft. by 1000 ft.

43. When any divisor has ciphers on the right.

ILLUSTRATIVE EXAMPLE.

Divide $37268 by 2300.

SOLUTION.

23 | 00) $372 | 68 ($16, Quo.
23
—
142
138
—
$468, Rem.

EXPLANATION.—The divisor (2300) equals 100 × 23. Applying **41**, divide by one of these factors (100) as shown in **42**, obtaining 372 as one twenty-third of the true quotient, and $68 as part of the true remainder.

Divide $372 by 23, obtaining the true quotient ($16) and a second remainder of $4, which is one-hundredth of the true remainder. Multiply $4 by 100, and add 68 to the product, to find the total true remainder.

RULE.—I. *Cut off the ciphers at the right of the divisor and an equal number of figures at the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor to find the true quotient.*

III. *Prefix the remainder, if any, to the figures cut off from the dividend, to obtain the true remainder.*

EXAMPLES FOR PRACTICE.

Divide

1. 7182365 lb. by 4000.
2. 5382976 yr. by 600 yr.
3. 682847 yd. by 8000.
4. 2592768 bu. by 3700 bu.
5. 687195 gal. by 53000.
6. 4928136 ft. by 67000 ft.

Divide

7. $48267391 by 176000.
8. $59483726 by 31900.
9. $172836492 by $5173000.
10. $6580000 by 239000.
11. $27180000 by 15100.
12. $82935100 by $249000.

UNITED STATES MONEY.

44. In **United States Money,** 10 mills equal 1 cent; 10 cents equal 1 dime; and 10 dimes or 100 cents equal 1 dollar.

REM. 1.—The *Sign* of U. S. money is $.

REM. 2.—A point (.) is used to distinguish the several denominations of U. S. money; all the figures on the left of the point expressing dollars; the first figure at the right of the point, *tens of cents* or dimes; the second figure at the right of the point, *units* of cents (less than ten cents); and the third figure at the right of the point denotes mills.

REM. 3.—If the cents are less than 10, a cipher should be written before the cents and after the point, to denote the absence of tens of cents or dimes.

REM. 4.—U. S. money is usually expressed in two denominations, *dollars* and *cents*, the mills being omitted if less than 5, and considered an additional cent if 5 or more.

Write
1. 95 dollars and 18 cents.
2. 216 dollars and 65 cents.
3. 809 dollars and 40 cents.
4. 59 dollars and 7 cents.
5. 419 dollars and 4 cents.

Write
6. 5219 dollars and 43 cents.
7. 7814 dollars and 5 cents.
8. 920 dollars and 85 cents.
9. 2473 dollars and 3 cents.
10. 6825 dollars and 30 cents.
11. 5783 dollars, 19 cents, and 8 mills.
12. 2596 dollars, 8 cents, and 3 mills.

Read
13. $.08.
14. $.38.
15. $.04.
16. $28.36.
17. $815.09.
18. $7268.35.
19. $5837.65.
20. $9164.53.
21. $1927.05.
22. $613.583.
23. $4285.062.
24. $7300.906.

45. Since there are 1000 mills or 100 cents in 1 dollar, dollars may be changed to mills by annexing 3 ciphers (**32**); and to cents by annexing 2 ciphers. Applying **42,** the lower denominations may be changed to dollars by pointing off 2 figures from the right of the cents, or 3 figures from the right of the mills.

Change to cents.

1. $735.
2. $69.
3. $2138.
4. $5175.
5. $163.
6. $2500.

Change to dollars.

7. 3589 cents.
8. 62583 cents.
9. 985 cents.
10. 41783 cents.
11. 5275 mills.
12. 38279 mills.

46. Addition of United States money.

ILLUSTRATIVE EXAMPLE.

Add $419, $376.853, $7186.23, and $685.529.

SOLUTION.

$419.
376.853
7186.23
685.529
$8667.612

EXPLANATION.—Only similar units can be added (Prin. 1, **15**). Therefore arrange the several addends so that mills shall fall under mills, cents under cents, units of dollars under units of dollars, etc.

Add as in ordinary addition: The sum of the dimes' orders being 16 dimes (Prin. 2 and 3, **15**), or 1 dollar and 6 dimes, place a point to the left of the 6 dimes to denote the denomination of its units (Rem. 2, **44**).

RULE.—I. *Arrange the addends so that their respective points shall fall in a perpendicular column.*

II. *Add and carry as in ordinary addition.*

III. *Place a point in the sum directly under those of the addends.*

EXAMPLES FOR PRACTICE.

1. Add $735.60, $5189.25, $819, $23.70, and $298.45.
2. Add $8195.35, $625.82, $345.83, and $2185.
3. Add $49.50, $68.758, $317.286, $5.41, and $81.374.
4. Add $425, $683.35, $29.415, $8, $.73, and $69.40.
5. Add 18 dollars and 32 cents, 75 dollars and 8 cents, 419 dollars and 5 cents, 87 dollars and 30 cents, and 36 dollars, 14 cents, and 3 mills.

47. Subtraction of United States money.

ILLUSTRATIVE EXAMPLE.

Subtract 25 dollars and 6 cents from 83 dollars and 35 cents.

SOLUTION.

$83.35
25.06
$58.29

EXPLANATION.—Only similar units can be subtracted (Prin. 1, **19**). Hence, arrange the terms so that the point in the subtrahend shall fall under the point in the minuend, causing similar orders to fall in the same column.

RULE.—I. *Arrange the subtrahend so that its point shall fall under the point in the minuend.*

II. *Subtract, borrow, and carry as in ordinary subtraction.*

III. *Place a point in the remainder directly under those of the minuend and subtrahend.*

EXAMPLES FOR PRACTICE.

Subtract

1. $413.45 from $709.82.
2. $6128.13 from $8475.48.
3. $519 from $617.30.
4. $249.85 from $325.

Subtract

5. $18 and 4 cents from $25.
6. 19 cents from $3 and 8 cents.
7. $49 from $316 and 75 cents.
8. $318 and 2 cents from $475.

48. Multiplication of United States money.

ILLUSTRATIVE EXAMPLE.

Multiply 582 dollars and 8 cents by 67.

SOLUTION.

$582.08
67
4074 56
34924 8
$38999.36

EXPLANATION.—Since the multiplicand expresses 58208 cents, 67 times the multiplicand, or 3899936, must also express cents (Prin. 3, **25**). Therefore, point off two figures from the right of the product to make it express dollars (**45**).

RULE.—I. *Arrange the factors and multiply as in ordinary multiplication.*

II. *Point off from the right of the product as many figures as are pointed off in the multiplicand.*

EXAMPLES FOR PRACTICE.

Multiply

1. $5173 by 37.
2. $7385.15 by 148.
3. $592.45 by 62.
4. $4823.32 by 247.
5. $91.75 by 87.

Multiply

6. $419 and 6 cents by 31.
7. $57, 18 cts., and 3 mills by 43.
8. $91, 4 cts., and 5 mills by 89.
9. $4183 and 70 cents by 418.
10. $973.09 by 526.

49. Division of U. S. money by an abstract divisor.

ILLUSTRATIVE EXAMPLE.

Divide $425.79 by 83.

SOLUTION.

83) $425.79 ($5.13
415
107
83
249
249

Explanation.—Since the dividend expresses 42579 cents, one eighty-third of the dividend or the quotient (513) must also express cents (Rem. 2, **34**). Hence, point off two figures from the right of the quotient to make it express dollars (**45**).

Rule.—*Divide as in ordinary division. If the dividend contains cents, point off two figures from the right of the quotient; and if the dividend contains mills, point off three figures.*

Note 1.—If the dividend expresses dollars only, and is not exactly divisible by the divisor, annex two ciphers to the dividend to make it express cents and continue the division until these ciphers have been brought down.

Note 2.—It is not customary to carry the quotient beyond the cents' order. If the final remainder is one-half the divisor, or more than one-half, add 1 to the cents' order of the quotient; if less than one-half the divisor, omit it.

EXAMPLES FOR PRACTICE.

Divide
1. $2280.46 by 7.
2. $977.04 by 12.
3. $1767.22 by 91.
4. $72471 by 147.

Divide
5. $9352 by 32 (Note 1).
6. $5183 by 67 (Note 2).
7. $817.52 by 513.
8. $5128.35 by 271.

50. Division of U. S. money by U. S. money.

ILLUSTRATIVE EXAMPLES.

Divide $1323 by $4.50.

SOLUTION.

$4.50) $1323.00 (294

Explanation.—Since the divisor ($4.50) expresses cents, annex two ciphers to the dividend that it may also express cents (Rem. 2, **34**). Divide as in ordinary division, finding that 450 cents are contained in 132300 cents, 294 times.

RULE.—*If necessary, make both divisor and dividend express the lowest unit contained in either. Divide as in ordinary division.*

EXAMPLES FOR PRACTICE.

Divide

1. $1635.90 by $.05.
2. $2358.23 by $4.13.
3. $39570.50 by $682.25.
4. $52808 by $5.74.
5. $72275 by $1.75.

Divide

6. $9875 by 5 cents.
7. $111 by $1.48.
8. $18991.15 by $283.45.
9. $108 by 25 cents.
10. $406602 by 9 cents.

RELATION OF INTEGERS.

51. All arithmetical operations involve either an increase or decrease of some given number. The law of increase and decrease of integral numbers is very simple in character and uniform in application. By understanding this general law, the learner will be practically independent of the hundreds of topical rules and special analyses upon which he would otherwise be compelled to rely.

52. A given integer is increased by considering it a component and augmenting it by one or more other components; and is decreased by considering it the sum of two or more components and taking away one or more of those components. If the increase is by unequal components, the process requires addition; if by equal components, multiplication. If the decrease is by unequal components, the process requires subtraction; and if by equal components, division.

53. Increase or decrease by unequal components.

ILLUSTRATIVE EXAMPLES.

1. A has $75 and B $50. How much have both?
2. A and B have $125. If A has $75, how much has B?

SOLUTION.

$$\begin{array}{r} \text{Components of} \\ \text{given minuend.} \end{array} \left\{ \begin{array}{lcr} \text{subtrahend, if given} & = & \$75 \\ \text{remainder, if required} & = & \underline{50} \end{array} \right. \begin{array}{l} = \text{unequal addend} \\ = \text{unequal addend} \end{array} \left\} \begin{array}{l} \text{Components of} \\ \text{required sum.} \end{array} \right.$$

$$\text{Minuend, if given} = \$125 = \text{total of unequal addends} = \text{sum, if required.}$$

Deduction.—The total of two or more unequal addends is the ***sum if required***, or the ***minuend if given***; and the components of a required sum or given minuend are the addends of which it is composed. Hence, add the components to find a required total; and subtract the given component (or the sum of the given components, if more than one) from a given total to find the remaining component.

Definitions.

1. A *Component* is the weight, measure or value of *one* considered unequal unit or quantity.

2. A *Total* is the weight, measure or value of *all* the considered unequal units or quantities.

Formulas.

1. Given component (addend) + given component (addend) = required total (sum).

2. Given total (minuend) — given component (subtrahend) = required component (remainder).

54. Increase or decrease by equal components.

ILLUSTRATIVE EXAMPLES.

1. Find the total weight of 35 bu. wheat, allowing 60 lb. per bu.
2. If 35 bu. of wheat weigh 2100 lb., what is the weight of 1 bu.?
3. How many bushels in 2100 lb. wheat, allowing 60 lb. per bu.?

SOLUTION.

Factors of given dividend.	concrete divisor =	60 lb.	= weight of 1 equal addend = multiplicand	Factors of required product.
	abstract quotient =	35	= number of equal addends = multiplier	
	Dividend, if given =	2100 lb.	= total of equal addends = product, if required.	

Deduction.—The total of two or more equal components is the *product if required*, or the *dividend if given*. *One factor* of a required product or given dividend is the weight, measure, value, etc., of *one equal component;* and the *other factor* is the *number* of such equal components. Hence, if a product is required, multiply its two given factors; and if a product is given, divide it by its given factor, and the quotient will be its remaining factor.

Definitions.

1. The *Concrete Factor* is the weight, measure or value of *one* considered equal unit or quantity (Prin. 1, **25**).

2. The *Abstract Factor* is the *number* of considered equal units or quantities (Prin. 2, **25**).

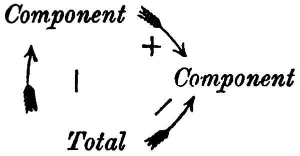
3. The *Product* is the weight, measure or value of *all* the considered units or quantities.

FORMULAS.

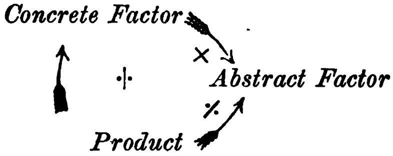
1. Concrete factor × abstract factor = required product.
2. Given product ÷ concrete factor = abstract factor.
3. Given product ÷ abstract factor = concrete factor.

55. The two separate formulas for unequal components, and the three for equal components may, for the ease with which they can be remembered and applied, be simplified into the following general formulas:

GENERAL FORMULA I.



GENERAL FORMULA II.



The sign of operation between any two of the above terms in either general formula indicates the process to be employed, if they are given, in obtaining the remaining term. Or, *vice versa,* if any one of the above terms is required, perform the operation indicated by the sign between the other two terms, which must then be given.

REVIEW EXAMPLES.

56. 1. A cyclopedia is divided into three volumes, the first containing 718 pages, the second 695 pages, and the third 748 pages. How many pages does the cyclopedia contain?

Component + component + component = total.

2. An army went into battle with 75830 men, and came out of the battle with 59275 men. How many men were lost?

Given total − given component = lost component.

3. Grover Cleveland was born in the year 1837, and inaugurated President of the United States when 48 years old. In what year was he inaugurated?

Earlier date (component) + interval (component) = later date (total).

4. What is the cost of 6275 pounds sugar at 8 cents per pound ?

Concrete factor × abstract factor = required product.

5. If 85 sheep cost $267.75, at what price per head were they bought ?

Given product ÷ given abstract factor = required concrete factor.

6. I bought 18 yards of muslin at 8 cents per yard, 25 yards alpaca at 45 cents per yard, 13 yards sheeting at 30 cents per yard, 7 yards of cloth at $2.15 per yard, and in payment gave the clerk 2 twenty-dollar notes. How much change should he return ?

7. A man bought 113 acres of land for $3955.85, sold 74 acres at $38 per acre, and the remainder at $42 per acre. What was his total gain ?

8. I deposited in bank at various times the following sums: $715.80, $498.30, $518.25, $68.48, and $1918.87. After making the last deposit, I found there was a balance to my credit of $1395.15. How much had I withdrawn ?

9. A speculator bought 375 acres of land at $18.75 per acre, expended $1280 in improvements, and then sold it at $23.25 per acre. What was his gain ?

10. How much must be paid for 183960 pounds of corn bought at 63 cents per bushel, allowing 56 pounds to the bushel ?

11. If a man buy 230 acres of land for $6210, sells 50 acres at $29 per acre, 76 acres at $28.50 per acre, and 41 acres at $31.25 per acre, at what price per acre must he sell the remainder to make a total profit of $1349 ?

12. A merchant's sales were as follows: Monday, $275.80; Tuesday, $316.62; Wednesday, $248.90; Thursday, $305.50; Friday, $237.40; and Saturday, $462.70. What were his average sales per day ?

To average two or more unequal components means simply to change them to the same number of equal components. Hence, product (total sales) ÷ abstract factor (No. of days) = concrete factor (average or equal sales per day).

13. If there are 32 quarts in a bushel, how much will be received for 3 bushels of chestnuts by retailing them at 8 cents per quart ?

14. A man bought a lot for $1867, paid $395 to have it graded, $83 to have it enclosed, and then sold it at a loss of $325. How much did he receive for the lot ?

15. I bought a farm for \$6737.50 and sold it for \$7218.75, gaining \$2.75 per acre. How many acres did the farm contain ?

16. A grocer mixed 350 pounds of Rio coffee costing 12 cents per pound with 175 pounds of Laguayra coffee costing 16 cents per pound. At what price per pound must he sell the mixture to gain \$8.75 ?

17. If 23 men can complete a contract in 38 days, in how many days can one man complete it ?

38 days' work of one man = concrete factor. 23 men = abstract factor. 38 × 23 = 874 days' work of one man performed by all the men = required product.

18. If one man can do a job of work in 234 days, in how many days can 13 men do the same job ?

19. If 30 men can perform a task in 48 days, in how many days can 40 men perform it ?

20. If a man was born in 1835 and died in 1889, how old was he at the time of his death ?

21. An army consisted of 5 divisions, each division of 6 brigades, each brigade of 4 regiments, each regiment of 10 companies, and each company of 87 men. How many men did the army contain ?

22. A man has \$2016.62, bought 370 bushels of wheat at \$1.12 per bushel, 980 bushels of corn at 74 cents per bushel, 536 bushels of rye at 82 cents per bushel, and invested the remainder of his money in flour at \$6.25 per barrel. How many barrels of flour did he buy ?

23. The temperature of a certain city averaged 75° Fahrenheit for the month of June, 88° for July, and 92° for August. What was the average temperature for the three months ?

24. A merchant had \$91.20 remaining after buying 2180 bushels of rye at 75 cents per bushel and 5237 bushels of wheat at \$1.05 per bushel. How many bushels could he have purchased if he had bought no rye and invested all his money in wheat ?

25. A man bought 3500 bushels of wheat and 2763 bushels of rye for \$5778.55. If the cost of the wheat was 98 cents per bushel, what was paid per bushel for the rye ?

26. A farmer sold to a merchant 15 pounds of butter at 28 cents per pound, 25 dozen eggs at 14 cents per dozen, 5 dozen chickens at \$2.20 per dozen, and in payment received 4 gallons of molasses at 55 cents per gallon, 9 pounds of coffee at 18 cents per pound, 1 bbl. of flour worth \$6.40, and the remainder in sugar at 8 cents per pound. How many pounds of sugar did the farmer receive ?

FACTORING.

57. The **Factors** of a number are such integers as will produce it when multiplied together.

Thus, \$3 and 5 are factors of \$15; \$2, 3 and 7 are factors of \$42.

58. A **Prime Number** is one which has no integral factors except itself and 1.

59. A **Composite Number** is one which has *other* integral factors than itself and 1.

Thus, \$33 is a composite number, because \$3 × 11 = \$33; but 13 is a prime number because its only factors are \$1 × 13.

60. A **Prime Factor** of a number is any prime number taken as a factor.

Thus, \$3 × 5 are prime factors of \$15, since each of these factors is a prime number. \$4 × 9 are composite factors of \$36, since each of these factors is a composite number.

61. An **Exact Divisor** of a number is one which will divide it and leave no remainder.

Rem.—All the prime or composite factors of a number are exact divisors of that number. Therefore, any prime number that is an exact divisor of a given number must also be a prime factor of that number.

62. One number is said to be *divisible* by another when it contains that other an *exact* number of times. Any number is divisible by itself and 1; and it will be divisible—

1. By 2, if it is an *even* number.
2. By 3, if the sum of all its figures is divisible by 3.

Thus, 285 is divisible by 3 because 2 + 8 + 5, or 15, is so divisible.

3. By 4, if its two right-hand figures are either ciphers or express a number which is divisible by 4.

Thus, 4 is an exact divisor of 900 because its two right-hand figures are ciphers; and of 1756 because the number expressed by its two right-hand figures (56) is divisible by 4.

4. By 5, if its right-hand figure is either 5 or 0.

5. By 6, if it is an even number and the sum of its figures is divisible by 3.

Thus, 6 is an exact divisor of 348, because the latter is an even number, and the sum of its orders (3+4+8 = 15) is divisible by 3.

6. By 8, if its three right-hand figures are either ciphers or express a number which is divisible by 8.

Thus, 8 is an exact divisor of 5000 because its three right-hand figures are ciphers; and of 62176 because the number expressed by its three right-hand figures (176) is divisible by 8.

7. By 9, if the sum of its figures is divisible by 9.

Thus, 56178 is divisible by 9 because 5+6+1+7+8, or 27, is so divisible.

8. By 10, if its right-hand figure is a cipher.

9. By 12, if the sum of its figures is divisible by 3, and the number expressed by its *two* right-hand figures is divisible by 4.

63. To find the prime factors of a composite number.

ILLUSTRATIVE EXAMPLE.

What are the prime factors of 420 yards?

SOLUTION.

2) 420 yd.
2) 210 "
3) 105 "
5) 35 "
7 "

EXPLANATION.—Since 420 is an even number, divide it by 2, a prime number (1, **62**), obtaining 210. Since 210 is another even number, divide it by 2, obtaining 105. Since the sum of the figures of 105 (1+0+5 = 6) is divisible by 3, a prime number, take 3 as the next divisor (2, **62**), obtaining 35. Since the right-hand figure of 35 is 5, take 5, a prime number, as the next divisor (4, **62**), obtaining 7. Hence, the several prime divisors (2, 2, 3, 5) and the prime quotient (7 yd.) are the prime factors of 420 yd.

RULE.—*Divide the given number by its least prime factor; similarly divide the resulting quotient; and so continue until a prime quotient is obtained. The several divisors and the last quotient are the prime factors.*

PROOF.—*The continued product of all the prime factors should equal the given number.*

EXAMPLES FOR PRACTICE.

Find the prime factors of

1. 45 ft.
2. 132 gal.
3. 274 days.
4. 392 miles.
5. 525 men.
6. 924 lb.
7. 6435 tons.
8. 3240 yd.
9. 4375 hours.
10. 18216 oz.
11. 29184 ft.
12. $108900.

COMMON DIVIDENDS.

64. A **Dividend** of a number is any number that is exactly divisible by it.

65. A **Common Dividend** of two or more numbers is *any* number that is exactly divisible by each of them.

66. The **Least Common Dividend** of two or more numbers is the *least* number that is exactly divisible by each of them.

67. PRINCIPLES.—1. *A dividend of a number must contain all the prime factors of that number.*

2. *A common dividend of two or more numbers must contain all the prime factors of each of those numbers.*

3. *The least common dividend of two or more numbers is the least number that contains all the prime factors of each of those numbers.*

68. To find the least common dividend of two or more numbers.

ILLUSTRATIVE EXAMPLE.

Find the least common dividend of $18, $30, and $36.

SOLUTION.

$$\begin{array}{r|l} \$2 & \$18-\$30-\$36 \\ \hline 3 & 9-\ 15-\ 18 \\ \hline 3 & 3-\ \ 5-\ \ 6 \\ \hline & 1-\ \ 5-\ \ 2 \end{array}$$

$$\$2\times3\times3\times5\times2=\$180.$$

EXPLANATION.—Each of the given numbers contains the prime factor $2, hence, their least common dividend must contain the factor $2 *at least once.*

Since 3 is a prime factor of the unfactored parts of $18 (9), of $30 (15), and of $36 (18), their least common dividend must also contain the factor 3 *at least once.*

Since 3 is a prime factor of the yet unfactored parts of $18 (3) and of $36 (6), their least common dividend must contain the factor 3 *at least one more time, or twice.*

In addition to the factors already obtained ($2 × 3 × 3), the required least common dividend must contain the factor 5 (the unfactored part of $30), else it will not be divisible by $30 ; and it must also contain the factor 2 (the unfactored part of $36), else it will not be divisible by $36. Hence the least common dividend of $18, $30, and $36 must be $2 × 3 × 3 × 5 × 2, or $180.

RULE.—I. *Omit such of the smaller numbers as may be exact divisors of the larger numbers.*

II. *Write the remaining numbers in a horizontal line, and divide by any prime number that will exactly divide two or more of them, placing the quotients and the undivided numbers in a line beneath.*

III. *Divide the quotients and undivided numbers by any prime number that will exactly divide two or more of them; and so continue until all the numbers in the lower line are prime to each other.*

IV. *The product of the divisors and the numbers in the lower line will be the least common dividend.*

NOTE.—When no two of the given numbers have a common prime factor, the *product of all the given numbers* will be their least common dividend.

REM.—The least common dividend is only of practical use in finding the least common denominator of common fractions.

EXAMPLES FOR PRACTICE.

Find the least common dividend of

1. 24 bu., 36 bu., and 42 bu.
2. 12 gal., 27 gal., and 30 gal.
3. 15 hr., 35 hr., and 45 hr.
4. 18 lb., 24 lb., and 42 lb.
5. 8 doz., 15 doz., and 24 doz.
6. 5 yd., 12 yd., and 15 yd.
7. 27 yr., 36 yr., and 48 yr.
8. $315, $525, and $735.
9. 12 ft., 15 ft., 18 ft., and 21 ft.
10. $27, $36, $54, $63, and $72.
11. $18, $21, $24, $30, and $42.
12. $5, $15, $25, $40, $45, $50.
13. $3, $5, and $7. (Note.)
14. 7 lb., 11 lb., and 13 lb.

15. The circumference of the front wheels of a wagon is 12 feet and of the hind wheels 16 feet. If at starting, a rivet in each wheel be directly over the centre of the axle, what distance will the wagon go before both rivets again occupy the same position.

COMMON DIVISORS.

69. A **Common Divisor** of two or more numbers is any number that will exactly divide each of them.

70. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

REM.—When numbers have no common divisor except 1, they are said to be *prime to each other.*

71. PRINCIPLE.—*The greatest common divisor of two numbers is a divisor of their sum, of their difference, and of the sum or difference of one number and any number of times the other.*

72. To find the greatest common divisor of two numbers.

ILLUSTRATIVE EXAMPLE.

Find the greatest common divisor of $387 and $473.

SOLUTION.

$387) $473 (1
387
$86) $387 (4
344
$43) $86 (2
86

$43 = G. C. D.

EXPLANATION.—The G.C.D. of $387 and $473 must divide the difference between $387 and $473 (**71**), and that difference is $86.

The G.C.D. of $86 and $387 must divide the difference between $387 and four times $86, and that difference is $43.

The G.C.D. of $43 and $86 must be $43, for $43 will divide itself, and is contained exactly two times in $86. Hence, $43 is the G.C.D. of $387 and $473.

RULE.—I. *Divide the greater number by the less. If there is no remainder, the less number will be the greatest common divisor.*

II. *If there should be a remainder, divide the preceding divisor by this remainder; and continue to divide the last divisor by the last remainder until there is no remainder. The last divisor will be the greatest common divisor.*

NOTE.—If, at any time, 1 should be obtained as a remainder, the numbers considered have no common divisor.

REM.—The only practical application of the above rule is in reducing common fractions to their lowest terms.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of

1. 172 yr. and 258 yr.
2. $235 and $495.
3. 779 lb. and 1653 lb.
4. 462 yd. and 714 yd.
5. 1377 bu. and 1581 bu.
6. 1083 bbl. and 1387 bbl.
7. 13661 ft. and 32355 ft.
8. 16499 doz. and 25927 doz.
9. 1980 tons and 3120 tons.
10. 1179 gal. and 1703 gal.
11. $323 and $589.
12. $1938 and $2074.

CANCELLATION.

73. **Cancellation** is the omission of the same factor from both divisor and dividend to shorten the labor of division.

74. PRINCIPLES.—1. *To cancel any factor of a number is equivalent to dividing the number by the canceled factor.*

2. *To cancel the same factor, or equal factors, from both divisor and dividend does not affect the quotient* (Prin. 3, **35**).

75. To find a quotient by cancellation.

ILLUSTRATIVE EXAMPLE.

A farmer traded 160 pounds of butter, worth 21 cents per pound, for 2 barrels of sugar averaging 240 pounds per barrel. How much per pound did the farmer pay for the sugar?

SOLUTION.

$$\frac{\overset{\$.07}{\$.\cancel{21}} \times \overset{80}{\cancel{160}}}{\underset{3}{\cancel{240}} \times \cancel{2}} = \$.07.$$

EXPLANATION.—$.21 × 160 = the total value of the butter; and since the butter was exchanged for sugar, the total value of the butter is also the total cost of the sugar. Therefore, divide the cost of all the pounds of sugar ($.21 × 160) by the number of pounds (240 lb. × 2), obtaining 7 cents as the required price of one pound.

The solution may be simplified by canceling all factors which are common to both divisor (240 × 2) and dividend ($.21 × 160), and placing over the canceled numbers their uncanceled factors, if any. Thus, cancel the common factor 2 from the divisor ($\cancel{2}$) and the dividend ($\overset{80}{\cancel{160}}$); then the common factor 80 from the divisor ($\overset{3}{\cancel{240}}$) and the dividend ($\cancel{80}$); then the common factor 3 from the divisor ($\cancel{3}$) and the dividend ($\overset{7}{\cancel{21}}$), leaving only the uncanceled factor $.07 in the dividend, which is the required quotient.

RULE.—I. *Place the factors of the dividend above a horizontal line and the factors of the divisor below it.*

II. *Cancel all factors which are common to both divisor and dividend; and divide the product of the uncanceled factors of the dividend by the product of the uncanceled factors of the divisor.*

NOTE.—If, in canceling, the quotient 1 is obtained, it should be expressed in the dividend (above the horizontal line), but not necessarily in the divisor (below the line).

EXAMPLES FOR PRACTICE.

1. Divide $350 × 9 by 25 × 6.
2. Divide $480 × 160 by $60 × 32.
3. Divide 75 bbl. × 504 by 15 × 8 × 9.
4. Divide 315 gal. × 64 × 56 by 360 gal. × 392.
5. Divide 630 yd. × 352 × 65 by 90 × 88 × 91.
6. How many horses at $175 each are worth as much as 7 horses at $125 each?
7. How many barrels of flour worth $8 per barrel should a miller give for 40 bushels of wheat worth $1.40 per bushel?
8. A merchant sold 12 barrels of pork, averaging 200 pounds per barrel, at 8 cents per pound, and took in payment 160 hams, weighing 10 pounds each. At what price per pound were the hams valued?
9. A mechanic worked for a farmer at 15 cents per hour and averaged 8 hours per day for 24 days. He accepted his wages in wheat at 90 cents per bushel. How many bushels did he receive?
10. A seamstress received 8 pieces of muslin, measuring 45 yards to the piece, from which she made 10 bundles of shirts with 12 shirts in each bundle. How many yards of muslin were used in making a shirt?

COMMON FRACTIONS.

76. A **Fraction** expresses one or more of the equal parts of a unit.

Rem.—*One of the equal parts* into which a unit is divided is called a *fractional unit*, to distinguish it from whole or integral units.

77. The **Denominator** of a fraction is written below a horizontal line, and expresses the number of equal parts into which the integral unit is divided.

Rem. 1.—In the fraction $\$\frac{3}{4}$, the denominator 4 denotes that the integral unit (\$1) is conceived to be divided into 4 equal parts, thus *denominating* (naming) the *value* or *size* of the fractional units.

Rem. 2.—The greater the denominator (the more the parts into which any unit is divided), the less the value of each part; and the less the denominator (the fewer the parts), the greater their value.

78. The **Numerator** of a fraction is written above a horizontal line, and expresses the *number* of equal parts of an integral unit which are taken to form the fraction.

Rem.—In the fraction $\$\frac{3}{4}$, the num(*b*)erator 3 expresses the *number* of fractional units. Hence, $\$\frac{3}{4}$ denotes 3 fractional units, each of which is the fourth of a dollar.

79. The **Terms** of a fraction are its numerator and denominator.

Thus, in $\$\frac{3}{4}$, the numerator 3 is one term, and the denominator 4 is the other.

EXERCISE.

Express with figures the following fractions, and state what each term denotes:

1. One-half of an apple.
2. One-third of an orange.
3. Two-thirds of a melon.
4. One-fourth of a mile.
5. Three-fourths of an inch.
6. Two-fifths of a bushel.
7. Five-sixths of a gallon.
8. Three-eighths of a dollar.
9. Seven-twelfths of a yard.
10. Three-sixteenths of an acre.
11. Five-thirtieths of a foot.
12. Nine forty-fifths of a rod.

80. A **Common Fraction** is one which requires a written denominator.

81. Common Fractions may be **Proper** or **Improper**.

82. A **Proper Fraction** is one whose numerator is less than its denominator.

Rem.—The *value* of a proper fraction is less than a whole unit. $\$\frac{1}{2}$, $\$\frac{2}{3}$, $\$\frac{5}{8}$ are proper fractions.

83. An **Improper Fraction** is one whose numerator is either equal to, or greater than its denominator.

Rem.—The *value* of an improper fraction is either equal to, or greater than a whole unit. $\$\frac{5}{5}$, $\$\frac{5}{4}$, $\$\frac{9}{7}$, $\$\frac{13}{8}$ are improper fractions; the first because its numerator is equal to its denominator, and the others because their numerators are greater than their denominators.

84. A **Mixed Number** is a whole number and a fraction united.

Rem.—$\$3\frac{1}{4}$, $15\frac{3}{8}$ pounds, $48\frac{5}{8}$ gallons are mixed numbers.

85. The **Value of a Fraction** is the quotient arising from dividing its numerator by its denominator.

Rem.—A fraction expresses an *unexecuted* division, its numerator being the dividend, and its denominator the divisor.

86. Since a fraction expresses an unexecuted division, any change made in its numerator or denominator will produce the same change in the value of the fraction as a like change in a dividend or divisor will produce in the value of a quotient (Gen. Prin., **35**). Hence,

87. Principles.—1. *To multiply the numerator or divide the denominator by any number multiplies the fraction by the same number.*

2. *To divide the numerator or multiply the denominator by any number divides the fraction by the same number.*

3. *To multiply or divide both numerator and denominator by the same number does not change the value of the fraction* (Prin. 3, **35**).

REDUCTION OF FRACTIONS.

88. Reduction of Fractions means to change their *form* without changing their value.

Rem.—A fraction is said to be reduced to *higher terms* when Prin. 3, **87**, is employed to express its numerator and denominator in *larger* numbers; to *lower terms* when the same principle is employed to express its numerator and denominator in *smaller* numbers; and to *lowest terms* when its numerator and denominator are thus made *prime to each other* (Rem., **70**).

89. Reduction of fractions to higher or lower terms of a required denomination.

ILLUSTRATIVE EXAMPLES.

1. Reduce $\$\frac{2}{3}$ to twelfths of a dollar.

SOLUTION.

$$\frac{\$2\times4}{3\times4}=\frac{\$8}{12}$$

Explanation.—The required denominator (12) is 4 times the given denominator (3); hence, not to change the value of the fraction, the required numerator must also be 4 times the given numerator (2), and $2\times4=8$ (Prin. 3, **87**).

2. Reduce $\$\frac{24}{72}$ to ninths of a dollar.

SOLUTION.

$$\frac{\$24\div8}{72\div8}=\frac{\$3}{9}$$

Explanation.—The required denominator (9) is one-eighth of the given denominator (72); therefore, without changing the value of the fraction, the required numerator must also be one-eighth of the given numerator (24); and $24+8=3$ (Prin. 3, **87**).

3. Reduce $5 to sevenths of a dollar.

SOLUTION.

$$\frac{\$5\times7}{1\times7}=\frac{\$35}{7}$$

Explanation.—$\$5=\$\frac{5}{1}$. To reduce $\$\frac{5}{1}$ to sevenths, that is, to change a fraction with the denominator 1 to an equivalent fraction with the denominator 7, multiply both terms by 7, as explained in Ex. 1.

Rule.—*Multiply or divide both numerator and denominator by such a number as will change the given denominator to the required denominator.*

Note 1.—To find the common multiplier or divisor of both terms, divide the greater denominator by the less.

Note 2.—Before applying the rule to an integer, change it to the form of a fraction by writing the denominator 1 under it. Thus, $\$2=\$\frac{2}{1}$; 8 lb. $=\frac{8}{1}$ lb.

EXAMPLES FOR PRACTICE.

Reduce

1. $\frac{4}{5}$ lb. to 15ths.
2. $\frac{5}{7}$ yd. to 28ths.
3. $\frac{3}{8}$ qt. to 56ths.
4. $\frac{3}{7}$ bu. to 35ths.
5. $\frac{5}{16}$ ft. to 48ths.
6. $\frac{11}{15}$ yr. to 165ths.
7. $\frac{18}{19}$ oz. to 209ths.
8. $\frac{5}{31}$ lb. to 217ths.

Reduce

9. $\frac{6}{27}$ ft. to 9ths.
10. $\frac{15}{36}$ lb. to 12ths.
11. $\frac{24}{48}$ yd. to 6ths.
12. $\frac{15}{40}$ bu. to 8ths.
13. $\frac{36}{66}$ yr. to 11ths.
14. $\frac{185}{296}$ oz. to 8ths.
15. $\frac{366}{915}$ qt. to 5ths.
16. $\frac{190}{285}$ yd. to 3ds.

Reduce

17. 9 lb. to 4ths.
18. 38 yr. to 9ths.
19. 15 yd. to 6ths.
20. 19 gal. to 12ths.
21. 43 bu. to 8ths.
22. 71 oz. to 19ths.
23. 137 ft. to 45ths.
24. $518 to 78ths.

90. Reduction of fractions to lowest terms.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{72}{108}$ and $\$\frac{205}{328}$ to their lowest terms.

FIRST SOLUTION.

$$\frac{\$72 \div 9}{108 \div 9} = \frac{\$8}{12}$$

$$\frac{\$8 \div 4}{12 \div 4} = \frac{\$2}{3}, \textit{ Ans.}$$

Explanation.—Divide both terms of $\$\frac{72}{108}$ by any common factor, say 9, obtaining $\$\frac{8}{12}$ as an equivalent fraction in lower terms (Prin. 3, **87**). Divide both terms of the obtained fraction ($\$\frac{8}{12}$) by 4, a common factor, obtaining $\$\frac{2}{3}$. Since the terms of $\$\frac{2}{3}$ are prime to each other, it must be the required fraction in lowest terms (Rem., **88**).

SECOND SOLUTION.

G.C.D. of 205 and 328 is 41.

$$\frac{\$205 \div 41}{328 \div 41} = \frac{\$5}{8}$$

Explanation.—If the common factor cannot readily be discovered by inspection, it will be more convenient to find the greatest common divisor of both terms, as shown in **72**, obtaining 41.

Divide both terms by 41, obtaining 5 for the numerator and 8 for the denominator of the equivalent fraction in lowest terms.

First Rule.—*Divide both terms by any common factor; divide both terms of the resulting fraction by any common factor; and continue the operation until a fraction is obtained whose terms are prime to each other.*

Second Rule.—*Divide both terms of the fraction by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

Reduce to equivalent fractions in lowest terms:

1. $\frac{8}{16}$ gal.
2. $\frac{32}{40}$ yd.
3. $\frac{63}{81}$ yr.
4. $\frac{28}{98}$ ft.
5. $\frac{36}{132}$ bu.
6. $\frac{64}{160}$ tons.
7. $\frac{72}{168}$ oz.
8. $\frac{135}{216}$ bbl.
9. $\frac{112}{280}$ yr.
10. $\frac{504}{792}$ gal.
11. $\frac{39}{143}$ qt.
12. $\frac{114}{133}$ bbl.
13. $\frac{498}{830}$ yd.
14. $\frac{485}{679}$ gal.
15. $\frac{549}{671}$ bu.
16. $\frac{604}{1359}$ lb.
17. $\frac{3062}{4593}$ bbl.
18. $\frac{1623}{3787}$ oz.
19. $\frac{5608}{7711}$ gal.
20. $\frac{14987}{17128}$ yd.

91. Reduction of mixed numbers to improper fractions.

ILLUSTRATIVE EXAMPLE.

Reduce \$$29\frac{3}{5}$ to an improper fraction.

SOLUTION.

$$\begin{array}{rl} 29 & \\ 5 & \\ \hline 145 & \text{fifths.} \\ 3 & \text{“} \\ \hline 148 & \text{“} \end{array}$$

\$$29\frac{3}{5}$ = \$$\frac{148}{5}$

EXPLANATION.—There are \$$\frac{5}{5}$ in \$1, and 29 times \$$\frac{5}{5}$, or \$$\frac{145}{5}$, in \$29. Hence, in \$29 and \$$\frac{3}{5}$ there must be \$$\frac{145}{5}$ + \$$\frac{3}{5}$, or \$$\frac{148}{5}$.

PRINCIPLES.—1. *One whole unit is expressed as a fraction by making both numerator and denominator the same.*

2. *Two or more whole units are expressed as a fraction by making the numerator as many times the denominator as the number of whole units is times 1.*

3. *A mixed number contains as many fractional units as there are fractional units in the integer + the fractional units in the terminal fraction.*

RULE.—*Multiply the integer by the denominator, and to the product add the numerator for a new numerator. Keep the same denominator.*

EXAMPLES FOR PRACTICE.

Reduce to improper fractions:

1. $5\frac{3}{8}$ gal.
2. $9\frac{5}{6}$ yd.
3. $12\frac{2}{3}$ tons.
4. $23\frac{4}{5}$ yr.
5. $38\frac{3}{7}$ cords.
6. $135\frac{5}{12}$ miles.
7. $672\frac{3}{4}$ melons.
8. $287\frac{7}{11}$ bbl.
9. $518\frac{5}{16}$ qt.
10. $816\frac{3}{25}$ ft.
11. $716\frac{9}{25}$ bu.
12. $406\frac{5}{128}$ yr.
13. $8265\frac{37}{151}$ oz.
14. $5073\frac{43}{315}$ yd.
15. $6918\frac{57}{419}$ yr.
16. \$$7164\frac{61}{3189}$.

92. Reduction of improper fractions to whole or mixed numbers.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{93}{4}$ to a mixed number.

SOLUTION.

$\$93 \div 4 = \$23\frac{1}{4}$

EXPLANATION.—$\$\frac{4}{4}$ equal 1 whole dollar, and there must be as many whole dollars in 93 fourths as 4 fourths are contained times in 93 fourths, or 23 whole dollars and 1 fourth remaining.

RULE.—*Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

Reduce the following to integers or mixed numbers:

1. $\frac{16}{4}$ yd.
2. $\frac{48}{6}$ bbl.
3. $\frac{161}{7}$ yr.
4. $\frac{257}{9}$ oz.
5. $\frac{182}{13}$ bu.
6. $\frac{294}{5}$ gal.
7. $\frac{253}{59}$ oz.
8. $\frac{458}{23}$ qt.
9. $\frac{893}{17}$ yr.
10. $\frac{511}{31}$ bbl.
11. $\frac{1623}{371}$ yd.
12. $\frac{1956}{23}$ bu.
13. $\frac{19126}{517}$ gal.
14. $\frac{2935}{417}$ oz.
15. $\frac{2753}{113}$ bu.
16. $\frac{4190}{329}$ bbl.

COMMON DENOMINATOR.

93. A **Common Denominator** of two or more fractions is *any* denominator to which all of them can be reduced.

94. The **Least Common Denominator** of two or more fractions is the *least* denominator to which all of them can be reduced.

REM.—A fraction can be reduced to only such higher denominators as its own denominator can exactly divide (Note 1, **89**). Hence, a common denominator of two or more fractions must be a dividend of the several denominators of those fractions; and the least common denominator of two or more fractions must be the least common dividend of their several denominators.

95. Reduction of two or more fractions to their least common denominator.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{7}{8}$, $\$\frac{4}{9}$, and $\$\frac{5}{6}$ to their least common denominator.

SOLUTION.

L.C.D. of 8, 9, 6 = 72.

$\$\frac{7}{8} = (72 \div 8) \times 7 = \$\frac{63}{72}$

$\$\frac{4}{9} = (72 \div 9) \times 4 = \$\frac{32}{72}$

$\$\frac{5}{6} = (72 \div 6) \times 5 = \$\frac{60}{72}$

EXPLANATION.—First find the least common dividend of the denominators (8, 9, 6), as shown in **68**, and take the result (72) as the required least common denominator.

Then reduce the several fractions to this common denominator, as shown in Ex. 1, **89**, finding that $\$\frac{7}{8} = \$\frac{63}{72}$, $\$\frac{4}{9} = \$\frac{32}{72}$, and $\$\frac{5}{6} = \$\frac{60}{72}$.

RULE.—I. *The least common dividend* (**68**) *of the given denominators will be the required least common denominator.*

II. *Reduce each fraction to the required common denominator as shown in* Ex. 1, **89**.

NOTE.—If necessary, reduce all fractions to their lowest terms (**90**) before applying the above rule.

EXAMPLES FOR PRACTICE.

Reduce to fractions having the least common denominator:

1. $\frac{2}{3}$ yd., $\frac{3}{4}$ yd., $\frac{5}{6}$ yd.
2. $\frac{1}{2}$ ft., $\frac{1}{3}$ ft., $\frac{3}{8}$ ft.
3. $\frac{7}{8}$ bu., $\frac{11}{12}$ bu., $\frac{5}{9}$ bu.
4. $\frac{3}{5}$ yr., $\frac{7}{10}$ yr., $\frac{8}{15}$ yr.
5. $\frac{3}{4}$ gal., $\frac{2}{3}$ gal., $\frac{7}{12}$ gal.
6. $\frac{2}{3}$ oz., $\frac{3}{7}$ oz., $\frac{4}{5}$ oz. (Note, **68**.)
7. $\frac{5}{8}$ ft., $\frac{3}{7}$ ft., $\frac{9}{14}$ ft., $\frac{8}{21}$ ft.
8. $\frac{3}{5}$ bu., $\frac{5}{9}$ bu., $\frac{4}{15}$ bu., $\frac{9}{25}$ bu.
9. $\frac{7}{18}$ yr., $\frac{11}{24}$ yr., $\frac{13}{36}$ yr., $\frac{8}{63}$ yr.
10. $\frac{1}{2}$ yd., $\frac{2}{3}$ yd., $\frac{4}{7}$ yd., $\frac{1}{5}$ yd.
11. $\frac{2}{3}$ ft., $\frac{4}{7}$ ft., $\frac{5}{9}$ ft., $\frac{11}{28}$ ft.
12. $\frac{7}{36}$ bu., $\frac{25}{96}$ bu., $\frac{13}{112}$ bu., $\frac{37}{132}$ bu.

ADDITION OF FRACTIONS.

96. Addition of Fractions is the process of finding the sum of two or more fractions.

97. PRINCIPLES.—1. *To be added, fractions must express similar fractional units, and therefore have a common denominator.*

2. *The numbers of fractional units* (the numerators) *are the addends.*

3. *The sum of the several numerators expresses fractional units of the same name* (same denominator) *as the fractions added.*

98. To find the sum of two or more fractions.

ILLUSTRATIVE EXAMPLE.

1. Find the sum of $\$\frac{1}{3}$, $\$\frac{3}{4}$, and $\$\frac{5}{6}$.

SOLUTION.

$\underline{12}$ = least com. denom.
$\$\frac{1}{3}$... 4 = new numerator.
$\$\frac{3}{4}$... 9 = " "
$\$\frac{5}{6}$... $\underline{10}$ = " "
$\$\frac{23}{12} = \$1\frac{11}{12}$

EXPLANATION.—Fractions must have a common denominator before they can be added (Prin. 1, **97**). The least common denominator of the several fractions is 12 (Rem., **94**). Reduce each fraction to this common denominator (**95**), and write the corresponding numerators in a column. Add these numerators (Prin. 2, **97**), obtaining 23 as the numerator of the sum, under which write the common denominator 12 (Prin. 3, **97**), finding $\$\frac{23}{12} = \$1\frac{11}{12}$ to be the required sum.

2. Find the sum of $\$95\frac{2}{3}$, $\$28\frac{5}{8}$, $\$137\frac{1}{6}$, $\$45\frac{5}{9}$.

SOLUTION.

$\underline{72}$
$\$95\frac{2}{3}$...48
$28\frac{5}{8}$...45
$137\frac{1}{6}$...12
$45\frac{5}{9}$...$\underline{40}$
2 ..$\frac{145}{72} = 2\frac{1}{72}$
$\$307\frac{1}{72}$

EXPLANATION.—First add the fractions as in the preceding example, obtaining $\$\frac{145}{72} = \$2\frac{1}{72}$. Carry the integral part of this result (2) and add the integers, obtaining $307, to the right of which annex the fractional part $\frac{1}{72}$, obtaining $\$307\frac{1}{72}$ as the required sum.

RULE.—*Reduce to equivalent fractions of a common denominator; add the resulting numerators; and place the sum over the common denominator.*

NOTE 1.—To find the sum of two or more mixed numbers: *add the fractions and integers separately, and combine the two results.*

NOTE 2.—When necessary, all results if proper fractions should be reduced to lowest terms; and if improper fractions, to equivalent mixed numbers.

EXAMPLES FOR PRACTICE.

Find the sum of

1. $\frac{2}{3}$ yd., $\frac{1}{4}$ yd., $\frac{5}{6}$ yd.
2. $\frac{1}{6}$ gal., $\frac{5}{9}$ gal., $\frac{7}{12}$ gal.
3. $\frac{4}{5}$ lb., $\frac{2}{3}$ lb., $\frac{11}{15}$ lb.
4. $\frac{4}{9}$ bbl., $\frac{5}{12}$ bbl., $\frac{3}{8}$ bbl.
5. $\frac{1}{2}$ oz., $\frac{3}{4}$ oz., $\frac{2}{3}$ oz., $\frac{1}{6}$ oz.
6. $\frac{11}{12}$ bu., $\frac{5}{8}$ bu., $\frac{5}{6}$ bu.
7. $\frac{1}{3}$ yr., $\frac{6}{7}$ yr., $\frac{3}{4}$ yr. (Note, **68**).
8. $\frac{3}{5}$ qt., $\frac{5}{7}$ qt., $\frac{7}{8}$ qt.

Find the sum of

9. $\frac{9}{14}$ lb., $\frac{4}{21}$ lb., $\frac{13}{27}$ lb., $\frac{7}{45}$ lb.
10. $\frac{5}{9}$ yr., $\frac{3}{5}$ yr., $\frac{7}{15}$ yr., $\frac{3}{20}$ yr.
11. $41\frac{2}{3}$ yd., $73\frac{3}{4}$ yd., $67\frac{5}{6}$ yd.
12. $61\frac{2}{3}$ bu., $318\frac{5}{6}$ bu., $68\frac{5}{8}$ bu.
13. $417\frac{3}{8}$ yr., $249\frac{13}{16}$ yr., $385\frac{17}{32}$ yr.
14. $519\frac{17}{35}$ lb., $182\frac{5}{7}$ lb., $327\frac{9}{25}$ lb.
15. $285\frac{4}{5}$ oz., $173\frac{9}{10}$ oz., $341\frac{7}{15}$ oz.
16. $19\frac{8}{35}$ yd., $23\frac{4}{7}$ yd., $185\frac{8}{21}$ yd.

98

REM. 1.—The small figures at the right of the following addends in yards express fourths. This mode of expressing fractions of a yard is common in the dry-goods trade.

17. Add 13^1 yd., 48 yd., 19^3 yd., 32^2 yd., 41^1 yd., 39^3 yd., 28^2 yd.
18. Add 27^3 yd., 34^2 yd., 29^1 yd., 32^1 yd., 37^2 yd., 48 yd., 35^3 yd.
19. Add 38^2 yd., 41^2 yd., 29^1 yd., 35^2 yd., 40^3 yd., 36^2 yd., 43^3 yd.

REM. 2.—Grocers frequently express ounces in small figures at the right of addends in pounds. These small figures then denote sixteenths.

20. Add 9^{14} lb., 17^3 lb., 28^{12} lb., 25^1 lb., 43^{15} lb., 32^5 lb., 14^8 lb.
21. Add 28^{13} lb., 48^{10} lb., 119^5 lb., 173^{11} lb., 81^8 lb., 9^{15} lb., 26^4 lb.

SUBTRACTION OF FRACTIONS.

99. Subtraction of Fractions is the process of finding the difference between two fractions.

100. PRINCIPLES.—1. *To be subtracted, the fraction of the subtrahend must express fractional units of the same name as the fraction of the minuend; and both fractions must therefore have a common denominator.*

2. *The numbers of fractional units* (the numerators) *are the terms of subtraction.*

3. *The difference of the numerators expresses fractional units of the same name* (same denominator) *as those of the subtrahend and minuend.*

101. To find the difference between two fractions.

ILLUSTRATIVE EXAMPLE.

Subtract \$$\frac{4}{9}$ from \$$\frac{5}{6}$.

SOLUTION.

$$\begin{array}{rl} & \underline{18} = \text{com. denom.} \\ \$\frac{5}{6} \ldots & 15 = \text{new numerator.} \\ \$\frac{4}{9} \ldots & \underline{\ \ 8} = \text{“} \quad \text{“} \\ & \$\frac{7}{18} = \textit{Answer.} \end{array}$$

EXPLANATION.—First find the least common denominator (Prin. 1, **100**). Reduce each fraction to this common denominator; write the new numerator of the subtrahend under the new numerator of the minuend; and subtract (Prin. 2, **100**), obtaining 7 as the numerator of the difference, under which write the common denominator (Prin. 3, **100**).

RULE.—*Reduce to equivalent fractions of a common denom-*

inator, and place the difference of the resulting numerators over the common denominator.

NOTE.—If *small* mixed numbers are given, reduce to improper fractions before applying the rule.

EXAMPLES FOR PRACTICE.

Find the difference between

1. $\frac{2}{3}$ ft. and $\frac{3}{4}$ ft.
2. $\frac{8}{9}$ yd. and $\frac{5}{21}$ yd.
3. $\frac{3}{4}$ bbl. and $\frac{7}{12}$ bbl.
4. $\frac{9}{16}$ oz. and $\frac{11}{24}$ oz.
5. $\frac{5}{8}$ gal. and $\frac{1}{6}$ gal.
6. $\frac{2}{3}$ doz. and $\frac{1}{2}$ doz.
7. $\frac{2}{5}$ yr. and $\frac{7}{15}$ yr.
8. $\frac{5}{6}$ lb. and $\frac{7}{9}$ lb.
9. $\frac{3}{8}$ qt. and $\frac{5}{7}$ qt.
10. $\frac{3}{4}$ yd. and $\frac{4}{5}$ yd.
11. $3\frac{1}{2}$ yr. and $2\frac{3}{4}$ yr.
12. $5\frac{2}{3}$ bu. and $1\frac{4}{5}$ bu.
13. $8\frac{5}{6}$ oz. and $9\frac{3}{4}$ oz.
14. $6\frac{4}{5}$ ft. and $4\frac{3}{8}$ ft.
15. $5\frac{4}{9}$ lb. and $7\frac{1}{4}$ lb.

102. Subtraction of mixed numbers.

ILLUSTRATIVE EXAMPLES.

1. Subtract \$357 from \$415$\frac{2}{3}$.

SOLUTION.

$$\begin{array}{r} \$415\frac{2}{3} \\ 357 \\ \hline \$58\frac{2}{3} \end{array}$$

EXPLANATION.—Since *no* part of a dollar is to be subtracted from \$$\frac{2}{3}$, the fractional part of the remainder will be \$$\frac{2}{3}$, to which prefix the difference of the integral terms, obtaining \$58$\frac{2}{3}$.

2. Subtract \$194$\frac{2}{3}$ from \$436.

SOLUTION.

$$\begin{array}{r} \$436 \\ 194\frac{2}{3} \\ \hline \$241\frac{1}{3} \end{array}$$

EXPLANATION.—\$$\frac{2}{3}$ cannot be taken from *no* thirds of a dollar; therefore, borrow \$1 or \$$\frac{3}{3}$ from \$436, leaving \$435, and then subtract \$$\frac{2}{3}$ from \$$\frac{3}{3}$, leaving \$$\frac{1}{3}$, which should be annexed to \$435—\$194, obtaining \$241$\frac{1}{3}$.

3. Subtract \$527$\frac{5}{6}$ from \$862$\frac{3}{4}$.

SOLUTION.

$$\begin{array}{rl} & \underline{12} \\ \$862\frac{3}{4}\ldots & 9+12 \\ 527\frac{5}{6}\ldots & 10 \\ \hline \$334\frac{11}{12} & \end{array}$$

EXPLANATION.—Since \$$\frac{10}{12}$ cannot be taken from \$$\frac{9}{12}$, borrow \$1 or \$$\frac{12}{12}$ from \$862, leaving \$861, and then subtract \$$\frac{10}{12}$ from (\$$\frac{9}{12}$+\$$\frac{12}{12}$ =) \$$\frac{21}{12}$, leaving \$$\frac{11}{12}$, which annex to \$861—\$527, obtaining \$334$\frac{11}{12}$.

RULE.—*Subtract the fractions and integers separately, and combine the two results.*

101—102

EXAMPLES FOR PRACTICE.

Find the difference between

1. $\$723$ and $\$419\frac{4}{5}$.
2. $\$526$ and $\$839\frac{2}{3}$.
3. $\$238\frac{5}{9}$ and $\$462$.
4. $\$571\frac{3}{8}$ and $\$167$.
5. $\$618\frac{2}{3}$ and $\$327\frac{1}{4}$.
6. $\$907\frac{4}{5}$ and $\$645\frac{3}{4}$.
7. $\$718\frac{1}{12}$ and $\$286\frac{3}{5}$.
8. $\$413\frac{1}{3}$ and $\$325\frac{4}{7}$.
9. $\$175\frac{7}{8}$ and $\$243\frac{2}{3}$.
10. $\$841\frac{3}{4}$ and $\$615\frac{5}{6}$.
11. $\$291\frac{1}{3}$ and $\$612\frac{4}{7}$.
12. $\$438\frac{1}{6}$ and $\$287\frac{4}{9}$.
13. 16^{1} yd. and 43^{8} yd.
14. 72^{2} yd. and 25^{3} yd.
15. 43^{11} lb. and 37^{8} lb.

MULTIPLICATION OF FRACTIONS.

103. Multiplication of Fractions is the process of finding the product of two factors when one or both of them are fractions.

REM.—By referring to the relation of integers (Sol., **54**) it will be seen that one factor of a product (the multiplicand) denotes the weight, measure, value, etc., of one whole addend; and the other factor (the multiplier) the number of such addends. It follows, therefore, that if the multiplier is a proper fraction (less than 1) it denotes that the weight, measure, value, etc., of the required product is less than that of one whole addend (less than the multiplicand); and that if the multiplier is an integer, a mixed number, or an improper fraction (greater than 1) it denotes that the weight, measure, value, etc., of the required product is greater than that of one whole addend (greater than the multiplicand). Hence,

104. PRINCIPLE.—*Multiplication implies increase of the multiplicand when the multiplier is greater than 1; and decrease when the multiplier is less than 1.*

NOTE.—Prin. 1, 2, 3, and 4, **25**, relating to integers, are equally applicable to fractions.

105. To multiply when one or both factors are fractions.

ILLUSTRATIVE EXAMPLES.

1. What is the cost of $\frac{4}{15}$ of a yard of cloth at $\$\frac{5}{8}$ per yard?

SOLUTION.

$$\$\frac{5}{8} \times \frac{4}{15} = \$\frac{20}{120} = \$\frac{1}{6}.$$

$$\text{Or, } \$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \$\frac{1}{6}.$$

EXPLANATION.—$\frac{4}{15}$ = 4 times $\frac{1}{15}$. If 1 yard ($\frac{15}{15}$) cost $\$\frac{5}{8}$, $\frac{1}{15}$ of a yard will cost $\frac{1}{15}$ of $\$\frac{5}{8}$, or $\frac{5}{120}$ (Prin. 2, **87**); and $\frac{4}{15}$ of a yard will cost 4 times $\$\frac{5}{120}$, or $\$\frac{20}{120}$ (Prin. 1, **87**), which reduced to lowest terms equals $\$\frac{1}{6}$.

For convenience, cancel all factors common to opposite terms, as in the second solution, obtaining the result in lowest terms without further reduction.

2. Multiply (1) $\$\frac{3}{16}$ by 4; (2) $\$\frac{7}{8}$ by 32; and (3) $4 by $\frac{5}{6}$.

SOLUTIONS.

(1)

$$\$\frac{3}{\underset{4}{\cancel{16}}}\times\overset{1}{\cancel{4}}=\$\frac{3}{4}.$$

(2)

$$\$\frac{7}{\underset{1}{\cancel{8}}}\times\overset{4}{\cancel{32}}=\$28.$$

(3)

$$\$\overset{2}{\cancel{4}}\times\frac{5}{\underset{3}{\cancel{6}}}=\$\frac{10}{3}=\$3\tfrac{1}{3}.$$

EXPLANATION.—The solution of these examples is similar to that of Ill. Ex. 1, as will be evident by writing the integral factors in the form of fractions with 1 as their denominators. Thus, (1) $\$\frac{3}{16}\times\frac{4}{1}=?$; (2) $\$\frac{7}{8}\times\frac{32}{1}=?$; (3) $\$\frac{4}{1}\times\frac{5}{6}=?$

RULE.—*Multiply the numerators for a new numerator and the denominators for a new denominator, first canceling all factors which are common to opposite terms.*

NOTE 1.—If one of the factors is an integer, change it to the form of a fraction by writing 1 under it for a denominator, or by imagining 1 to be thus written, and then apply the rule.

NOTE 2.—If one or both factors are *small* mixed numbers, reduce them to improper fractions (**91**) before employing the rule.

NOTE 3.—When the integral factor is seen to be an exact divisor of the denominator of the fractional factor, apply Prin. 1, **87**, and avoid a written solution involving formal cancellation. Thus, $\$\frac{3}{16}\times 4=\$\frac{3}{16\div 4}=\$\frac{3}{4}$.

NOTE 4.—If the integer is large, business men do not usually trouble themselves about making formal statements as in the illustrative examples, or about discovering common factors for cancellation. They simply *multiply the integer by the numerator of the fraction and divide the product by the denominator.* Thus, $\$1838\times\frac{3}{4}=(1838\times 3)\div 4$.

NOTE 5.—The word *of* written between fractions or between fractions and integers, indicates that they are to be multiplied. Fractions thus connected are called *compound fractions.*

EXAMPLES FOR PRACTICE.

Multiply
1. $\$\frac{9}{17}$ by $\frac{34}{45}$.
2. $\$\frac{5}{8}$ by $\frac{8}{20}$.
3. $\$\frac{3}{4}$ by $\frac{2}{5}$.
4. $\$\frac{2}{3}$ by $\frac{3}{4}$.
5. $\$\frac{1}{3}$ by $\frac{4}{5}$.
6. $\$\frac{5}{7}$ by $\frac{7}{15}$.
7. $\$\frac{1}{2}$ by $\frac{1}{3}$.
8. $\$\frac{5}{6}$ by $\frac{7}{8}$.

Multiply
9. $\$\frac{2}{3}$ by $1\frac{1}{4}$.
10. $\$3\frac{3}{5}$ by $\frac{11}{15}$.
11. $\$2\frac{1}{4}$ by $1\frac{1}{3}$.
12. $\$3\frac{1}{2}$ by $5\frac{1}{4}$.
13. $\$4\frac{1}{3}$ by $2\frac{3}{4}$.
14. $\frac{1}{6}$ of $\$\frac{1}{2}$.
15. $\frac{2}{3}$ of $\$\frac{3}{4}$.
16. $\frac{1}{2}$ of $\frac{3}{4}$ of $\$\frac{4}{5}$.

Multiply
17. $689 by $\frac{3}{4}$.
18. $158 by $\frac{5}{8}$.
19. $625 by $\frac{2}{3}$.
20. $\$\frac{4}{5}$ by 718.
21. $\$\frac{5}{9}$ by 925.
22. $\$\frac{13}{18}$ by 6.
23. $44 by $\frac{4}{11}$.
24. $\$\frac{8}{15}$ by 45.

Multiply
25. $\$\frac{5}{36}$ by 9.
26. $\$\frac{11}{63}$ by 14.
27. $15 by $\frac{9}{35}$.
28. $\$\frac{7}{16}$ by 28.
29. $258 by $\frac{5}{7}$.
30. $48 by $\frac{7}{8}$.
31. $\$\frac{13}{14}$ by 56.
32. $\$\frac{4}{5}$ by 25.

106. Multiplication of integers and mixed numbers.

ILLUSTRATIVE EXAMPLE.

Multiply (1) $\$179\frac{3}{4}$ by 7; and (2) $\$325$ by $18\frac{5}{8}$.

SOLUTIONS.

$$\begin{array}{rr} (1) & (2)\\ \$179\frac{3}{4} & \$325\\ 7 & 18\frac{5}{8}\\ \hline \$1258\frac{1}{4} & 8)1625\\ \cline{2-2} & 203\frac{1}{8}\\ & 2600\\ & 325\\ \cline{2-2} & \$6053\frac{1}{8}\end{array}$$

EXPLANATION.—(1.) 7 times $\frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$. Write $\frac{1}{4}$ and mentally carry 5. 7 times 9 are 63, and 5 to carry are 68. Write 8, carry 6 to 7 times 7, and continue as with other integers.

(2.) If the integer is large, as in second solution, multiply it by the fractional part of the mixed number as shown in Note 4, **105**; then by the integral part of the mixed number.

RULE.—*Separately multiply the integer by the fractional and the integral part of the mixed number; then add the results.*

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply
1. $\$236\frac{5}{8}$ by 9.	5. $\$481\frac{13}{16}$ by 9.	9. $\$987$ by $27\frac{3}{4}$.
2. $\$425\frac{3}{8}$ by 4.	6. $\$394\frac{2}{15}$ by 5.	10. $\$826$ by $32\frac{7}{8}$.
3. $\$48$ by $8\frac{5}{6}$.	7. $\$478$ by $38\frac{4}{5}$.	11. $\$1245$ by $43\frac{4}{7}$.
4. $\$63$ by $5\frac{4}{9}$.	8. $\$5172$ by $25\frac{5}{6}$.	12. $\$938$ by $213\frac{3}{5}$.

107. Multiplication of mixed numbers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Find the cost of $25\frac{3}{4}$ pounds of coffee at $18\frac{5}{8}$ cents per pound.

SOLUTION.

$$\begin{array}{rll} & \$.18\frac{5}{8} & \\ & 25\frac{3}{4} & \underline{32}\\ \hline (\frac{5}{8}\times\frac{3}{4}=) & \frac{15}{32} & \ldots 15\\ (18\times\frac{3}{4}=) & 13\frac{1}{2} & \ldots 16\\ (\frac{5}{8}\times 24=) & 15\frac{5}{8} & \ldots \underline{20}\\ (18\times 25=) & \left\{\begin{array}{l} 90\\ 36\end{array}\right. & \frac{51}{32} = 1\frac{19}{32}\\ & \underline{\quad 1} & \\ & \$4.79\frac{19}{32}. & \end{array}$$

EXPLANATION.—Separately multiply the integral and fractional parts of the multiplicand (18 and $\frac{5}{8}$) by the fractional part of the multiplier ($\frac{3}{4}$); then separately multiply the fractional and integral parts of the multiplicand (18 and $\frac{5}{8}$) by the integral part of the multiplier (25). The sum of the several partial products thus obtained will be the total product.

RULE.—I. *Separately multiply the fractional and integral parts of the multiplicand by the fractional part of the multiplier.*

II. *Separately multiply the fractional and integral parts of the multiplicand by the integral part of the multiplier.*

III. *Add the several partial products thus obtained.*

EXAMPLES FOR PRACTICE.

Multiply
1. $\$72\frac{3}{4}$ by $24\frac{5}{8}$.
2. $\$16\frac{3}{5}$ by $25\frac{3}{8}$.
3. $\$36\frac{2}{3}$ by $18\frac{5}{6}$.
4. $\$63\frac{3}{4}$ by $32\frac{4}{7}$.

Multiply
5. $\$6154\frac{4}{5}$ by $41\frac{2}{3}$.
6. $\$473\frac{5}{8}$ by $19\frac{3}{4}$.
7. $\$571\frac{3}{4}$ by $15\frac{2}{3}$.
8. $\$739\frac{3}{7}$ by $85\frac{5}{6}$.

Multiply
9. $\$27184\frac{4}{9}$ by $35\frac{6}{11}$.
10. $\$492\frac{5}{12}$ by $281\frac{2}{5}$.
11. $\$75\frac{5}{16}$ by $64\frac{7}{15}$.
12. $\$1825\frac{3}{4}$ by $25\frac{5}{8}$.

DIVISION OF FRACTIONS.

108. Division of Fractions is the process of finding the quotient when the dividend, the divisor, or both of them, contain fractions.

REM.—Since the dividend corresponds with the product, and the divisor and quotient to the multiplicand and multiplier (**34**), Rem., **103**, and Prin., **104**, taken reversely, also apply to division; that is, if the divisor (multiplier) is an integer, a mixed number, or an improper fraction whose value is greater than 1, the quotient (multiplicand) will be less than the dividend (product); but if the divisor is a proper fraction, the quotient will be greater than the dividend. Hence,

109. PRINCIPLE.—*Division implies decrease of the dividend when the divisor is greater than 1, and increase of the dividend when the divisor is less than 1.*

110. Division when the divisor or dividend is a fraction.

ILLUSTRATIVE EXAMPLES.

Divide (1) $\$\frac{5}{6}$ by $\$\frac{7}{9}$; (2) $12 by $\$\frac{3}{4}$; (3) $\$\frac{10}{21}$ by $5.

SOLUTIONS.

(1)

$$\frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{7} = \frac{15}{14} = 1\frac{1}{14}.$$

(2)

$$\frac{\overset{4}{\cancel{12}}}{1} \times \frac{4}{\cancel{3}} = 16.$$

(3)

$$\frac{\overset{2}{\cancel{10}}}{21} \times \frac{1}{\cancel{5}} = \frac{2}{21}.$$

FIRST SOLUTION.—$1 ÷ $\$\frac{7}{9}$, or 9 *ninths of a dollar* divided by 7 *ninths* of a dollar $= \frac{9}{7}$ (Rem., **85**) or the divisor $\frac{7}{9}$ with its terms inverted. Hence, $\$\frac{5}{6} \div \$\frac{7}{9}$

must equal $\frac{5}{6}$ of $\frac{9}{7}$, or $\frac{15}{14}$ (Prin. 1, **35**). The product of $\frac{5}{6} \times \frac{9}{7}$ is obtained by Rule, **105.**

SECOND SOLUTION.—$\$1 \div \$\frac{3}{4} = \$\frac{4}{3}$; and \$12 divided by $\$\frac{3}{4}$ must therefore be 12 times $\frac{4}{3}$, or 16 (Rule, **105**).

THIRD SOLUTION.—$\$1 \div \$5 = \frac{1}{5}$; and $\$\frac{10}{21} \div \$5 = \frac{10}{21}$ of $\frac{1}{5}$, or $\frac{2}{21}$.

RULE.—*Invert the divisor, and multiply the dividend by the resulting fraction, applying Rule,* **105.**

NOTE 1.—If the divisor is an integer, imagine it to have the form of a fraction with the denominator 1. Thus, in the 3d solution, the integral divisor 5 was imagined to be $\frac{5}{1}$, which, when inverted, would be $\frac{1}{5}$.

NOTE 2.—If either the divisor or dividend is a small mixed number, reduce it to an improper fraction before applying the rule.

NOTE 3.—If the dividend is a large integer, invert the divisor and apply Note 4, **105.**

EXAMPLES FOR PRACTICE.

Divide	Divide	Divide	Divide
1. $\$1$ by $\$\frac{4}{5}$.	**9.** $\$\frac{5}{8}$ by $\$1\frac{7}{8}$.	**17.** $\$\frac{3}{4}$ by \$18.	**25.** $\$2\frac{1}{3}$ by \$5.
2. $\$\frac{1}{6}$ by $\frac{2}{3}$.	**10.** $\$3\frac{1}{2}$ by $1\frac{1}{4}$.	**18.** $\$2\frac{3}{4}$ by $3\frac{1}{2}$.	**26.** $\$1\frac{3}{7}$ by \$25.
3. $\$\frac{3}{4}$ by $\$\frac{2}{3}$.	**11.** $\$2\frac{2}{3}$ by $\$1\frac{1}{6}$.	**19.** \$1218 by $\frac{5}{8}$.	**27.** \$24 by $\frac{8}{9}$.
4. $\$\frac{6}{7}$ by $\frac{5}{8}$.	**12.** \$56 by $5\frac{1}{3}$.	**20.** $\$\frac{2}{3}$ by \$12.	**28.** $\$\frac{2}{15}$ by $\frac{5}{6}$.
5. \$18 by $\frac{6}{11}$.	**13.** $\$1\frac{5}{7}$ by \$36.	**21.** $\$1\frac{1}{4}$ by 15.	**29.** $\$2\frac{3}{4}$ by $\frac{3}{8}$.
6. \$5 by $\frac{40}{41}$.	**14.** $\$5\frac{5}{8}$ by 9.	**22.** $\$7\frac{3}{5}$ by 4.	**30.** $\$\frac{3}{4}$ by \$7.
7. $\$\frac{6}{7}$ by 3.	**15.** \$519 by $\$\frac{6}{7}$.	**23.** $\$\frac{12}{17}$ by 8.	**31.** \$256 by $3\frac{5}{6}$.
8. $\$\frac{5}{6}$ by \$5.	**16.** $\$3\frac{3}{5}$ by $4\frac{1}{5}$.	**24.** $\$5\frac{1}{3}$ by $\frac{7}{12}$.	**32.** \$67 by $\$\frac{5}{8}$.

111. Division of mixed numbers by integers, or of integers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Divide $\$217\frac{5}{8}$ by 13.

SOLUTION.

```
 13 ) $217 5/8
  8      8
---   -------
104 ) 1741 ( $16 77/104
      104
      ----
       701
       624
       ---
        77
```

EXPLANATION.—To multiply both divisor and dividend by the same number produces no change in the quotient (Prin. 3, **35**). Hence, multiply both dividend and divisor by the denominator of the fractional part of the dividend (8), obtaining 104 as the integral divisor and 1741 as the integral dividend; then divide as with other integers.

RULE.—*Multiply both divisor and dividend by the denominator of the terminal fraction; then divide as with integers.*

EXAMPLES FOR PRACTICE.

Divide

1. $\$18491$ by $2\frac{1}{3}$.
2. $\$75834$ by $\$3\frac{1}{2}$.
3. $\$415283\frac{2}{3}$ by 4.
4. $\$918273$ by $1\frac{3}{4}$.
5. $\$627345$ by $\$8\frac{2}{3}$.

Divide

6. $\$1927348\frac{4}{5}$ by $18.
7. $\$6357182\frac{3}{8}$ by 9.
8. $\$5879306\frac{5}{6}$ by 35.
9. $\$428391$ by $28\frac{2}{3}$.
10. $\$687132$ by $\$14\frac{5}{6}$.

Divide

11. $\$359834\frac{4}{15}$ by 87.
12. $\$2739186$ by $\$193\frac{4}{13}$.
13. $\$907382\frac{4}{5}$ by 25.
14. $\$763528\frac{15}{31}$ by 16.
15. $\$2879376$ by $\$18\frac{5}{28}$.

16. A retail shoe dealer bought 24 pairs of shoes for $\$72\frac{3}{4}$. What was the average price per pair?

17. How many casks containing $36\frac{2}{3}$ gallons each are capable of holding 29920 gallons of cider?

112. Division of mixed numbers by mixed numbers.

ILLUSTRATIVE EXAMPLE.

Divide $\$793\frac{3}{4}$ by $\$23\frac{5}{6}$.

SOLUTION.

$$
\begin{array}{rl}
\$23\frac{5}{6}\ ) & \$793\frac{3}{4} \\
\underline{12} & \underline{\quad 12} \\
286\ ) & 9525\ (\ 33\frac{87}{286} \\
 & \underline{858} \\
 & \ \ 945 \\
 & \ \ \underline{858} \\
 & \ \ \ \ 87
\end{array}
$$

EXPLANATION.—Multiply both mixed numbers by the least common denominator of their terminal fractions (L.C.D. of 6 and 4 is 12), obtaining $9525 as the integral dividend and $286 as the integral divisor (Prin. 3, **35**). Divide as with other integers, producing $33\frac{87}{286}$ as the required quotient.

RULE.—*Multiply both divisor and dividend by the least common denominator of their terminal common fractions to obtain an integral divisor and dividend; then divide as with other integers.*

EXAMPLES FOR PRACTICE.

Divide

1. $\$75\frac{2}{3}$ by $14\frac{3}{4}$.
2. $\$465\frac{1}{2}$ by $8\frac{5}{8}$.
3. $\$219\frac{4}{5}$ by $\$12\frac{5}{8}$.
4. $\$926\frac{1}{4}$ by $9\frac{5}{6}$.

Divide

5. $\$7834\frac{2}{5}$ by $\$28\frac{1}{2}$.
6. $\$5167\frac{5}{12}$ by $35\frac{2}{3}$.
7. $\$7916\frac{5}{8}$ by $\$16\frac{5}{6}$.
8. $\$4153\frac{1}{2}$ by $71\frac{1}{3}$.

Divide

9. $\$517\frac{2}{15}$ by $92\frac{4}{25}$.
10. $\$7821\frac{7}{12}$ by $\$38\frac{5}{18}$.
11. $\$591\frac{5}{7}$ by $128\frac{2}{3}$.
12. $\$9418\frac{3}{11}$ by $59\frac{5}{8}$.

RELATION OF COMMON FRACTIONS.

113. So much of **51, 52, 53, 54,** and **55** as pertains to addition and subtraction of integers also applies without modification to addition and subtraction of fractions; and so much as relates to multiplication and division of integers will also apply to multiplication and division of fractions, *if the abstract factor* (denoting the number of equal components) *is an integer, a mixed number, or an improper fraction.* If, however, as explained in Rem., **103,** and Rem., **108,** the abstract factor is a proper fraction (denoting one or more *parts* of a component), the following modification of **54** may be substituted.

114. Fractional multiplication and division compared.

1. Find the cost of ⅛ of a pound of tea at 48 cents per pound.

2. If ⅛ of a pound of tea cost 6 cents, what is the price per pound?

3. How much tea at 48 cents per pound can be bought for 6 cents?

SOLUTION.

Factors of dividend. { concrete divisor = $.48 = cost of 1 *whole* pound = concrete multiplicand } Factors of product.
abstract quotient = × ⅛ = fractional number of lbs. = abstract multiplier
Dividend, if given = $.06 = cost of fractional number of lbs. = product, if required.

DEDUCTION.—The weight, measure, value, etc., of any part of a whole unit or quantity is the product if required, or the dividend if given. *One factor* of a required product or given dividend is the weight, measure, value, etc., of *one whole unit or quantity;* and the other factor is any considered *fractional part* of that whole unit or quantity. Hence, if a product is required, multiply its two given factors; and if a product is given, divide it by its given factor and the quotient will be its remaining factor.

DEFINITIONS.

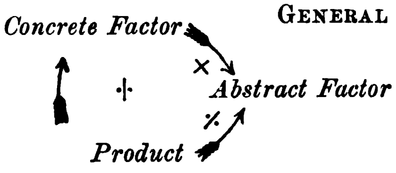
1. The *Concrete Factor* is the weight, measure, or value of *one* considered whole unit or quantity.

2. The *Abstract Factor* is the integral or fractional *number* of considered units or quantities.

REM.—⅛ of a pound, in the above solution, is, theoretically, as much a number of pounds, as 35 bu. in the solution, **54,** is a number of bushels.

3. The *Product* is the weight, measure, or value of *more* or *less* than the considered whole unit or quantity.

GENERAL FORMULA.



Explanation.—If any two of the three terms of this formula are given, perform the operation indicated by the sign between them, and the result will be the third term.

EXAMPLES FOR PRACTICE.

115.—1. What is the cost of $\frac{3}{4}$ of a pound of tea at \$$\frac{7}{8}$ per pound?

SOLUTION.

\$$\frac{7}{8} \times \frac{3}{4} =$ \$$\frac{21}{32}$.

Explanation.—The cost of *one* whole pound, \$$\frac{7}{8}$ (concrete factor), and the fractional number of pounds, $\frac{3}{4}$ (abstract factor), are given. Hence, multiply these two factors, obtaining \$$\frac{21}{32}$ as the cost of $\frac{3}{4}$ of a pound, or the cost of less than one whole pound (product).

2. At \$$\frac{3}{4}$ per gallon, how much molasses can be bought for \$$\frac{1}{3}$?

SOLUTION.

\$$\frac{1}{3} \div$ \$$\frac{3}{4} = \frac{4}{9}$.

Explanation.—The cost of *one* whole gallon, \$$\frac{3}{4}$ (concrete factor), and the cost of less than one whole gallon, \$$\frac{1}{3}$ (product), are given. Hence, divide the product (\$$\frac{1}{3}$) by its concrete factor (\$$\frac{3}{4}$), obtaining $\frac{4}{9}$ gal. as the fractional number of gallons, or the abstract factor.

3. If $\frac{2}{3}$ of a bushel of corn can be bought for \$$\frac{5}{8}$, what is the price per bushel?

SOLUTION.

\$$\frac{5}{8} \div \frac{2}{3} =$ \$$\frac{15}{16}$.

Explanation.—The fractional number of bushels, $\frac{2}{3}$ bu. (abstract factor), and the cost of less than one whole bushel, \$$\frac{5}{8}$ (product), are given. Hence, divide the product (\$$\frac{5}{8}$) by its abstract factor ($\frac{2}{3}$ bu.), obtaining \$$\frac{15}{16}$ as the cost of one whole bushel, or the concrete factor.

4. If $\frac{5}{6}$ of a cord of wood contains $106\frac{2}{3}$ cubic feet, how many cubic feet will $\frac{2}{3}$ of a cord contain?

SOLUTION.

$106\frac{2}{3} \div \frac{5}{6} = 128$ cu. ft.
$128 \times \frac{2}{3} = 85\frac{1}{3}$ cu. ft.

Explanation.—The fractional number of cords, $\frac{5}{6}$ of a cord (abstract factor), and the measure of less than one whole cord, $106\frac{2}{3}$ cu. ft. (product), are given. Hence, divide the product ($106\frac{2}{3}$ cu. ft.) by its abstract factor ($\frac{5}{6}$ cd.), obtaining 128 cu. ft. as the measure of one whole cord, or the concrete factor. Next multiply the obtained concrete factor (128 cu. ft.) by the fractional number of cords, $\frac{2}{3}$ cd. (the abstract factor of the required answer), obtaining $85\frac{1}{3}$ cu. ft. as the measure of less than one whole cord, or the product.

Note.—If a problem contains two similar terms, as in Ex. 4, where two abstract factors ($\frac{5}{6}$ of a cord and $\frac{2}{3}$ of a cord) are given, the answer will usually require two operations, that is, as many operations as there are similar terms.

5. At $\$\frac{5}{6}$ per yard, how many yards of cloth can be bought for $\$7\frac{2}{3}$?

6. What is the cost of $\frac{4}{5}$ of a ton of iron at $\$117\frac{3}{8}$ per ton?

7. At $\$\frac{1}{8}$ per lb., how many pounds of sugar can be bought for $\$\frac{15}{16}$?

8. I bought $\frac{4}{7}$ lb. coffee for $\$\frac{1}{7}$. What was the price per pound?

9. If a furnace consume $\frac{15}{16}$ of a ton of coal per month, in what time will it consume $3\frac{4}{5}$ tons?

10. What amount of honey worth $30\frac{5}{8}$ cents per pound can be bought for $25\frac{3}{4}$ cents?

11. How many vegetable beds of $\frac{3}{16}$ of an acre each can be obtained from $1\frac{1}{2}$ acres of land?

12. I owe $\$29\frac{7}{8}$ and wish to pay the debt with oats worth $\$\frac{3}{8}$ per bushel. How many bushels should I deliver?

13. If a pupil can write $\frac{2}{3}$ of a composition in $\frac{3}{5}$ of an hour, in what time will he be able to write his entire composition?

14. If a cow eats $\frac{4}{7}$ of a bushel of bran in one day, in what time will she eat $4\frac{5}{7}$ bushels?

15. If a ship can sail $268\frac{13}{14}$ miles in one day, how far can she sail in $3\frac{5}{8}$ days?

16. At the rate of $3\frac{12}{13}$ miles per hour, how long will a man require to walk $15\frac{1}{3}$ miles?

17. A piece of muslin containing $40\frac{3}{4}$ yards, cost $\$2.03\frac{3}{4}$. At what price per yard was it bought?

18. If $\frac{2}{3}$ bu. wheat cost $\$\frac{5}{8}$, what is the cost of $\frac{3}{4}$ bu. (Ex. 4)?

19. If I pay $8\frac{1}{4}$ cents for $\frac{3}{4}$ of a pound of sugar, how much of the same kind of sugar can I buy for $60\frac{1}{2}$ cents?

20. If $3\frac{3}{5}$ tons of hay are worth $\$62\frac{1}{20}$, how much hay will be worth $\$45\frac{5}{8}$?

21. If 15 men can do a piece of work in $\frac{5}{6}$ of a day, how long will it take one man to do it (Ex. 17, **56**)?

22. If 5 men can do a piece of work in $3\frac{1}{4}$ days, how long will it take 7 men to do it?

116. In comparison of numbers, the word *of* immediately follows the abstract factor and immediately precedes the concrete factor, making these two factors of any given or required product easy of identification.

NOTE.—The phrases *more than* and *less than* also precede the concrete factor, but follow fractional parts which must be respectively added to or subtracted from 1 to find the abstract factor of the given or required product.

EXAMPLES FOR PRACTICE.

1. ⅔ of 7⅘ yards of silk equal how many yards?

SOLUTION.
7⅘ × ⅔ = 5⅕ yd.

EXPLANATION.—The measure of one whole quantity, 7⅘ yd. (concrete factor), and the fractional number of quantities, ⅔ (abstract factor), are given. Hence, multiply these two factors, obtaining 5⅕ yd. as the measure of ⅔ of the whole quantity, or the measure of less than one whole quantity (product).

REM.—In this example, *of* immediately follows the abstract factor (⅔) and immediately precedes the concrete factor (7⅘ yd.).

2. 2⅓ lb. butter are ⅚ of how many pounds?

SOLUTION.
2⅓ lb. ÷ ⅚ = 2⅘ lb.

EXPLANATION.—The weight of less than the considered whole quantity, 2⅓ lb. (product), and the fractional number of quantities, ⅚ (abstract factor), are given. Divide the product (2⅓ lb.) by its abstract factor (⅚), obtaining 2⅘ lb. as the weight of the considered whole quantity, or the concrete factor.

REM.—In Ex. 2, *of* immediately follows the abstract factor (⅚) and immediately precedes the concrete factor (how many pounds, or the required whole quantity).

3. What part of $1800 are $450?

SOLUTION.
$1800 ÷ $450 = $\frac{450}{1800}$ = ¼.

EXPLANATION.—The value of the considered whole quantity, $1800 (concrete factor), and the value of less than the considered whole quantity, $450 (product), are given. Divide the product ($450) by its concrete factor ($1800), using the fractional form to denote the unexecuted division (Rem., **85**), obtaining ¼ as the considered fractional number of quantities (the abstract factor).

REM.—In Ex. 3, *of* immediately follows the required abstract factor (what part) and immediately precedes the concrete factor ($1800).

4. 350 miles are ⅔ more than what distance?

SOLUTION.
350 ÷ $\frac{5}{3}$ = 210 mi.

EXPLANATION.—The measure of more than the considered whole distance, 350 mi. (product), and the fractional number of considered distances, $\frac{5}{3}$ (abstract factor), are given. Divide the product (350 mi.) by its abstract factor ($\frac{5}{3}$), obtaining 210 miles as the measure of the considered whole distance (required concrete factor).

REM.—In Ex. 4, *more than* immediately follows the given component of the abstract factor, ⅔ (Note, **116**), and immediately precedes the required concrete factor (what distance).

116

5. A merchant drew $1800 from bank, which was $\frac{3}{7}$ of his deposit. What was his deposit ?

6. What part of 150 acres of land are 25 acres?

7. 35 gallons of cider were sold from a barrel containing 42 gallons. What part of the barrel was sold ?

8. If $\frac{5}{8}$ of a farm is worth $6000, what is the value of the whole farm ?

9. How many yards of cloth in $\frac{5}{12}$ of 1524 yards ?

10. I paid $1800 for a house, and afterwards sold it for $\frac{11}{18}$ of its cost. What did I receive for the house ?

11. A farmer sold 75 acres from a farm of 320 acres. What part of the farm did he sell ?

12. A man had $12000, and spent $\frac{3}{8}$ of what he had in building a house. What was the cost of the house?

13. In a certain school are 375 pupils, of whom 150 are boys and the remainder girls. What part of the school are girls ?

14. A gentleman bought $18500 worth of real estate, paid $\frac{18}{25}$ of the purchase money in cash, and gave his note for the remainder. What was the face of the note ?

15. A contributed $3500 to the capital of a firm, B $7000, and C $4500. What part of the total capital did each contribute ?

16. A firm has $\frac{1}{3}$ of its capital in merchandise, $\frac{3}{8}$ in real estate, and the remainder, or $1680, in cash. What is the capital of the firm ?

17. A farm consists of 35 acres in wheat, 48 acres in corn, 32 acres in oats, 30 acres in grass, 25 acres in woodland, and the remaining 5 acres in orchard, etc. What part of the farm is in grass ?

REVIEW OF COMMON FRACTIONS.

117.—**1.** A farm wagon carrying $6\frac{5}{7}$ barrels of corn at a load, delivered all the corn grown in a certain field in 58 loads. What was the total yield of that field ?

2. I bought $2\frac{1}{4}$ acres of land and divided it into building lots of $\frac{3}{8}$ of an acre each. How many building lots did I obtain ?

3. A vegetable garden contains $132\frac{3}{28}$ square rods in potatoes, $178\frac{5}{21}$ square rods in cabbage plants, $47\frac{19}{36}$ square rods in onions, and $415\frac{11}{12}$ square rods in other vegetables. What is the area of the garden ?

4. A grocer bought 4 bags of coffee, the gross weight of each being respectively $127\frac{5}{8}$, $132\frac{11}{16}$, $129\frac{1}{8}$, and $136\frac{7}{8}$ pounds. Allowing $1\frac{1}{8}$ pounds for the weight of each bag, what was the total net weight of the coffee?

5. A merchant had $132\frac{5}{8}$ pounds of coffee, and afterwards bought $75\frac{3}{4}$ pounds at one time and $218\frac{11}{16}$ pounds at another. Of this coffee he sold at one time $47\frac{3}{8}$ pounds, and at another $125\frac{1}{2}$ pounds. How much coffee had he then remaining?

6. A contractor agreed to dig $2583\frac{5}{8}$ cubic yards of earth. How much of his contract remained after digging $1625\frac{7}{12}$ cubic yards?

7. The average yield of a field of corn was $49\frac{5}{8}$ bushels per acre. What was the total yield if the field contained 45 acres?

8. A merchant sold 80 pounds of butter at $28\frac{3}{4}$ cents per pound, 58 dozen eggs at $16\frac{5}{8}$ cents per dozen, and $28\frac{3}{4}$ gallons of milk at 22 cents per gallon. What was the total amount of sales?

9. A man exchanged with a grocer $\frac{15}{16}$ of a bushel of corn worth 64 cents per bushel, for $6\frac{3}{8}$ pounds of bacon. What did the grocer charge per pound for the bacon?

10. A farmer sold to a merchant $460\frac{3}{4}$ pounds of pork at $5\frac{1}{2}$ cents per pound, and 50 bushels of corn at $65\frac{1}{2}$ cents per bushel, receiving in part payment 30 yards of calico at $5\frac{3}{4}$ cents per yard, 25 yards of sheeting at $32\frac{5}{8}$ cents per yard, $9\frac{5}{8}$ yards of alpaca at 25 cents per yard, and 75 pounds of sugar at $6\frac{1}{2}$ cents per pound. What is the balance due the farmer?

11. A merchant bought 5 hams, weighing respectively $12\frac{1}{2}$, $18\frac{5}{8}$, $15\frac{3}{4}$, $14\frac{15}{16}$, and 16 pounds, at $11\frac{1}{2}$ cents per pound. What was the total cost?

12. A dealer bought 46 bales of cotton, averaging 435 pounds per bale, at $11\frac{13}{32}$ cents per pound, and sold the entire quantity at $12\frac{3}{4}$ cents per pound. If his expenses for freight, insurance, commission, etc., amounted to $95.30, what was his total net profit?

13. I bought 9 bags of wheat weighing respectively $123\frac{1}{2}$, $119\frac{3}{4}$, $125\frac{5}{8}$, $120\frac{7}{8}$, $117\frac{15}{16}$, $119\frac{5}{16}$, $122\frac{1}{4}$, $120\frac{1}{2}$, and $119\frac{1}{4}$ pounds, at $$1\frac{3}{8}$ per bushel of 60 pounds. Allowing 1 pound for the weight of each bag, what was the total cost?

14. A pedestrian traveled $27\frac{3}{8}$ miles on Monday, $31\frac{5}{8}$ miles on Tuesday, $35\frac{5}{8}$ miles on Wednesday, $24\frac{5}{12}$ miles on Thursday, $36\frac{1}{2}$ miles on Friday, and $24\frac{3}{8}$ miles on Saturday. What was the average distance traveled per day?

15. I sold $\frac{5}{8}$ of a piece of goods, containing $39\frac{3}{4}$ yards. How many yards remained in the piece?

16. A merchant sold $18\frac{3}{4}$ yards of muslin from a piece containing $38\frac{7}{8}$ yards. What fractional part of the piece did he sell?

17. $21\frac{3}{8}$ gallons were sold from a barrel of molasses containing $43\frac{7}{32}$ gallons. How many gallons remained in the barrel?

18. How many acres does a farm contain, if $\frac{3}{8}$ of it is in grass, $\frac{5}{16}$ in corn, $\frac{1}{4}$ in wheat, and the remaining 16 acres in oats?

19. A, B, and C engage in partnership, A investing $3000, B $4000, and C $5000. What part of the total capital did each invest?

20. A gentleman invested $\frac{3}{4}$ of his money in city property, $\frac{1}{2}$ of the remainder in a farm, $\frac{3}{4}$ of what still remained in improvements upon the farm, and then had $960 left. What did he have at first?

REM.—$(1-\frac{3}{4})$ of $(1-\frac{1}{2})$ of $(1-\frac{3}{4})$ = abstract factor, denoting fractional part of all the money remaining.

21. A man owed $4200 and paid $1575. What part of his original debt did he still owe?

22. A can do a piece of work in 15 days and B in 18 days. In what time can it be done if both work together?

23. A merchant sold 468 barrels of a consignment of potatoes and had 312 barrels remaining. What part of the consignment did he sell?

24. If a miller charges $\frac{1}{8}$ of a grist for toll, how much corn must I carry to his mill to entitle me to receive 203 pounds of corn meal?

25. If coffee loses $\frac{1}{15}$ of its weight in roasting, how much green coffee will be required to make 252 pounds of roasted coffee?

26. A house and lot cost $7200, and the house cost $\frac{2}{3}$ more than the lot. What was the cost of each?

27. The cost of a carriage and a span of horses is $1260, and the cost of the horses is $\frac{2}{7}$ less than the cost of the carriage. What is the cost of each?

28. A by working alone can finish a certain job of work in 9 days, and A and B by working together can finish the same job in 4 days. In what time can B alone finish the job?

29. A can complete a job of work in 4 days, B in 6 days, and C in 8 days. In what time can the job be completed if all work together?

30. A man bought a farm, paid $\frac{3}{8}$ of the purchase money in cash, and in payment of the balance gave his note secured by mortgage for $3400. How much did he pay in cash?

DECIMAL FRACTIONS.

118. A **Decimal Fraction** expresses one or more of the equal decimal parts of a unit.

Rem.—The term *decimal* is derived from the Latin *decem*, signifying *ten;* and is applied to this kind of fractions because the denominator is either 10, or the product of two or more tens, as 100, 1000, 10000, etc.

119. The **Decimal Point** is a period (.) written to the left of the numerator of a decimal to indicate its denominator.

Rem.—The denominator of a decimal is indicated by writing the decimal point as many places to the left of the right-hand figure of the numerator as there are ciphers in the denominator. Thus, \$$\frac{3}{10}$, \$$\frac{15}{100}$, and \$$\frac{95}{1000}$, by omitting their denominators, may be written \$.3, \$.15, and \$.095.

120. The **Decimal Order** or **Decimal Value** of a figure is its position with respect to the decimal point.

Rem.—The *nearer* any figure of a decimal fraction is to its decimal point, the *higher* its value; and the *further* it is removed from the decimal point, the *less* its value.

121. The **Notation of Decimals** is simply an extension on a descending scale of the system of notation of integers, in which the value of any order is ten times the value of the next order at the right, and one-tenth of the next order at the left, as is shown in the following

Decimal Notation Table.

Etc., etc.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Etc., etc.
	9	7	5	3	9	1	8	3	9	2	.	5	1	2	4	6	7	9	1	5	
	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	

Integral Orders. *Decimal Orders.*

EXPLANATION.—It will be noticed in the preceding table that, commencing at the decimal point, the integral orders are enumerated from the right to the left, and the decimal orders from the left to the right; that the first integral order at the left of the units' place is *tens*, and the first decimal order at the right of the units' place is *tenths*; that the second integral order at the left of the units' place is *hundreds*, and the second decimal order at the right of the units' place is *hundredths*, etc., etc.; and, generally, that the name of any decimal order corresponds with the name of that integral order which is equally distant from the units' place.

122. A **Complex Decimal** is one which has a common fraction at the right of its lowest order; as $\$.5\frac{2}{3}$; $\$.003\frac{4}{7}$.

123. A **Simple Decimal** is one which has no common fraction at the right of its lowest order; as \$.25; \$.0098.

REM.—Complex Decimals are said to be *terminate* when they can be reduced to simple decimals, and *interminate* when they cannot be so reduced.

124. A **Mixed Decimal** is one which is composed of an integer and a decimal fraction; as 5.18 lb.; 98.005 bu.

125. A **Decimal Unit** is *one* of the equal decimal parts into which a whole unit is divided.

REM.—The *name* of the decimal unit, or denominator of the decimal, is the same as the name or order of its right-hand figure. Thus, in the decimal \$.018, 1 *thousandth of a dollar* is the decimal unit.

126. PRINCIPLES.—1. *The denominator of a decimal is 1 followed by as many ciphers as it has places to the right of the decimal point.*

Thus, in \$.0015, the denominator is 1 followed by four ciphers (10000) or *ten-thousandths;* and the decimal is read 15 ten-thousandths dollars.

2. *Annexing ciphers to, or omitting them from, the right of a decimal does not change its value.*

To annex a cipher to a decimal also annexes a cipher to its denominator (Prin. 1), and hence multiplies both numerator and denominator by 10, and does not change the value of the fraction (Prin. 3, **87**); and to omit a cipher from the right of a decimal also omits a cipher from its denominator, and therefore divides both numerator and denominator by 10.

3. *Prefixing a cipher to a decimal and moving the decimal*

point to the left of the cipher prefixed, divides the decimal by 10.

To prefix a cipher to a decimal does not change its numerator, but annexes another cipher to its denominator (Prin. 1), which is equivalent to multiplying the denominator by 10, and dividing the fraction by 10 (Prin. 2, **87**).

4. *Moving the decimal point one place to the right multiplies the decimal by 10; two places to the right multiplies by 100; etc.*

To move the decimal point one place to the right does not change the number of fractional units; but makes the numerator contain one decimal place, and the denominator one cipher, less than before (Prin. 1); which is equivalent to dividing the denominator by 10, and multiplying the fraction by 10 (Prin. 1, **87**).

5. *Moving the decimal point one place to the left divides the decimal by 10; two places to the left divides by 100; etc.*

To move the decimal point one place to the left does not change the number of fractional units; but makes the numerator contain one decimal place, and the denominator one cipher, more than before (Prin. 1); which is equivalent to multiplying the denominator by 10, and dividing the fraction by 10 (Prin. 2, **87**).

NUMERATION OF DECIMALS.

ILLUSTRATIVE EXERCISE.

127. Express, orally, the decimal .00378 of a pound.

Explanation.—First read the numerator as if it were an integer (three hundred and seventy-eight); then, applying Prin. 1, **126**, name the denominator, which is thus found to be 1 followed by five ciphers (100000), or hundred-thousandths.

Rule.—*Read the decimal as if it were an integer; and then state the name of its right-hand order.*

EXAMPLES FOR PRACTICE.

Express orally the following decimals:

1. .3 oz.
2. .03 lb.
3. .003 bu.
4. .48 yd.
5. .048 gal.
6. .0048 bbl.
7. .419 ft.
8. .0419 qt.
9. .00419 yr.

10. .6783 bu.
11. .06783 bbl.
12. .006783 gal.
13. .0006783 yr.
14. $.9182734.
15. $38.02187364.
16. $593.0070080.
17. $8275.10090008.
18. $4.0003180029.
19. 17.00000298\frac{3}{4}$.
20. 6.002009948\frac{1}{6}$.
21. 1.2090008107\frac{2}{3}$.

NOTATION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

128. Write decimally ninety-five millionths.

EXPLANATION.—First write the decimal point and 4 ciphers (thus, .0000), because the denominator *millionths* requires 6 decimal places (Prin. 1, **126**), and these 4 prefixed ciphers denote what the numerator 95 (containing 2 places) lacks of having the required number of 6 places; then write the numerator, obtaining .000095.

RULE.—*Write the numerator as if it were an integer, and so locate the decimal point that the right-hand figure of the numerator will occupy the decimal order denoted by the denominator.*

NOTE.—The learner should accustom himself to writing the decimal point and prefixed ciphers *before* writing the numerator, as shown in the Illustrative Example. The number of ciphers to prefix equals the number of ciphers in the denominator diminished by the number of orders in the numerator.

Write decimally the following:

1. 4 tenths.
2. 7 hundredths.
3. 13 hundredths.
4. 429 thousandths.
5. 35 thousandths.
6. 6 thousandths.
7. 24 hundredths.
8. 24 thousandths.
9. 5 hundredths.

10. 8163 ten-thousandths.
11. 75 ten-thousandths.
12. 248 hundred-thousandths.
13. 843 ten-thousandths.
14. 1235 hundred-thousandths.
15. 78 hundred-thousandths.
16. 627 ten-thousandths.
17. 7483 millionths.

18. Five, and three hundred eighty-two ten-thousandths.
19. Eight, and eight hundred seventy-one hundred-thousandths.
20. Nine, and five hundred sixty-three thousandths.
21. Seventy-nine, and two thousand four hundred thirty-seven hundred-thousandths.

22. Eight hundred ninety-six, and fifty-one thousand three hundred twenty-seven millionths.

23. One hundred seventy-five thousand seven hundred fifty-four hundred-millionths.

24. Seven hundred forty-eight, and one million twenty-three thousand four hundred sixty-nine billionths.

REDUCTION OF DECIMALS.

129. Reduction of common fractions to decimals.

ILLUSTRATIVE EXAMPLE.

Reduce $\$\frac{3}{25}$ to a decimal.

SOLUTION.

```
25 ) $3.00 ( $.12
     2 5
     ---
       50
       50
       --
```

EXPLANATION.—$\$\frac{3}{25}$ expresses an unexecuted division, $3 being the dividend and 25 the divisor (**85**). To annex ciphers to the dividend $3 multiplies it by 10 for every cipher annexed ; hence, if the numerator with two ciphers annexed ($300) be divided by the denominator (25), the quotient will be 100 times the true quotient (Prin. 1, **35**). Rectify this error by cutting off two places from the right of the quotient, which is equivalent to dividing the quotient by 100 (**42**).

RULE.—*Annex ciphers to the numerator, divide by the denominator, and point off from the right of the quotient as many decimal places as there have been ciphers annexed.*

NOTE 1.—Students should know the decimal equivalents of halves, fourths, and eighths, without reducing. Thus, $\frac{1}{2} = .5$; $\frac{1}{4} = .25$; $\frac{3}{4} = .75$; $\frac{1}{8} = .125$; $\frac{3}{8} = .375$; $\frac{5}{8} = .625$; and $\frac{7}{8} = .875$.

EXAMPLES FOR PRACTICE.

Reduce the following to equivalent decimals :

1. $\frac{3}{8}$ lb.
2. $\frac{7}{8}$ oz.
3. $\frac{3}{16}$ yr.
4. $\frac{7}{16}$ bbl.
5. $\frac{11}{32}$ yd.
6. $\frac{19}{32}$ qt.
7. $\frac{51}{64}$ gal.
8. $\frac{37}{64}$ bu.
9. $\frac{1}{16}$ ft.
10. $\frac{3}{64}$ yd.
11. $\frac{2}{125}$ lb.
12. $\frac{1}{128}$ yr.
13. $9\frac{3}{4}$ gal.
14. $25\frac{5}{16}$ miles.
15. $17\frac{5}{64}$ yd.
16. $4\frac{19}{256}$ bbl.

NOTE 2.—To avoid long division, separate the fraction into factors, the decimal equivalent of one of which is known without reduction (Note 1), and divide this decimal equivalent by the remaining factor or factors. Thus, $\frac{5}{32} = \frac{1}{4}$ of $\frac{5}{8}$; $\frac{5}{8} = .625$; hence, .625+4, or .15625, must be the decimal equivalent of $\frac{1}{4}$ of $\frac{5}{8}$, or $\frac{5}{32}$.

Reduce the following to decimals in accordance with Note 2:

17. $\frac{5}{16}$ gal.
18. $\frac{7}{64}$ bbl.
19. $\frac{5}{32}$ yr.
20. $\frac{17}{64}$ lb.
21. $\frac{19}{32}$ yd.
22. $\frac{25}{64}$ bu.
23. $18\frac{3}{16}$ rods.
24. $25\frac{5}{64}$ tons.

NOTE 3.—In business, even with terminate decimals, it is not customary to extend the division until there is no remainder. The business rule is: Annex to the numerator as many ciphers as there are decimal places required in the result, and, after the last cipher has been brought down, discard the remainder if less than half the divisor, or increase the last quotient figure by 1 if the remainder is half the divisor or more.

Applying Note 3, reduce the following to decimals of

2 dec. places.	*3 dec. places.*	*4 dec. places.*	*5 dec. places.*
25. $19\frac{2}{3}$ oz.	**27.** $75\frac{7}{12}$ yr.	**29.** $278\frac{3}{64}$ bu.	**31.** $8\frac{7}{128}$ yr.
26. $415\frac{5}{8}$ lb.	**28.** $91\frac{9}{32}$ yd.	**30.** $67\frac{5}{9}$ bbl.	**32.** $15\frac{6}{11}$ miles.

NOTE 4.—To expand a complex decimal to a given number of decimal places, annex to the numerator of the terminal common fraction a cipher for each lacking decimal place, and then divide by the denominator.

Expand the following complex decimals to

5 dec. places.	*6 dec. places.*	*7 dec. places.*
33. $46.342\frac{7}{8}$ bu.	**35.** $812.9312\frac{13}{16}$ gal.	**37.** $2.91\frac{11}{64}$ tons.
34. $75.19\frac{17}{32}$ lb.	**36.** $73.41\frac{5}{6}$ yd.	**38.** $18.273\frac{2}{3}$ miles.

130. Reduction of decimals to common fractions.

ILLUSTRATIVE EXAMPLE.

Reduce \$.625 and \$$.07\frac{2}{3}$ to common fractions.

FIRST SOLUTION.

$$\$.625 = \frac{\$625 \div 125}{1000 \div 125} = \frac{\$5}{8}$$

SECOND SOLUTION.

$$\$.07\tfrac{2}{3} = \frac{\$7\frac{2}{3} \times 3}{100 \times 3} = \frac{\$23}{300}$$

FIRST SOLUTION.—Change the decimal to the form of a common fraction by writing its denominator, obtaining \$$\frac{625}{1000}$; then reduce to lowest terms by **90**, obtaining \$$\frac{5}{8}$.

SECOND SOLUTION.—Supplying the denominator produces the complex fraction, $\frac{\$7\frac{2}{3}}{100}$; multiplying both terms by the denominator of the terminal common fraction (3) produces the equivalent simple fraction \$$\frac{23}{300}$ (Prin. 3, **87**).

RULE.—*Omit the decimal point and prefixed ciphers, write the denominator underneath, and reduce the resulting fraction to lowest terms.*

EXAMPLES FOR PRACTICE.

Reduce the following decimals to common fractions:

1. .25 bbl.
2. .875 lb.
3. .375 gal.
4. .075 yd.
5. .00625 oz.
6. 5.00125 yr.
7. 18.03125 bu.
8. 5.064 ft.
9. 3.00128 qt.
10. 7.31625 yd.
11. 5.34375 ft.
12. 19.06¼ bbl.
13. 25.16⅔ oz.
14. 58.02⅔ lb.
15. .008⅓ bu.
16. .0⅚ bbl.

ADDITION OF DECIMALS.

ILLUSTRATIVE EXAMPLES.

131.—1. Add 7.48 yr., 23.0415 yr., .032 yr., and 6.3138 yr.

SOLUTION.

$$\begin{array}{r} 7.48 \\ 23.0415 \\ .032 \\ 6.3138 \\ \hline 36.8673 \end{array}$$

EXPLANATION.—Only like orders can be added (Prin. 1, **15**), therefore arrange the addends so that like decimal orders shall fall in the same column. Add as with whole numbers. Since the sum of the tenths' column must be tenths (Prin. 2, **15**), place the decimal point in the sum at the left of 8 tenths to denote its decimal order.

2. Find the sum of $5.71⅝, $8.4¼, and $27.009⅚ to 3 dec. places.

SOLUTION.

$$\begin{array}{r} \$5.7163- \\ 8.425 \\ 27.0098+ \\ \hline \$41.1511 \\ \$41.151+ \end{array}$$

EXPLANATION.—Arrange the decimals, add, and point off as in Ex. 1; first expanding the complex decimals by Notes 3 and 4, **129**, to 4 decimal places, or one more decimal place than is required in the sum. The extra decimal place in the addends is intended to insure accuracy in the three decimal places required in the sum.

RULE.—*Arrange the addends so that their decimal points shall fall in a column. Add as with whole numbers; and place the decimal point in the result directly beneath the decimal points of the addends.*

EXAMPLES FOR PRACTICE.

What is the sum of

1. 5.75 bu., 28.415 bu., 19.5 bu., 7.01298 bu., 328.52 bu.?
2. 528.09 lb., 6.0047 lb., 98.193 lb., 514.6 lb., 2.015 lb.?

3. 8153.45 yd., 32.0028 yd., 237.483 yd., 5.00165 yd., .06 yd.?

4. $25.82\frac{3}{4}$ yr., $75.219\frac{7}{8}$ yr., $8.4\frac{3}{5}$ yr., $16.52\frac{9}{16}$ yr., $23.81\frac{11}{32}$ yr., $1629.4\frac{3}{8}$ yr., and $16.219\frac{1}{4}$ yr., as a simple decimal?

5. $29.418\frac{3}{16}$ rods, $193.26\frac{3}{25}$ rods, $4.21\frac{23}{32}$ rods, $678.5\frac{5}{16}$ rods, .25 rods, $625.48\frac{1}{3}$ rods, .4 rods, and 19 rods, as a simple decimal?

6. $28.1\frac{3}{500}$ miles, $75\frac{1}{16}$ miles, $428.3\frac{1}{25}$ miles, 58 miles, $6138.72\frac{3}{32}$ miles, $65.37\frac{1}{20}$ miles, .19 miles, 68 miles, and $2.43\frac{3}{8}$ miles as a simple decimal?

7. $41.92\frac{5}{8}$ cords, $7.2\frac{15}{16}$ cords, $218.359\frac{12}{500}$ cords, 75 cords, 7.5 cords, $81.21\frac{21}{64}$ cords, .375 cords, and $4.17\frac{25}{32}$ cords, as a simple decimal?

8. 4.318 yr., $473.1\frac{3}{16}$ yr., $85.48\frac{2}{3}$ yr., 125.19 yr., $76.34\frac{13}{32}$ yr., 50 yr., and $5\frac{3}{7}$ yr., to 3 decimal places (See Ill. Ex. 2)?

9. $\frac{7}{16}$ bu., $\frac{5}{8}$ bu., $.02\frac{51}{64}$ bu., $.31\frac{11}{32}$ bu., to 4 decimal places?

10. $\$.58\frac{3}{8}$, $\$7.39\frac{5}{8}$, $\$250.81\frac{4}{5}$, $\$12\frac{2}{3}$, to 3 decimal places?

11. Nineteen, and forty-nine ten-thousandths acres; seventy-three, and one hundred fifty-six millionths acres; thirty-four, and eight hundred-thousandths acres; and three thousand nine hundred thirty-seven, and two hundred ninety-five ten-thousandths acres?

12. Three hundred four, and thirty-two thousandths miles; eighteen, and two thousand seventy-five hundred-thousandths miles; three, and fifteen ten-thousandths miles; and five thousand eighty-two, and one thousand nineteen hundred-thousandths miles.

SUBTRACTION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

132. Subtract $219.13\frac{2}{3}$ yd. from $562.7\frac{3}{16}$ yd. to 3 dec. places.

SOLUTION.

$$\begin{array}{l} 562.7188 - \\ \underline{219.1367 -} \\ 343.5821 \\ 343.582 + \text{yd.} \end{array}$$

EXPLANATION.—Expand the minuend and subtrahend as directed in Explanation, Ill. Ex. 2, **131**, for addends. Arrange the numbers so that the orders of the subtrahend shall fall under like orders of the minuend. As the difference of the tenths' orders must also be tenths (Prin. 2, **19**), place the decimal point in the remainder at the left of 5 tenths to denote its decimal order.

RULE.—*Arrange the decimals so that the decimal point of the subtrahend shall fall directly under the decimal point of the minuend. Subtract as with whole numbers; and place the* *decimal point in the remainder directly beneath those of the minuend and subtrahend.*

EXAMPLES FOR PRACTICE.

Find the difference between

1. 19.6205 lb. and 25.3 lb.
2. 12.283 yr. and 3.4278 yr.
3. 65.09 ft. and 23.17268 ft.
4. 9.0032 yd. and .35 yd.
5. 61.0026 bu. and 52.35 bu.
6. 418.37 oz. and 28.4137 oz.
7. 15 bbl. and .63 bbl.
8. .49 gal. and .2873 gal.
9. 67.84$\frac{1}{4}$ rods and 32.9$\frac{7}{8}$ rods, as a simple decimal.
10. 235.04$\frac{3}{16}$ acres and 109.2$\frac{3}{8}$ acres, as a simple decimal.
11. 72.0053$\frac{1}{16}$ miles and 82.19$\frac{3}{32}$ miles, as a simple decimal.
12. 42.07$\frac{3}{16}$ days and 16.5$\frac{7}{64}$ days, to 4 decimal places.
13. 8.19$\frac{3}{64}$ cords and 5.73$\frac{3}{128}$ cords, to 3 decimal places.
14. \$92.8$\frac{2}{3}$ and \$63$\frac{5}{6}$, to 3 decimal places.
15. From nine hundred and forty-eight years take three hundred and seventy-five ten-thousandths of a year.
16. From eight hundred fifty-four, and one hundred ninety-six hundred-thousandths fathoms take three hundred sixty-two, and one thousand two hundred thirty-nine millionths fathoms.

MULTIPLICATION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

133. Multiply 63.42 hours by .758.

SOLUTION.

$$\begin{array}{r} 63.42 \\ \underline{.758} \\ 50736 \\ 31710 \\ \underline{44394} \\ 48.07236 \end{array}$$

EXPLANATION. — 63.42 = $\frac{6342}{100}$, and .758 = $\frac{758}{1000}$. Multiply the numerator of one fraction (6342) by the numerator of the other fraction (758), obtaining 4807236 as the numerator of the required product (**105**). Since the denominator of $\frac{6342}{100}$ is 1 followed by *two* ciphers, and the denominator of $\frac{758}{1000}$ is 1 followed by *three* ciphers, the product of these denominators (100000), or 1 followed by 2 + 3, or 5 ciphers, must be the denominator of the required product (**105**); and to denote this denominator, point off 5 decimal places (Rem., **119**).

RULE. — *Multiply as with whole numbers; and from the right of the product point off as many decimal places as are contained in both factors.*

Note 1.—If the product contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

Note 2.—Terminate Complex Decimals (Rem., **123**) are usually expanded to simple decimals before applying the rule; and Interminate Complex Decimals are multiplied as they stand by Rule, **106** or **107.**

Note 3.—Decimals are multiplied by 10, 100, 1000, etc., by moving the decimal point of the multiplicand as many places to the right as there are ciphers in the multiplier.

EXAMPLES FOR PRACTICE.

Multiply

1. 619.358 yd. by 24.

2. 57.625 ft. by 2.75.

3. 82.6125 bu. by .418.

4. 207.045 bbl. by .0086.

5. 34.01832 yr. by .027.

6. .6832 lb. by 28.3.

7. .529 tons by .00071.

8. .00036 ft. by .0046.

9. $.0000915 by .0063.

Multiply

10. $\$.215\frac{3}{4}$ by .75.

11. $\$1.729\frac{5}{16}$ by $2.5\frac{4}{5}$.

12. $\$68.25\frac{3}{25}$ by $.9\frac{7}{32}$.

13. $\$42.12\frac{2}{3}$ by .032. **(106.)**

14. $\$65.938\frac{5}{8}$ by .026.

15. $\$417.52\frac{3}{7}$ by $.07\frac{3}{5}$. **(107.)**

16. $2.5873 by 100. (Note 3.)

17. $.58273 by 1000.

18. $3.289173 by 500. (100 × 5)

DIVISION OF DECIMALS.

ILLUSTRATIVE EXAMPLE.

134. Divide $13.244 by 4.73.

SOLUTION.

$$\begin{array}{r} 4.73\ )\ \$13.244\ (\ \$2.8 \\ \underline{9\ 46} \\ 3\ 784 \\ \underline{3\ 784} \end{array}$$

Explanation. — 473 *hundredths* are contained in 1324 *hundredths* 2 (whole) times; and if the first quotient figure (2) expresses units, the second quotient figure (8) must express the next lower order to units, or tenths. Hence, place the decimal point in the quotient at the left of 8, to make it denote tenths.

Rem.—Since the dividend is a product and the divisor and quotient are its factors (Rem. 1, **34**), the dividend of the Ill. Ex. ($13.244) must contain as many decimal places as the divisor and quotient together (Explanation, **133**). Hence, if the divisor contains only 2 of the 3 decimal places of the dividend, the lacking decimal place must be found in the quotient. From this are deduced the following directions:

1. If, before commencing the division, the dividend is seen to contain fewer decimal places than the divisor, supply the deficiency by annexing ciphers.

2. Subtract the number of decimal places in the divisor from those in the dividend, to find the number of decimal places to point off in the quotient.

RULE.—*Divide as with whole numbers; and from the right of the quotient point off as many figures as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.*

NOTE 1.—If the quotient contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

NOTE 2.—To obtain a given number of decimal places in the quotient, use as many decimal places of the dividend as will equal the number of decimal places in the divisor *plus* the number of decimal places required in the quotient, annexing ciphers to the dividend, if necessary.

NOTE 3.—Decimals are divided by 10, 100, 1000, etc., by moving the decimal point of the dividend as many places to the left as there are ciphers in the divisor.

NOTE 4.—Complex decimals, if terminate, are usually expanded to simple decimals before applying the rule; and if interminate, are usually divided by Rule, **111** or **112**.

EXAMPLES FOR PRACTICE.

Divide

(Obtaining simple decimals as quotients).

1. \$44.65 by .005.
2. 31 cords by .0004.
3. 43.2 acres by .0016.
4. .000875 yr. by 1.75 yr.
5. .0009 bu. by .003.
6. 18.72 yd. by .4.
7. 10 miles by .001.
8. .001 rods by 5 rods.
9. 18.57 months by 1.5.
10. 423 fathoms by .18.
11. .00018 yr. by .0075 yr.
12. 5 tons by .0512.

Divide

(Obtaining simple decimals as quotients).

13. 5 bbl. by .005 bbl.
14. \$72.0372 by \$41.64.
15. \$.31 by .0005.
16. \$90.288 by \$4.18.
17. 246.75 gal. by 32.9 gal.
18. 23204.88 yd. by .0324.
19. .0002784 ft. by .032 ft.
20. \$25.375 by \$5.33$\frac{1}{3}$.
21. \$198.455 by \$.66$\frac{2}{3}$.
22. 2.72$\frac{1}{8}$ months by .000$\frac{1}{16}$.
23. 457.62 rods by .058$\frac{1}{3}$.
24. 13.702$\frac{1}{3}$ miles by .16$\frac{2}{3}$.

(Carrying the quotients to as many decimal places as are specified in the parentheses).

25. 19 fathoms by .06 (3 places).
26. 578.3175 lb. by 53 lb. (4 pl.).
27. 672.51 miles by 34 (3 pl.).
28. 76.28128 yr. by .352 (3 pl.).
29. \$35.14 by \$7.1835 (2 pl.).
30. 35 months by 17 months (3 pl.).
31. \$475.8 by .148 (3 pl.).
32. .00176 yd. by 11 yd. (5 pl.).
33. 1 mile by 756 miles (3 pl.).
34. \$783 by \$.043 (4 pl.).
35. 58.1753 pt. by 1.9 (2 pl.).
36. .068134 ft. by 6.75 (3 pl.).
37. 1.482917 yd. by 372.6 (3 pl.).
38. 41$\frac{1}{4}$ cords by .05 (3 pl.).

134

Divide

39. 74.18$\frac{5}{16}$ rods by 12.5 (3 pl.).
40. 5.004$\frac{7}{8}$ fathoms by 38 (4 pl.).
41. .0$\frac{2}{3}$ bbl. by 75 (4 pl.).
42. 15\frac{3}{4}$ by 18\frac{5}{16}$ (2 pl.).
43. $\frac{5}{7}$ yr. by $\frac{1}{8}$ (3 pl.).

Divide

44. .00$\frac{1}{3}$ bu. by .04$\frac{1}{6}$ (2 pl.).
45. .018$\frac{4}{7}$ pt. by .1$\frac{2}{9}$ pt. (3 pl.).
46. .5$\frac{1}{4}$ gal. by .00$\frac{9}{32}$ (2 pl.).
47. 3.182\frac{5}{6}$ by 1.71\frac{2}{3}$ (3 pl.).
48. 5.18372$\frac{2}{3}$ yr. by .8$\frac{4}{7}$ (2 pl.).

[When the divisor terminates in one or more ciphers, apply Note 3.]

Divide

49. $18.475 by $10.
50. 345.1298 acres by 100.
51. 41523.85 miles by 1000.
52. $\frac{3}{4}$ bu. by 100.
53. 2135\frac{3}{16}$ by 10 (3 pl.).

Divide

54. $375.82 by $500 (2 pl.).
55. $172.81 by 300 (3 pl.).
56. 1867.8 acres by 7000 (4 pl.).
57. $18.25 by 1600 (5 pl.).
58. 18 yr. by 1100 (3 pl.).

REVIEW OF DECIMALS.

135. Decimal fractions have the same relation to each other as other fractions. Hence, **113**, **114**, and **116**, relating to common fractions, are equally applicable to decimals.

1. A farmer's yield of potatoes averaged 98.94 bushels per acre, of oats 34.65 bushels per acre, and of wheat 13.65 bushels per acre. If 110 acres were planted in potatoes, 25.2 acres in oats, and 17.8 acres in wheat, what was the total number of bushels harvested?

2. A contractor bargained to remove an embankment at $.425 per cart load, and in settlement received $5378.3325. How many loads had he removed?

3. A grocer buys four bags of coffee which average 132.08 pounds per bag. If three of the bags weigh respectively 128.05, 131.75, and 134.15 pounds, what is the weight of the fourth bag?

4. A farmer owns five tracts of land containing respectively eight hundred seventy-six, and eighteen thousandths; twenty-eight and seven-tenths; four hundred fifty-six, and five hundred thirty-nine ten-thousandths; seventy-two, and thirteen thousandths; and nine thousand three hundred twenty-four, and seven hundred sixteen hundred-thousandths acres. How much land does the farmer own?

5. A dealer in produce bought four wagon-loads of wheat, containing respectively 30.015, 31.5, 29.05, and 33.1 bushels. What was the total cost at $1.05625 per bushel?

6. What is the total cost of 22.75 tons of coal bought at $6.875 per ton, 15.625 tons at $6.375 per ton, 18.25 tons at $6.125 per ton, and 5.0625 tons at $5.1875 per ton?

7. I lack 15.035 bushels of having 24.08 bushels of wheat. How many bushels of wheat have I?

8. How many bushels of wheat at $1.20375 per bushel can be exchanged for 1926 bushels of apples at 40 cents per bushel?

9. What is the cost of 48.625 cords of wood at $7.06¼ per cord?

10. What is the cost of 58 barrels of cider, averaging 38.125 gallons per barrel, at $.18¾ per gallon?

11. The sum of five addends is 375.045 years. If four of the addends are 72.05, 31.009, .4158, and 8.3 years, what is the fifth?

12. A merchant deposited a certain amount of money in bank, and afterwards withdrew, at different times, .012, .034, .45, and .215 of his original deposit. What part of his original deposit had he still remaining in bank?

13. If a man has one hundred eighty-five and seventeen ten-thousandths acres; then buys sixty-eight and seventy-five thousandths acres; then sells ninety-five and forty-seven ten-thousandths acres; then buys forty-two and one hundred thirty-two hundred-thousandths acres; then buys one hundred twelve and thirty-nine thousandths acres; and then sells forty-eight and five hundredths acres; how much land will he have remaining?

14. The product of two factors is 2285.035 yards, and one of the factors is 318.25. What is the other factor?

15. The remainder is 18.2675 miles, and the subtrahend 375.1 miles. What is the minuend?

16. A, B, and C own a tract of land, A owning .258 of the tract, B .425, and C the remainder. What part of the tract does C own?

17. A farmer owned 140 sheep and sold .05 of the flock to his neighbor. How many sheep had he remaining?

18. A man sold .75 of his farm and had 115 acres remaining. How many acres did he sell?

19. After making a payment of .475 of his debt, a man finds that he still owes $210. How much did he originally owe?

20. A man's salary is $650 per annum, of which he pays $200 for board, $125 for clothes, and $97.50 for sundry other expenses. What decimal part of his salary does he save?

21. A merchant withdrew .375 of his deposit to invest in a farm which cost him $6890.25. What was his original deposit in bank?

22. I bought an invoice of coffee, and disposed of .25 of the purchase at one sale, .014 of the purchase at a second sale, and .75 of what was left of the preceding sales at a third sale. What part of the invoice remained after the third sale?

23. I paid $5280 for a house and .03125 as much for a lot. What was the cost of the lot?

24. A man sold off 204 acres from his farm and had 221 acres remaining. What decimal part of his farm did he sell?

25. I bought 6000 yards of muslin at 5 cents per yard, sold .018 of the purchase at one sale at 6 cents per yard, .25 of what was left after the first sale at 7 cents per yard, and the remainder at 8 cents per yard. What was my total gain?

26. A grocer bought sugar and coffee amounting to $114.80, the cost of the sugar being .435 of the cost of the coffee. What did he pay for each?

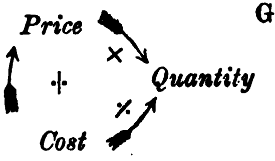
27. A man bequeathed .125 of his property to an orphan asylum, .3125 to his son, and the remaining $24120 to his wife. What was the value of his property?

COUNTING-HOUSE PRACTICE.

136. Counting-House Practice, as here treated, embraces the principal contractions employed in business computations.

Rem.—In business calculations, *quantity* refers to the *total number* of pounds, yards, dozens, etc., of a particular commodity; *price* to the money value of *one* pound, *one* yard, *one* dozen, etc.; and *cost* to the money value of *all* the pounds, yards, or dozens.

137. *Quantity* corresponds with the *abstract factor* of **54**; *price* with the *concrete factor;* and *cost* with the *product.* Hence,



General Formula.

Explanation.—If any two of the three terms of this formula are given, perform the operation indicated by the sign between them, and the result will be the third term.

138. An **Aliquot Part** of a number is such a part of it as will exactly divide that number.

REM.—Any decimal price is an aliquot part of a dollar if the numerator of its equivalent common fraction in lowest terms is 1.

Aliquot parts of one dollar.

50¢ $= \$\frac{1}{2}$.
33$\frac{1}{3}$¢ $= \$\frac{1}{3}$.
25¢ $= \$\frac{1}{4}$.

20¢ $= \$\frac{1}{5}$.
16$\frac{2}{3}$¢ $= \$\frac{1}{6}$.
12$\frac{1}{2}$¢ $= \$\frac{1}{8}$.

10¢ $= \$\frac{1}{10}$.
8$\frac{1}{3}$¢ $= \$\frac{1}{12}$.
6$\frac{1}{4}$¢ $= \$\frac{1}{16}$.

5¢ $= \$\frac{1}{20}$.
2$\frac{1}{2}$¢ $= \$\frac{1}{40}$.
2¢ $= \$\frac{1}{50}$.

Aliquot parts of aliquot parts of one dollar.

25¢ $= \frac{1}{2}$ of 50¢.
12$\frac{1}{2}$¢ $= \frac{1}{4}$ of 50¢.

6$\frac{1}{4}$¢ $= \frac{1}{8}$ of 50¢.
5¢ $= \frac{1}{10}$ of 50¢.

12$\frac{1}{2}$¢ $= \frac{1}{2}$ of 25¢.
6$\frac{1}{4}$¢ $= \frac{1}{4}$ of 25¢.

139. Decimal prices which are not aliquot parts of a dollar are frequently separable into two or more components which are. Thus,

15¢ $= \$\frac{1}{10} + \$\frac{1}{20}$ ($\frac{1}{2}$ of $\$\frac{1}{10}$).
17$\frac{1}{2}$¢ $= \$\frac{1}{8} + \$\frac{1}{20}$.
18$\frac{3}{4}$¢ $= \$\frac{1}{8} + \$\frac{1}{16}$ ($\frac{1}{2}$ of $\$\frac{1}{8}$).
22¢ $= \$\frac{1}{5} + \$\frac{1}{50}$ ($\frac{1}{10}$ of $\$\frac{1}{5}$).
22$\frac{1}{2}$¢ $= \$\frac{1}{5} + \$\frac{1}{40}$ ($\frac{1}{8}$ of $\$\frac{1}{5}$).
27$\frac{1}{2}$¢ $= \$\frac{1}{4} + \$\frac{1}{40}$ ($\frac{1}{10}$ of $\$\frac{1}{4}$).

31$\frac{1}{4}$¢ $= \$\frac{1}{4} + \$\frac{1}{16}$ ($\frac{1}{4}$ of $\$\frac{1}{4}$).
35¢ $= \$\frac{1}{4} + \$\frac{1}{10}$.
37$\frac{1}{2}$¢ $= \$\frac{1}{4} + \$\frac{1}{8}$ ($\frac{1}{2}$ of $\$\frac{1}{4}$).
45¢ $= \$\frac{1}{4} + \$\frac{1}{5}$.
55¢ $= \$\frac{1}{2} + \$\frac{1}{20}$ ($\frac{1}{10}$ of $\$\frac{1}{2}$).
62$\frac{1}{2}$¢ $= \$\frac{1}{2} + \$\frac{1}{8}$ ($\frac{1}{4}$ of $\$\frac{1}{2}$).

140. Advantage should be taken of all decimal prices which lack an aliquot part of being $1. Thus,

66$\frac{2}{3}$¢ $= \$1 - \$\frac{1}{3}$.
75¢ $= \$1 - \$\frac{1}{4}$.
80¢ $= \$1 - \$\frac{1}{5}$.

83$\frac{1}{3}$¢ $= \$1 - \$\frac{1}{6}$.
87$\frac{1}{2}$¢ $= \$1 - \$\frac{1}{8}$.
95¢ $= \$1 - \$\frac{1}{20}$.

97$\frac{1}{2}$¢ $= \$1 - \$\frac{1}{40}$.
98¢ $= \$1 - \$\frac{1}{50}$.
99¢ $= \$1 - \$\frac{1}{100}$.

141. To find the cost when the price is an aliquot part of $1.

ILLUSTRATIVE EXAMPLES.

1. What is the cost of 832 lb. coffee at 25¢ per pound ?

SOLUTION.—At $1 per pound, 832 lb. will cost $832.
At $\$\frac{1}{4}$ " " " " " $\frac{1}{4}$ of $832, or $208.

2. What is the cost of 760 gallons of wine at 37$\frac{1}{2}$¢ per gallon ?

SOLUTION.—At \$$\frac{1}{4}$ per gallon, 760 gal. will cost \$190 ($\frac{1}{4}$ of \$760).
At \$$\frac{1}{8}$ " " " " " 95 ($\frac{1}{2}$ of \$190).
At \$$\frac{3}{8}$ " " " " " \$285 (\$190 + \$95).

3. What is the cost of 528 yd. dress goods at 95¢ per yard?

SOLUTION.—At \$1 per yard, 528 yd. will cost \$528
At \$$\frac{1}{20}$ " " " " " 26.40 ($\frac{1}{20}$ of \$528).
At \$1 − \$$\frac{1}{20}$ " " " " " \$501.60 (\$528 − \$26.40).

REM.—To divide by 20, divide by 2 and place the figures of the quotient one place to the right of the orders divided.

4. What is the cost of 415 yd. silk at \$1.62½ per yard?

SOLUTION.—At \$1 per yard, 415 yd. will cost \$415.
At \$$\frac{1}{2}$ " " " " " " 207.50 ($\frac{1}{2}$ of \$415).
At \$$\frac{1}{8}$ " " " " " " 51.88 ($\frac{1}{4}$ of \$207.50).
At \$1$\frac{5}{8}$ " " " " " " \$674.38.

RULE.—*Divide the quantity by the number of its units that will cost \$1.*

NOTE.—When the final result can be obtained by one division, it need not be extended beyond the cents' order; but if two or more divisions are necessary, the partial quotients should be carried to the mills' order, to insure accuracy in the final result.

EXAMPLES FOR PRACTICE.

Find the cost of

1. 5308 lb. @ 25¢.	11. 892 yd. @ 87½¢.	21. 528 lb. @ \$1.25.
2. 324 gal. @ 12½¢.	12. 427 bu. @ 98¢.	22. 673 yd. @ \$1.75.
3. 918 yd. @ 16⅔¢.	13. 1782 lb. @ 83⅓¢.	23. 215 bu. @ \$1.12½.
4. 6715 lb. @ 20¢.	14. 372 gal. @ 97½¢.	24. 470 yd. @ \$1.37½.
5. 295 bu. @ 50¢.	15. 5173 lb. @ 99¢.	25. 825 bu. @ \$1.66⅔.
6. 683 gal. @ 33⅓¢.	16. 725 yd. @ 37½¢.	26. 95 bbl. @ \$6.18¾.
7. 7198 lb. @ 2½¢.	17. 2981 yd. @ 15¢.	27. 72 gal. @ \$3.87½.
8. 519 yd. @ 75¢.	18. 675 lb. @ 62½¢.	28. 48 bbl. @ \$5.75.
9. 3218 bu. @ 95¢.	19. 483 bu. @ 55¢.	29. 918 bu. @ \$2.95.
10. 629 gal. @ 66⅔¢.	20. 619 lb. @ 27½¢.	30. 571 yd. @ \$3.98.

142. Advantage should be taken of quantities which are dividends or aliquot parts of 10, 100, 1000, etc. Thus, Ex. 1. What is the cost of 10 lb. coffee at 37½¢ per pound?

Solution.—37½¢ = $.375; $.375 × 10 = $3.75 (Note 3, **133**).

Ex. 2. Find the cost of 400 yd. silk at $1.18¾ per yard.

Solution.—$1.18¾ = $1.1875; $1.1875 × 400 = $118.75 × 4 = $475.

Ex. 3. What is the cost of 50 qt. milk at 6¼¢ per quart?

Solution.—$.0625 × 100 = $6.25; hence, $.0625 × ½ of 100 = ½ of $6.25, or $3.13−.

Ex. 4. Find the cost of 87½ gal. syrup at 52⅝¢ per gallon.

Solution.—52⅝¢ = $.52625 (Note 1, **129**); 100 gal. will cost 100 times $.52625, or $52.625; and ⅛ less than 100 gal. or 87½ gal. will cost ⅛ less than $52.625, or $46.05−.

EXAMPLES FOR PRACTICE.

Find the cost of

1. 100 yd. @ 12½¢.
2. 1000 yd. @ 15⅝¢.
3. 50 yd. @ 32¢.
4. 75 yd. @ 64¢.
5. 300 yd. @ 46⅞¢.
6. 500 yd. @ 16¾¢.
7. 900 yd. @ 28½¢.
8. 250 yd. @ 73¢.
9. 33⅓ yd. @ 85¢.
10. 1250 yd. @ 54⅛¢.
11. 750 yd. @ 19¾¢.
12. 7½ yd. @ 48¢.
13. 2½ yd. @ 65¢.
14. 3⅓ yd. @ 93¢.
15. 12½ yd. @ 34⅛¢.

143. To find the quantity when the price is an aliquot part of $1.

ILLUSTRATIVE EXAMPLES.

1. How many yards of dress goods can be bought for $24 at 25¢ per yard? At 66⅔¢ per yard? At $1.25 per yard?

FIRST SOLUTION.	SECOND SOLUTION.	THIRD SOLUTION.
25¢ = $¼.	66⅔¢ = $⅔.	$1.25 = $5/4.
$\div \frac{1}{4} = \times \frac{4}{1}$ (Rule, **110**).	$\div \frac{2}{3} = \times \frac{3}{2} = \times (1 + \frac{1}{2})$.	$\div \frac{5}{4} = \times \frac{4}{5} = \times (1 - \frac{1}{5})$.
24 × 4 = 96 yd.	Hence, 24	Hence, 24
	(plus ½ of 24) 12	(less ⅕ of 24) 4⅘
	(= 3/2 of 24, or) 36 yd.	(= ⅘ of 24, or) 19⅕ yd.

First Solution.—At 25¢ per yard, $1 will buy 4 yards, and $24 will buy 24 times 4 yards, or 96 yards.

Second Solution.—At 66⅔¢ per yard, $1 will buy 1½ yards, and $24 will buy 24 times 1½ (= 24 plus ½ of 24), or 36 yards.

Third Solution.—At $1.25 per yard, $1 will buy ⅘ of a yard, and $24 will buy 24 times ⅘ (= 24 minus ⅕ of 24), or 19⅕ yards.

Rule.—*Multiply the cost by the quantity that $1 can buy.*

Note.—If the price expressed as a common fraction be inverted, it will denote the quantity that $1 can buy.

EXAMPLES FOR PRACTICE.

Find No. of yards.			Find No. of gallons.			Find No. of bushels.		
	Cost.	*Price.*		*Cost.*	*Price.*		*Cost.*	*Price.*
1.	$275	12½¢.	7.	$41.50	62½¢.	13.	$29.60	83⅓¢.
2.	$348	25¢.	8.	$819	50¢.	14.	$47.50	18¾¢.
3.	$267	16⅔¢.	9.	$4.17	20¢.	15.	$718	75¢.
4.	$19.08	33⅓¢.	10.	$620	$1.25.	16.	$825	62½¢.
5.	$213	75¢.	11.	$72.30	$1.50.	17.	$31.60	$2.50.
6.	$15.40	87½¢.	12.	$37.40	$1.66⅔.	18.	$45.30	$12.50.

144. To find the cost of a given weight in pounds, when the price is per bushel, barrel, ton, etc.

ILLUSTRATIVE EXAMPLES.

1. Find the cost of 874 lb. rye at 93¢ per bushel of 56 lb.

SOLUTION.

```
       874
       .93
      ----
      2622
     7866
    ------
8 ) $812.82
  ---------
 7 ) 101.603—
  ---------
   $14.514
```

Explanation.—874 times 93¢, or $812.82, would be the cost at 93¢ *per pound;* but at 93¢ per bushel of 56 lb., this obtained cost is 56 times the true cost. Hence, divide $812.82 by 56, obtaining $14.51+ as the true cost.

Rem.—Instead of an awkward long division by 56, employ Rule, **41**, and divide successively by its factors (8 × 7), carrying the division to mills, to secure accuracy in the cents of the final quotient.

2. What is the cost of 8978 ft. lumber at $1.67 per 100 ft.?

SOLUTION.

```
      8978
      1.67
     -----
     62846
    53868
    8978
  --------
 $149.93.26
```

Explanation.—8978 times $1.67, or $14993.26, is the cost at $1.67 per foot; but at $1.67 per 100 ft., this obtained cost is 100 times the true cost. Therefore, divide $14993.26 by 100, employing Rule, **42**, for dividing integers by 100, 1000, etc., and Note 3, **134**, for dividing decimals, obtaining $149.93+ as the required cost.

3. Find the cost of 8157 lb. coal at $5.39 per ton of 2000 lb.

SOLUTION.

8157
5.39
73413
24471
40785
2) 43.966.23
$21.983

Explanation.—$43966.23 is the cost at $5.39 per pound, or 2000 (= 1000 × 2) times the true cost. Dividing $43966.23 by 1000 according to Note 3, **134**, produces $43.96623 ; and dividing $43.966+ by 2, produces $21.98+ as the required cost.

4. Find the cost (1) of 943 lb. coal at $6.18 per long ton of 2240 lb.; (2) of 79 lb. flour at $7.28 per barrel of 196 lb.; (3) of 895 lb. corn in the ear at $2.91 per barrel of 350 lb.

SOLUTIONS.

(1)	(2)	(3)
2240 = 10 × 8 × 7 × 4.	196 = 7 × 7 × 4.	350 = 10 × 7 × 5.
943	$7.28	895
6.18	79	2.91
7544	6552	895
943	5096	8055
5658	7) 575.12	1790
8) 582.7.74	7) 82.16	7) 260.4.45
7) 72.847—	4) 11.737+	5) 37.206+
4) 10.407—	$2.934+, *Ans.*	$7.441+, *Ans.*
$2.601+, *Ans.*		

Rule.—*Multiply the weight in pounds by the price, and divide the result by the number of pounds in the denomination upon which the price is expressed.*

Note 1.—The weight of a bushel in pounds, as prescribed by law in most States, is as follows :

Barley,	(8 × 6)	48	Corn, in ear,	(10 × 7)	70	Potatoes,	(10 × 6)	60
Beans,	(10 × 6)	60	Corn Meal,	(10 × 5)	50	Rye,	(8 × 7)	56
Buckwheat,	(8 × 6)	48	Cotton Seed,	(10 × 3)	30	Timothy Seed,	(9 × 5)	45
Clover Seed,	(10 × 6)	60	Oats,	(8 × 4)	32	Wheat,	(10 × 6)	60
Corn, shelled,	(8 × 7)	56	Peas,	(10 × 6)	60	Wheat Bran,	(10 × 2)	20

144

NOTE 2.—The weight or measure of other standard units upon which prices are usually expressed, is as follows: Short Ton, 2000 lb.; Long Ton, 2240 lb.; Cental, 100 lb.; Barrel of Flour, 196 lb.; Barrel of Pork or Beef, 200 lb.; Barrel of Corn, in the ear, 350 lb.; C of Lumber, 100 ft.; M of Lumber, 1000 ft.; etc.

EXAMPLES FOR PRACTICE.

Find the cost.

	Quantity.		*Price per bu.*
1.	3125 lb.	rye	@ 85¢.
2.	4116 "	shelled corn	" 62¢.
3.	728 "	wheat	" 95¢.
4.	275 "	timothy seed	" $2.25.
5.	948 "	corn meal	" 75¢.

Find the cost.

	Quantity.		*Price per bu.*
6.	2350 lb.	beans	@ $1.75.
7.	5739 "	buckwheat	" $1.15.
8.	7134 "	potatoes	" 58¢.
9.	1725 "	wheat bran	" 24¢.
10.	7293 "	corn in ear	" 63¢.

		Per 100.
11.	587 lb.	@ 45¢.
12.	826 lb.	" $1.38.
13.	2987 ft.	" $8.26.
14.	6215 ft.	" $7.45.

		Per 1000.
15.	75 ft.	@ $15.25.
16.	8294 lb.	" $10.75.
17.	648 ft.	" $18.45.
18.	3125 lb.	" $25.36.

		Per Short Ton.
19.	628 lb.	@ $4.75.
20.	745 lb.	" $5.92.
21.	9276 lb.	" $8.25.

		Per Long Ton.
22.	5823 lb.	@ $7.56.
23.	3521 lb.	" $3.45.
24.	1569 lb.	" $6.75.

BILLS AND ACCOUNTS.

145. An **Account** is a record of one or more sums or items due from business transactions; such as purchases or sales, services rendered, etc.

146. The **Ledger** is the principal book of accounts, in which the several items of each account are systematically arranged under the head of debit or credit, or both.

147. The **Debit** side of an account (marked Dr.) exhibits the amounts of the items due *to* the party keeping the account for goods sold, services rendered, money paid, etc.

148. The **Credit** side of an account (marked Cr.) exhibits the amounts of the items due *from* the party keeping the account for goods bought, services or money received, etc.

Rem.—The difference between its debit and credit sides is called the *Balance* of the account.

149. A **Bill** or **Invoice** is a detailed statement of merchandise sold or of services rendered.

Rem. 1.—A Bill or Invoice should state the place and date of sale; the names of buyer and seller; the terms of the sale; the identifying mark of the case, bale, barrel, etc., in which the merchandise has been shipped; the distinguishing characters or numbers which are marked on the smaller packages of a case of miscellaneous articles so that each may be readily found; the quantity, name, and price of each item, with its aggregate amount; and other details, if any, of the sale.

Rem. 2.—When paid, the Bill or Invoice is usually receipted by writing at the bottom, under the words "Received Payment," the signature of the party or firm selling the goods.

Rem. 3.—An authorized employé may receipt a Bill by first signing his own name and, in the next line, that of his employer, preceded by the word *for*, as in Bill No. 3; or what is more customary, by writing his employer's name first and, in the next line, his own name, preceded by *per* or *by*, as in Bill No. 2.

150. A **Statement of Account** is a written exhibit of the *sum* of the items charged at different dates of purchase.

Rem.—A *Statement* is not itemized as a Bill, but embraces the date and the aggregate of each sale, the itemized Bills of which had been previously rendered, as in No. 7.

151. In making out Bills, etc., many terms, characters, and abbreviations are used, the principal of which will be found in the following list:

COMMERCIAL ABBREVIATIONS.

@	at.	Bbl.	Barrel.	Bu.	Bushels.
A 1	First quality.	Bdls.	Bundles.	Bxs	Boxes.
Acct or a/c	Account.	Bds.	Boards.		
Agt.	Agent.	Bgs.	Bags.	C	Hundred.
Amt.	Amount.	Bkts	Baskets.	¢ or cts	Cents.
Apr.	April.	Blk	Black.	Chgd.	Charged.
Arr.	Arrived.	Bls.	Bales.	Chts	Chests.
Ass'd	Assorted.	Bot.	Bought.	Cks.	Casks,
Aug	August.	Br	Brig.	Co.	Company.
		Bro.	Brown, or Brother.	C. O. D.	Collect on delivery.
Bal.	Balance.	Brot.	Brought.	Col'd	Colored.

Com	Commission.
Cr.	Creditor.
Cs.	Cases.
Cwt	Hundredweight.
d	Pence.
Dec	December.
Dft.	Draft.
Disc.	Discount.
Do., ditto, or "	The same.
Doz	Dozen.
Dr.	Debtor.
Dryge	Drayage.
Ds. or da	Days.
Ea	Each.
E. & O. E.	Errors and omissions excepted.
Emb'd	Embroidered
Exch	Exchange.
Feb	February.
Fig'd	Figured.
Fir.	Firkin.
Fol.	Folio or page.
Fr't	Freight.
Ft	Foot or feet.
Gal	Gallon.
Gr. or gro.	Gross.
Guar	Guarantee.
Hlf	Half.
Hhd	Hogshead.
i. e	That is.
In	Inches.
Ins	Insurance.
Inst.	Present month.
Int	Interest.
Jan	January.
Jr. or Jun.	Junior.
Lbs	Pounds.
Ledg	Ledger.
M	Thousand.
Mar	March.
Mdse.	Merchandise.
Mem	Memorandum.
Messrs	Gentlemen or Sirs.
Mo	Month.
Mr	Master or Mister.
Mrs	Mistress.
N. B.	Take particular notice.
No. or #	Number.
Nov	November.
Oct	October.
Oz	Ounces.
P	Page.
Pp	Pages.
Pay't or Paym't	Payment.
Pd	Paid.
Per, p., or ⅌	By, or by the.
Pkgs	Packages.
Plts	Plates.
P. O	Post Office.
Pr	Pair.
Prox	Next month.
Ps	Pieces.
Qr	Quarter.
Rec'd paym't	Received payment.
Rec't	Receipt.
R. R	Railroad.
Rs. or rls	Rolls.
Schr	Schooner.
Sh	Ship.
Shipt	Shipment.
Stbt	Steamboat.
Stor	Storage.
Str	Steamer.
Sunds	Sundries.
Super	Superfine.
Trcs	Tierces.
Ult	Last month.
Wt	Weight.
Yds	Yards.
Yr	Year.

COMMERCIAL CHARACTERS.

@	At.
a/c	Account.
¢	Cents.
c/o	Care of.
✓	Check mark.
$	Dollars.
"	Ditto, or the same.
%	Hundredths
n/c	New account.
#	Number.
o/c	Old account.
⅌	Per, or by the.
£	Pounds sterling.
×	By, as 7 × 8 inches.

REM. 1.—5^1 yards, 8^2 yards, 13^3 yards, respectively mean $5\frac{1}{4}$, $8\frac{1}{2}$, and $13\frac{3}{4}$ yards; 12^{34} bushels and 75^{18} bushels respectively mean 12 bushels 34 pounds and 75 bushels 18 pounds. 15^{12} pounds and 18^{11} pounds respectively mean 15 pounds 12 ounces and 18 pounds 11 ounces.

REM. 2.—24 doz., $\frac{8}{\$7}$, $\frac{14}{\$8.25}$, $\frac{2}{\$8.75}$, mean 24 doz.—8 doz. @ $7 per dozen, 14 doz. @ $8.25 per dozen, and 2 doz. @ $8.75 per dozen, the quantity being written above a horizontal line and its price below.

REM. 3.—16 doz., $\frac{5}{1}$ @ 80¢, $\frac{7}{2}$ @ 70¢, $\frac{4}{3}$ @ 62¢, mean 16 dozen — 5 doz. of No. 1 @ 80¢ per doz., 7 doz. No. 2 @ 70¢ per doz., and 4 doz. No. 3 @ 62¢ per doz., the quantity being written above a horizontal line, and the class or distinguishing number of the goods below.

REM. 4.—2 bbl. sugar $\frac{247}{258}=\frac{19}{20}$ means 2 barrels sugar, one barrel weighing

247 lbs. gross, from which has been deducted 19 lbs. for the weight of the barrel, the other barrel 258 lbs. gross, minus 20 lbs. for weight of barrel, leaving (247−19)+(258−20) or 466 lbs. as the net weight of the sugar charged.

152. The following embrace several forms of Bills, Invoices, Statements, etc. The pupil should carefully and neatly copy each with pen and ink, make the necessary extensions, and "foot up" the several columns.

Folio 843.

(Bill No. 1.)

BALTIMORE, *Dec. 18, 1888.*

MR. GEO. W. HAMMOND,
CITY.

Terms Cash. *Bought of* COURTNEY & BRO.

4	lbs. Old Govt. Java Coffee, - -	25¢.	$*	**		
½	" Oolong Tea, - - - - - -	70¢.		**		
15	" Granulated Sugar, - - -	7½¢.	*	**		
10	" A Sugar, - - - - - - - -	6½¢.		**		
8	" P. R. Rice, - - - - - - -	6¼¢.		**		
2	gal. Syrup, - - - - - - -	75¢.	*	**		
½	" N. O. Molasses, - - - - -	70¢.		**		
5/12	doz. Cans Tomatoes, - - -	$1.50.		**		
7/12	" " Peaches - - - -	$1.80.	*	**		
½	" " Pineapples, - - -	$2.50.	*	**		
¼	" " Strawberries, - -	$2.25.		**		
3	lbs. Cheese, - - - - - - - -	25¢.		**		
2½	" Butter, - - - - - - - -	40¢.	*	**		
7	" Bro. Sugar, - - - - - - -	6¢.		**	$**	**

(Bill No. 2.)

PHILADELPHIA, *Sept. 1, 1889.*

A. J. BRADSHAW.

Per Express. *Bought of* J. FAGAN & SON.

4	Doz. Hatchets, - - - - - - - - - -	@ $6	$**	**
3	" " - - - - - - - - - -	" 7	**	**
2	" Doorsprings, - - - - - - - -	" 3	*	**
5	" Chains, - - - - - - - - - -	" $2^{\underline{10}}$	**	**
12	" " - - - - - - - - - -	" $2^{\underline{15}}$	**	**
10	" A Bits, - - - - - - - - - -	" $3^{\underline{75}}$	**	**
10	Iron Awls, - - - - - - - - - - -	" 2	**	**
6	Doz. Grindstone Fixtures, - - - -	" $7^{\underline{50}}$	**	**
			$***	**

Received Payment,
J. FAGAN & SON,
Per W. S. HILL.

(Bill No. 3.)

Bradford, Pa., *Sept. 19, 1889.*

Geo. W. Bird & Co.,

Bought of The Bradford Paper Co.

No. of Case.	No. of Reams.	Kind of Paper.	Weight per Ream.	Net Weight Case. lbs.	Price.	Amount.	
7466	15	Congress Note, - - - - - - -	5	75	18½	$**	**
	10	" " - - - - - - -	6	60	18½	**	**
	10	" Letter, - - - - - - -	10	100	16½	**	**
	5	" " ½ pk., - - - -	12	60	16¾	**	**
	20	" Cap, - - - - - - - -	20	400	17½	**	**
7581	25	Capital City Broad Bill, - - -	12	300	17	**	**
	25	Baltimore Cap, - - - - - - -	14	350	18¾	**	**
	64	Dixie Note plate fin., - - - -	7	448	19	**	**
7583	10	Congress Legal Cap, - - - -	18	180	19	**	**
	10	" Broad Bill, - - - -	14	140	17	**	**
7587	30	Druid Mills Cap, - - - - - -	14	420	18	**	**
						$***	**

Received Payment,

J. R. James,

Sept. 29, '89. for The Bradford Paper Co.

(Bill No. 4.)

S. B. 102, Fol. 86.

Baltimore, *June 3, 1889.*

Geo. B. Westerfield,

Bought of Canby, Gilpin & Co.

Terms: 30 days. Shipped by Western Md. R. R.

	1	Keg Crystal Chlorate of Potash, 112 lbs.	16¼	$**	**
	1	bbl. Powd. Alum, - - - - 358 "	5½	**	**
	3	" Ground " - - - - 1200 "	2	**	**
	10	" Glauber Salts, - - - - 3260 "	¾	**	**
	1	Bale Gum Myrrh, - - - - 170 6 = 164 "	25	**	**
	100	oz. Quinine, - - - - - - - - - - - - -	1.40	***	**
	200	lb. Carb. Magnesia, - - - - - - - - - -	30	**	**
	1	Gross Rheumatic Cure, - - - - - - - -	72.00	**	**
	1	" Oatmeal Soap, - - - - - - - - -	14.00	**	**
	1	" Silver " - - - - - - - - -	9.00	*	**
	2	" Bath " - - - - - - - - -	11.00	**	**
	4	" Glycerine " - - - - - - - - -	21.00	**	**
	1	bbl. Alcohol, - - - - - - - - - 46 gal.	2.18	***	**
				$***	**
		Insurance, - - - - - - - - -		3	30
				$***	**

(Bill No. 5.)

NEW YORK, *July 20, 1889.*

MESSRS. HOOVER & Co.,

Folio 923, S. B. 127. *Bought of* A. T. STEWART & Co.

Case.			No. of Yards.	Price.	Amount.	
$27	24	Pcs. Merrimac Prints. 22^1 28^2 26^3 32^1 36^2 38^1 42 29^1 27 29^1 37^3 44^1 28^2 41^1 37^1 42 43^1 27^2 37^1 39^2 41^1 32 26 37^2	***			
$28	23	Pcs. Standard Prints. 36^1 38^2 37 39^2 41^3 46 29^1 42^3 46^2 43^3 44^1 44 39^1 36^2 28^1 33^3 29^1 28^3 27^1 35^2 36 37^1 35	***			
			****	5¢	$**	**

(Bill No. 6.)

BALTIMORE, *July 3, 1889.*

MR. T. W. JOHNSON,
City,

Folio 313. *Bought of* ELLIOTT & CARR.

Terms: No claims allowed unless made within ten days from date of invoice.

12	M 9960—5 Envelopes, - - - - - -	1.60	$**	**		
6	" 6 " - - - - - -	1.75	**	**		
3	" 7684 3 " - - - - - -	1.85	*	**		
3	" 5½ " - - - - - -	2.10	*	**		
8	Rms. Treasury 14 Legal, - - - -	2.80	*	**		
⅓	Doz. 066 Frames, - - - - - -	9.00	*	**		
¼	" 688 " - - - - - - - -	10.80	*	**		
½	" 501 " - - - - - - -	6.00	*	**		
1	" 031 " - - - - - - -		3	50		
⅙	" 449 " small, - - - -	6.75	*	**		
⅙	" 449 " large, - - - -	8.90	*	**		
3	" No. 3 Pads, - - - - - - -	35	*	**		
1	" " 10 " - - - - - - -			40		
1½	Gro. Penholders, - - - - - - -	3.50	*	**		
1	Doz. Photos $\overset{12.50}{124}$ $\overset{18.00}{219}$ $\overset{27.00}{160}$ - - -		**	**		
⅙	" " $\overset{1.00}{462}$ $\overset{2.00}{77}$ $\overset{1.20}{459}$ $\overset{2.25}{423}$		*	**		
⅓	" Arnold's Ink, quarts, - - -	5.50	*	**		
1½	" Spencerian Ink, quarts, - - -	6.25	*	**		
¼	" " " pints, - - -	3.75		**		
¾	" " " hlf-pints, - -	2.10	*	**		
					$***	**

(No. 7. Monthly Statement.)

BALTIMORE, *July 1, 1889.*

MR. W. E. SADLER,
Carroll, Md.

Folio 912.

Debtor to ALLEN S. WILL.

1889.						
June	10	To Invoice rendered, - - - - - -	$15	62		
"	15	" " " - - - - - -	23	85		
"	17	" " " - - - - - -	17	92		
"	25	" " " - - - - - -	2	91		
"	27	" " " - - - - - -	17	22		
					$**	**

8. W. H. Smith bought of Geo. S. Tyler & Co., July 3, 5 doz. W. I. bolts, 6 in., @ $\$2^{\underline{50}}$; 3⅛ doz. 10 in. drawing-knives @ $\$9^{\underline{75}}$; 2¼ doz. socket chisels, 1½ in., @ $\$7^{\underline{50}}$; 2¾ doz. 10 in. screw-drivers, @ $\$6^{\underline{00}}$; $2\frac{7}{12}$ doz. Ames cast-steel shovels @$\$13^{\underline{50}}$; 3¾ doz. nail gimlets @ 50¢; $2\frac{1}{12}$ doz. spike gimlets @ 75¢; 1¾ doz. claw-hammers @ $\$5^{\underline{75}}$; ¾ doz. smoothing planes @ $\$9^{\underline{00}}$; ¾ doz. jack-planes @ $\$10^{\underline{80}}$; 1½ doz. garden rakes @ $\$6^{\underline{00}}$; and ¾ doz. lawn rakes @ $\$4^{\underline{50}}$. What is the amount of bill (Form of Bill No. 2)?

9. Geo. H. Lewis bought of the Mt. Holly Paper Co., July 15, 1889, case # 8322 : 22 reams Congress note, 5 lbs., @ 16¾¢; 24 reams same, 6 lbs., @ 16¾¢; 18 reams Congress letter, 10 lbs., @ 17¢; 8 reams same, ¼ packets, 12 lbs., @ 17½¢. Case # 8391 : 28 reams Druid Mills cap, 12 lbs., @ 19¢; 32 reams same, 14 lbs., @ 19¢; 16 reams Druid Mills broad bill, 12 lbs., @ 18¾¢; 12 reams same, 10 lbs., @ 18¾¢; 16¾ reams Congress legal, 12 lbs., @ 19½¢; 8 reams same, 14 lbs., @ 19½¢. What is the amount of bill (Form 3)?

10. Frank Church bought of W. H. Patrick & Co., Sept. 9, 5 lbs. O. G. Java coffee @ 32¢; 3 lbs. Mocha do. @ 28¢; 25 lbs. granulated sugar @ 8¼¢; 20 lbs. coffee A do. @ 7½¢; 5 lbs. crushed do. @ 9¢; 1 gal. N. O. molasses @ 65¢; 3 lbs. Heno tea @ 50¢; 4½ lbs. butter @ 45¢; 1½ oz. nutmegs @ 6¢; 7 lbs. Oswego starch @ 17½¢; 8 bars soap @ 7½¢; 2¼ lbs. cheese @ 22¢; ¾ doz. canned corn at $\$2^{\underline{25}}$; ¾ doz. canned tomatoes @ $\$1^{\underline{75}}$; ½ gal. maple syrup @ $\$4^{\underline{50}}$; 2 lbs. sperm candles @ 35¢; 3 bottles yeast powder @ 13¢. What is the amount of bill (Form 6)?

11. Jos. S. Sinclair bought of Jno. A. Harrison & Co., the following goods : Mar. 3, 1889, 5 doz. plates : 2 doz. 5 in. @ 85¢, 2 doz. 6 in. @ 95¢, 1 doz. 7 in. @ $\$1^{\underline{12}}$; 5 sets tea ware @ $\$3^{\underline{75}}$; and 3 doz.

pitchers: 1 doz. 4 qt. @ $\$3^{\underline{50}}$, 2 doz. 6 qt. @ $\$4^{\underline{50}}$. Mar. 18, 1889, 4 doz. tureens: 1 doz. # 1 @ $\$3^{\underline{25}}$, 1 doz. # 2 @ $\$3^{\underline{75}}$, 2 doz. # 3 @ $\$4^{\underline{25}}$; and 5 doz. bowls @ $\$2^{\underline{30}}$. Mar. 23, 1889, 7 doz. dishes @ $\$4^{\underline{20}}$; 8 sets dinner ware @ $\$6^{\underline{25}}$; and 5 bowls and covers @ 95¢. Mar. 27, 1889, 5 doz. syrup cans @ $\$3^{\underline{50}}$. Render separate bill for each sale, and monthly statement for March (Form 7).

12. M. S. Chapman bought of H. B. Claflin & Co., Sept. 1, case # 143, —— pieces American fancy prints, 28^2 26^1 33^3 36^1 38^2 41 42^3 44^1 45 38 37^3 36^2 39^1 28^1 34^2 27^1 29^3 36^2 35 37 39^2 40 41 43^2 44^1 36^1 29^2 37^2 43^1 28^2 29^1 34^2 37^1 38 40 42^1 43^2 37^1 28^1 @ $5\frac{1}{4}$¢.

Case # 149, —— pieces Manchester delaines, 36^1 32 34^1 37^2 38^1 37 42^1 43^2 44^1 47^3 37^2 31^1 41^2 28^1 33^2 29^1 26^3 38^1 37 35^1 41^2 46^3 29^1 39^2 45^1 @ $17\frac{3}{4}$¢.

Case # 157, —— pieces Amoskeag sheetings, 28^1 29^2 31^3 27^2 28^1 33^1 37 36^2 41^3 29^2 28^1 27^3 26^2 33^1 @ $11\frac{7}{8}$¢. What is the amount of bill (Form 5)?

DENOMINATE NUMBERS.

153. A Denominate Number is a concrete number whose units are used to measure, to weigh, or to value; as 5 yards (units of measure), 3 pounds (units of weight), and $16 (units of value).

REM.—Denominate numbers are distinguished as *simple* when they express their units in a single denomination, as 2 feet; and as *compound* when their units are expressed in two or more denominations, as 2 feet 6 inches.

MEASURES OF VALUE.

154. United States Money is the legal currency of the United States. *Scale:* 10 mills (m.) = 1 cent (¢); 10 cents = 1 dime (d.); 10 dimes = 1 dollar ($); 10 dollars = 1 eagle (E.).

155. English or Sterling Money is the legal currency of Great Britain. *Scale:* 4 farthings (far.) = 1 penny (d.); 12 pence = 1 shilling (s.); 20 shillings = 1 pound (£).

156. The following Table will show the units of currency of the principal nations of the world and their intrinsic value in U. S. money, as proclaimed by the Secretary of the Treasury on Jan. 1, 1889.

Country.	Unit of Currency.	Standard.	Value in U. S. money.
Argentine Republic.	Peso of 100 centavos..........	Gold and Silver.	$.965
Austria............	Florin of 100 kreutzers........	Silver..........	.336
Belgium...........	Franc of 100 centimes.........	Gold and Silver.	.193
Bolivia............	Boliviano of 100 centavos.....	Silver..........	.68
Brazil.............	Milreis of 1000 reis............	Gold...........	.546
British America....	Dollar of 100 cents............	Gold...........	1.00
Chili...............	Peso of 100 centavos..........	Gold and Silver.	.912
Cuba..............	Peso of 100 centavos..........	Gold and Silver.	.926
Denmark..........	Crown of 100 öre.............	Gold...........	.268
Ecuador...........	Sucre of 100 centavos.........	Silver..........	.68
Egypt.............	Pound of 100 piastres.........	Gold...........	4.943
France............	Franc of 100 centimes.........	Gold and Silver.	.193
Great Britain......	Pound sterling of 20 shillings..	Gold...........	4.8665
Greece............	Drachma of 100 lepta.........	Gold and Silver.	.193
German Empire....	Mark of 100 pfennige.........	Gold...........	.238
Guatemala.........	Peso of 100 centavos..........	Silver..........	.68
Hayti.............	Gourde......................	Gold and Silver.	.965
Honduras.........	Peso of 100 centavos.........	Silver..........	.68
India..............	Rupee of 16 annas............	Silver..........	.323
Italy..............	Lira of 100 centesimi..........	Gold and Silver.	.193
Japan.............	Yen of 100 sen...............	Gold...........	.997
Liberia............	Dollar of 100 cents............	Gold...........	1.00
Mexico............	Dollar of 100 centavos........	Silver..........	.739
Netherlands........	Florin or Guilder of 100 cents..	Gold and Silver.	.402
Nicaragua.........	Peso of 100 centavos..........	Silver..........	.68
Norway............	Crown of 100 öre..............	Gold...........	.268
Peru..............	Sol of 100 centavos............	Silver..........	.68
Portugal..........	Milreis of 1000 reis............	Gold...........	1.08
Russia............	Rouble of 100 copecks.........	Silver..........	.544
Sandwich Islands...	Dollar of 100 cents............	Gold...........	1.00
Spain.............	Peseta of 100 centimes.... ...	Gold and Silver.	.193
Sweden...........	Crown of 100 öre.............	Gold...........	.268
Switzerland........	Franc of 100 centimes.........	Gold and Silver.	.193
Tripoli............	Mahbub of 20 piastres.........	Silver..........	.614
Turkey............	Piaster of 40 paras............	Gold...........	.044
U. S. of Colombia..	Peso of 100 centavos..........	Silver..........	.68
Venezuela.........	Bolivar......................	Silver..........	.136

MEASURES OF WEIGHT.

157. Troy Weight is used in weighing gold, silver, jewels, etc. *Scale:* 24 grains (gr.) = 1 pennyweight (pwt.); 20 pennyweights = 1 ounce (oz.); 12 ounces = 1 pound (lb.)

158. Apothecaries' Weight is used by physicians and apothecaries in compounding medicines which are not liquid. *Scale:* 20 grains (gr.) = 1 scruple (℈); 3 scruples = 1 dram (ʒ); 8 drams = 1 ounce (℥); 12 ounces = 1 pound (℔).

159. Avoirdupois Weight is the customary weight of commerce. *Scale:* 16 ounces (oz.) = 1 pound (lb.); 100 pounds = 1 hundredweight (cwt.); 20 hundredweights = 1 ton (T.).

REM.—While the corresponding denominations of Troy Weight and Apothecaries' Weight are the same, both differ from the corresponding denominations of Avoirdupois Weight, as shown in the following

COMPARISON OF WEIGHTS.

7000 Troy grains = 1 lb. Avoir.	437½ Troy grains = 1 oz. Avoir.
5760 " " = 1 " Troy.	480 " " = 1 " Troy.

MEASURES OF EXTENSION.

160. Linear Measure is used in measuring lengths, breadths, depths, heights, or distances. *Scale:* 12 inches (in.) = 1 foot (ft.); 3 feet = 1 yard (yd.); 5½ yards = 1 rod (rd.); 320 rods = 1 mile (mi.).

161. Surveyors' Linear Measure is used by surveyors in measuring boundaries of land, length of roads, etc. *Scale:* 7.92 inches (in.) = 1 link (l.); 100 links = 1 chain (ch.); 80 chains = 1 mile (mi.).

MEASURES OF SURFACE.

162. Square Measure is used in measuring surfaces, such as land, boards, plastering, painting, paving, etc. *Scale:* 144 square inches (sq. in.) = 1 square foot (sq. ft.); 9 square feet = 1 square yard (sq. yd.); 30¼ square yards = 1 square rod (sq. rd.); 160 square rods = 1 acre (A.); 640 acres = 1 square mile (sq. mi.)

163. Surveyors' Square Measure is used by surveyors in measuring land. *Scale:* 10000 square links (sq. l.) = 1 square chain (sq. ch.); 10 square chains = 1 acre (A); 640 acres = 1 square mile (sq. mi.).

MEASURES OF VOLUME.

164. Cubic Measure is used in measuring the contents of hollow bodies or the volume (bulk, size) of solid bodies. *Scale:* 1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.); 27 cubic feet = 1 cubic yard (cu. yd.).

165. Wood Measure is used in measuring wood. *Scale:* 16 cubic feet (cu. ft.) = 1 cord foot (cd. ft.); 8 cord feet or 128 cubic feet = 1 cord (cd.)

MEASURES OF CAPACITY.

166. Liquid Measure is used in measuring liquids. *Scale:* 4 gills (gi.) = 1 pint (pt.); 2 pints = 1 quart (qt.); 4 quarts = 1 gallon (gal.).

167. Apothecaries' Fluid Measure is used by physicians in prescribing and by apothecaries in compounding liquid medicines. *Scale:* 60 minims (♏) = 1 fluiddrachm (f ʒ); 8 fluiddrachms = 1 fluidounce (f ℥); 16 fluid ounces = 1 pint (O); 8 pints = 1 gallon (Cong.).

168. Dry Measure is used in measuring grain, fruit, and vegetables. *Scale:* 2 pints (pt.) = 1 quart (qt.); 8 quarts = 1 peck (pk.); 4 pecks = 1 bushel (bu.).

MISCELLANEOUS MEASURES.

169. Circular Measure is used in estimating latitude and longitude, and in measuring angles. *Scale:* 60 seconds (″) = 1 minute (′); 60 minutes = 1 degree (°); 360 degrees = 1 circle (C).

170. In counting certain articles, the following *Scale* is used:

12 things = 1 dozen (doz.); 12 dozen = 1 gross (gro.); 12 gross = 1 great gross (g. gro.); 20 things = 1 score.

171. In measuring paper the following *Scale* is used: 24 sheets = 1 quire (qu.); 20 quires = 1 ream (rm.); 2 reams = 1 bundle (bdl.); 5 bundles = 1 bale (bl.).

MEASURE OF TIME.

172. *Scale:* 60 seconds (sec.) = 1 minute (min.); 60 minutes = 1 hour (hr.); 24 hours = 1 day (da.); 365 days = 1 common year (yr.); 366 days = 1 leap year (yr.). Also, 7 days = 1 week (wk.); 12 months (mo.) = 1 year; 100 years = 1 century.

REM. 1.—A *Solar Year* is the exact time required by the earth to make a complete revolution around the sun. It consists of 365 days, 5 hours, 48 minutes, and 49.7 seconds.

REM. 2.—A *Common Year*, for convenience, is made to consist of 365 days; hence, 4 common years are nearly one day less than 4 solar years, and 1 day is added to the fourth common year to rectify the error. The extra day is added to the month of February, and the year so increased is called a *leap year.*

REM. 3.—*Years divisible by 4, except centennial years, are leap years. Centennial years divisible by 400 are leap years.*

REM. 4.—The order and name of each month and its length in days are:

1st,	January	31 days.	7th,	July	31 days.
2d,	February . . .	28 or 29 "	8th,	August	31 "
3d,	March	31 "	9th,	September	30 "
4th,	April	30 "	10th,	October	31 "
5th,	May	31 "	11th,	November	30 "
6th,	June	30 "	12th,	December	31 "

173. Reduction of compound numbers to lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce 19 bu., 2 pk., 5 qt., to quarts.

SOLUTION.

bu. pk. qt.
19 2 5
4
78 (19 × 4 + 2)
8
629 (78 × 8 + 5)

EXPLANATION.—First reduce the highest denomination (19 bu.) to the next lower denomination (pk.) as follows: Since 1 bu. = 4 pk., 19 bu. must equal 19 times 4 pk., or 76 pk., and 19 bu. 2 pk. must equal 76+2, or 78 pk.

Then reduce the obtained result (78 pk.) to the next lower denomination (qt.), as follows: Since 1 pk. = 8 qt., 78 pk. must equal 78 times 8 qt., or 624 qt., and 78 pk. 5 qt. must equal 624+5, or 629 qt.

RULE.—I. *Multiply the highest denomination by the number of units of the next lower denomination, which will equal one unit of the highest denomination, and to the product add the given number, if any, of such lower denomination.*

II. *Similarly reduce the result thus obtained to units of the next lower denomination; and so continue until the given compound number has been reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

Reduce

1. $5 and 3¢ to mills.
2. £75 15*s.* 9*d.* to farthings.
3. £125 3*s.* to pence.
4. £47 12*s.* to farthings.
5. 31 bu. 1 pk. 7 qt. to pints.
6. 12 gal. 3 qt. 1 pt. 3 gi. to gills.
7. 2 yr. 15 da. 7 hr. 34 sec. to sec.
8. 195 T. 18 lb. to ounces.
9. 5 lb. 17 pwt. to grains.
10. 15 T. 14 cwt. 83 lb. to ounces.

Reduce

11. 5 mi. 137 rd. 1 ft. to inches.
12. 73 A. 15 sq. rd. 18 sq. yd. to sq. in.
13. 19 rd. 4 yd. 2 ft. to feet.
14. 7 cd. 5 cd. ft. to cubic feet.
15. 19 mi. 13 rd. to feet.
16. 19 cd. 32 cu. ft. to cu. ft.
17. 8 T. 3 cwt. 7 lb. to pounds.
18. 1 yr. 271 da. 45 min. to sec.
19. 5 reams of paper to sheets.
20. 75 lb. 8 oz. to pennyweights.

174. Reduction of denominate numbers to higher denominations.

ILLUSTRATIVE EXAMPLE.

Reduce 5749 pence to the higher denominations.

SOLUTION.

12) 5749

20) 479 + 1d.

£23 + 19s.

Ans. £23 19s. 1d.

EXPLANATION.—First reduce the given denomination (d.) to the next higher denomination (s.) as follows: if 12d. = 1s., there are as many shillings in 5749d. as 12d. are contained times in 5749d., or 479s. and 1d. remainder.

Then reduce the obtained result (479s.) to the next higher denomination (£), as follows: if 20s. = £1, there are as many £'s in 479s. as 20s. are contained times in 479s., or £23 and 19s. remainder. Hence, 5749d. equal £23 19s. 1d.

RULE.—I. *Divide the given number by the units of its denomination which equal 1 unit of the next higher denomination, and reserve the remainder, if any, as a part of the answer.*

II. *Proceed in like manner with this and each succeeding quotient until the required higher denomination is reached.*

III. *The last quotient with the several remainders annexed will be the answer.*

NOTE.—In reducing to higher denominations, observe (1) that each successive divisor should be of the same denomination as its dividend; (2) that each remainder is of the same denomination as the dividend from which it came; (3) that each quotient is of the denomination that its divisor equals in the scale.

EXAMPLES FOR PRACTICE.

Reduce

1. 31473 gi. to gallons.
2. 716 qt. to bushels.
3. 7318 sheets to reams.
4. 5178 pt. to bushels.
5. 3812718 min. to com. years.
6. 78 years to scores.
7. 17365 farthings to £'s.
8. 5678 mills to dollars.
9. 153812 oz., Avoir., to tons.
10. 58376 gr. to pounds, Troy.

Reduce

11. 17532 pence to £'s.
12. 183712 in. to miles.
13. 4918 cu. ft. to cords.
14. 357182 sq. ft. to acres.
15. 938713 ft. to rods.
16. 1325173 sec. to days.
17. 5273 pwt. to pounds, Troy.
18. 57186 oz., Avoir., to cwt.
19. 37821 sq. yd. to acres.
20. 2016 lb., Avoir., to lb., Troy.

175. Reduction of denominate common fractions to integers of lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce £$\frac{7}{8}$ to integers of the lower denominations.

SOLUTION.

7
20
8) 140 (17s.
8
60
56
4
12
8) 48 (6d.
48

EXPLANATION.—Rule, **173**, is equally applicable to denominate fractions. Hence, reduce £$\frac{7}{8}$ to the next lower denomination (s) by muliplying the numerator (7) by 20s. and dividing the product (140s.) by the denominator 8, (Note 4, **105**), obtaining 17 integral shillings and $\frac{4}{8}$ of a shilling.

In the same manner, reduce $\frac{4}{8}$s. to the next lower denomination (d) by multiplying the numerator (4) by 12d. and dividing the product (48d.) by the denominator (8), obtaining 6d. Answer, 17s. 6d.

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations.

1. $\frac{8}{9}$ da.
2. £$\frac{2}{3}$.
3. $\frac{3}{8}$ gal.
4. $\frac{7}{16}$ bu.
5. $\frac{2}{3}$ yr.
6. $\frac{5}{6}$ pk.
7. £$\frac{5}{16}$.
8. $\frac{5}{12}$ mi.
9. $\frac{2}{3}$ lb., Troy.
10. $\frac{3}{11}$ A.
11. $\frac{5}{16}$ T.
12. $\frac{5}{7}$ yd.

176. Reduction of denominate decimals to integers of lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce .475 of a year to integers of lower denominations.

SOLUTION.

$$\begin{array}{r} .475 \\ \underline{12} \\ \text{mo. } 5.700 \\ \underline{30} \\ \text{da. } 21.0 \end{array}$$

EXPLANATION.—Applying Rule, **173,** reduce .475 yr. to the next lower denomination (mo.), obtaining 5.700 mo. Reserve the integral part of this product (5 mo.) as the first term of the answer, and reduce the decimal part (.7 mo.) to the next lower denomination (da.), obtaining 21 da. *Ans.* 5 mo. 21 da.

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations.

1. £.375.
2. .1375 bu.
3. .45 yr.
4. .46875 gal.
5. £.00625.
6. .14225 yr.
7. .875 lb., Troy.
8. .15915625 T.
9. .378125 A.

177. Reduction of compound numbers to decimals of specified higher denominations.

ILLUSTRATIVE EXAMPLE.

Reduce 18s. 9d. to a decimal of a £.

SOLUTION.

$$\begin{array}{r} 12\,\underline{)\;9.00} \\ 20\,\underline{)\;18.7500} \\ £.9375 \end{array}$$

EXPLANATION.—Commencing with the lowest denomination (9d.), reduce it to the next higher denomination (s.) by dividing it by 12d. (Note, **174**), obtaining .75s.

Prefix 18s. and similarly reduce the result (18.75s.) to the next higher and specified denomination (£) by dividing it by 20, obtaining £.9375.

EXAMPLES FOR PRACTICE.

What decimal of
1. A £ are 13s. 9d.?
2. An hour are 4 min. 30 sec.?
3. A yard are 2 ft. 3 in.?
4. A ton are 1 cwt. 20 lb.?

What decimal of
5. A day are 21 hr. 54 min.?
6. A bushel are 3 pk. 5 qt. 1 pt.?
7. A £ are 1s. 3d.?
8. A gallon are 1 qt. 1 pt. 2 gi.?

ADDITION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

178. Add £15 12s. 8d.; £7 6d.; and 15s. 7d.

SOLUTION.

£	s.	d.
15	12	8
7	0	6
	15	7
£23	8s.	9d.

EXPLANATION.—Arrange the addends so that their similar denominations shall fall in the same column.

Beginning with the lowest denomination, add the column of pence, obtaining 21d., or 1s. 9d. Write 9d. under the column of pence, and add 1s. with the column of shillings, obtaining 28s., or £1 8s. Write 8s. under the column of shillings, etc., etc.

EXAMPLES FOR PRACTICE.

(1.)

£	s.	d.	far.
75	5	8	3
13	16	5	1
96	8	11	2
52	13	9	1
7	2	4	2
33	18	8	3

(2.)

lb.	oz.	pwt.	gr.
5	9	12	9
13	4	16	8
41	6	8	15
71	11	18	22
56	7	13	19
72	8	14	16

(3.)

mi.	rd.	yd.	ft.	in.
5	175	3	1	8
17	248	4	2	6
25	315	1	1	11
41	214	3	1	7
50	16	2	2	9
48	296	4	1	5

4. Add 5 gal. 3 qt. 1 pt.; 7 gal. 2 qt.; 2 qt. 1 pt.; and 8 gal. 1 pt.

5. An importer paid £375 8s. 6d. for an invoice of cutlery; £251 5s. 8d. for an invoice of files; and £73 17s. 6d. for an invoice of saws. The charges for transportation amounted to £19 2s. 4d. What was the total cost of the goods in sterling money?

6. Add 5 T. 13 cwt. 19 lb. 4 oz.; 7 T. 18 cwt. 39 lb. 5 oz.; 2 T. 12 cwt. 78 lb. 11 oz.; and 5 T. 12 cwt. 94 lb. 7 oz.

7. What is the sum of $\frac{3}{4}$ bu., $\frac{7}{8}$ pk., and $\frac{1}{2}$ qt.?

SUBTRACTION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

179. Subtract 3 bu. 2 pk. 4 qt. from 8 bu. 1 pk. 6 qt.

SOLUTION.

bu.	pk.	qt.
8	1	6
3	2	4
4 bu.	3 pk.	2 qt.

EXPLANATION.—Write the several denominations of the subtrahend under similar denominations of the minuend. Beginning with the lowest denomination, subtract 4 qt. from 6 qt., obtaining 2 qt. Then, since 2 pk. cannot be subtracted from 1 pk., borrow 1 bu. or 4 pk., and 2 pk. from 1+4, or 5 pk., leaves 3 pk. Next subtract 3 bu. from 8—1, or 7 bu., obtaining 4 bu.

EXAMPLES FOR PRACTICE.

1. A merchant sold 25 gal. 2 qt. 1 pt. 3 gi. of vinegar from a barrel containing 46 gal. 1 qt. 1 pt. 2 gi. What quantity remained?

2. A farmer engaged to deliver 15 T. 16 cwt. 75 lb. of hay, and subsequently delivered 9 T. 18 cwt. 34 lb. How much hay had he yet to deliver?

3. An English merchant's sales for a year amounted to £15396 5s. 8d. 2 far., and his purchases £18352 2s. 9d. 1 far. How much greater were his purchases than his sales?

180. To find the interval between two dates.

ILLUSTRATIVE EXAMPLE.

Find the interval between Sept. 25, 1863, and July 28, 1889.

SOLUTION.

yr.	mo.	da.
1889	7	28
1863	9	25
25	10	3

EXPLANATION.—Write the later date, expressing the *greater* period, as the minuend, and the earlier date as the subtrahend. Subtract as in **179.**

The minuend denotes the 1889th year of the Christian era, the 7th month of the year, and the 28th day of the month. The subtrahend denotes the 1863d year of the Christian era, the 9th month of the year, and the 25th day of the month.

NOTE.—In compound subtraction, it is usual to consider the borrowed month as equal to 30 days.

EXAMPLES FOR PRACTICE.

By compound subtraction find the interval between

1. Feb. 3, 1875, and Aug. 1, 1883.
2. June 7, 1853, and Mar. 11, 1881.
3. Jan. 25, 1815, and Sept. 3, 1836.
4. Mar. 3, 1848, and Mar. 2, 1861.

By compound subtraction find the interval between

5. Oct. 15, 1825, and Apr. 5, 1886.
6. Nov. 3, 1849, and Aug. 15, 1880.
7. Apr. 15, 1871, and Feb. 2, 1879.
8. Dec. 5, 1843, and Jan. 19, 1847.
9. Oct. 25, 1875, and Oct. 15, 1884.
10. May 19, 1807, and Feb. 13, 1887.

181. To find the number of days between two dates.

ILLUSTRATIVE EXAMPLE.

Find the number of days between Jan. 12, 1889, and May 16, 1889.

SOLUTION.

19	da.,	included part of Jan.
28	"	in February.
31	"	in March.
30	"	in April.
16	"	included part of May.
124	"	in the interval.

EXPLANATION.—The required interval includes the remainder of January (31−12 = 19 da.); the whole of the following months of February (28 da.); of March (31 da.); of April (30 da.); and 16 days in May, amounting to 124 days.

NOTE.—In finding the exact number of days, the first day of the interval is omitted and the last day counted.

EXAMPLES FOR PRACTICE.

Find the exact number of days between

1. May 18, 1888, and Aug. 20, 1888.
2. Oct. 13, 1888, and Mar. 7, 1889.
3. Jan. 25, 1889, and Sept. 3, 1889.
4. Nov. 16, 1888, and May 23, 1889.
5. Dec. 25, 1887, and Apr. 9, 1889.
6. Mar. 6, 1889, and Oct. 21, 1889.
7. June 9, 1888, and Mar. 25, 1889.
8. Feb. 13, 1888, and Nov. 8, 1888.
9. Sept. 25, 1888, and Aug. 3, 1889.
10. July 18, 1888, and Mar. 2, 1889.
11. Apr. 10, 1889, and Oct. 1, 1889.
12. Sept. 15, 1888, and Dec. 3, 1889.

MULTIPLICATION OF COMPOUND NUMBERS.

ILLUSTRATIVE EXAMPLE.

182. Multiply £15 9s. 5d by 7.

SOLUTION.

£15	9s.	5d.
		7
£108	5s.	11d.

EXPLANATION.—7 times 5d. = 35d. = 2s. 11d. Write 11d. as a term of the product, and carry 2s. to the product of shillings.

7 times 9s. = 63s. plus 2s. carried = 65s. = £3 5s. Write 5s. as the second term of the product, and carry £3 to the product of £'s.

(£15 × 7) + £3 to carry = £108, the last term of the product.

EXAMPLES FOR PRACTICE.

Multiply

1. 17 bu. 2 pk. 5 qt. 1pt. by 8.
2. 7 da. 18 hr. 35 min. 41 sec. by 6.
3. 5 lb. 7 oz. 18 pwt. 13 gr. by 7.
4. 9 gal. 1 qt. 1 pt. 3 gi. by 12.

Multiply

5. £25 14s. 10d. by 64 (8 × 8).
6. 6 rd. 4 yd. 1 ft. by 42 (7 × 6).
7. 3 yr. 175 da. by 317.
8. £18 9s. 6d. by 73.

9. What is the cost of 12 barrels of vinegar, averaging 41 gal. 3 qt. 1 pt. 3 gi. per barrel, at 18 cents per gallon?

10. I paid $35 for 8 bbl. of chestnuts, averaging 2 bu. 3 pk. 2 qt. per bbl., and retailed them at 5 cents per pt. What was my gain?

11. In a certain voyage of 28 days, a vessel averaged 12 mi. 56 rd. 4 yd. per hour. What was the total distance traversed?

12. The average weight of 37 barrels of pork is 197 lb. 8 oz. The full weight of each barrel should be 200 pounds. How much less than the required quantity of pork did the 37 barrels contain?

DIVISION OF COMPOUND NUMBERS.

183. Division of compound numbers by abstract numbers.

ILLUSTRATIVE EXAMPLE.

Divide 275 bu. 2 pk. 4 qt. by 6.

SOLUTION.

$$\begin{array}{r} 6\,\underline{)\,275 \text{ bu. } 2 \text{ pk. } 4 \text{ qt.}} \\ 45 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.} \end{array}$$

EXPLANATION.—⅙ of 275 bu. = 45 bu. and 5 bu. over. Write 45 bu. as the first term of the quotient, and carry the remainder 5 bu. or 20 pk. to the next lower term of the dividend (2 pk.), obtaining 22 pk. as the next partial dividend.

⅙ of 22 pk. = 3 pk. and 4 pk. over. Write 3 pk. as the second term of the quotient, and carry 4 pk. or 32 qt. to the next lower term of the dividend (4 qt.), obtaining 36 qt. as the next partial dividend. ⅙ of 36 qt. = 6 qt., the third term of the quotient.

EXAMPLES FOR PRACTICE.

Divide

1. 117 lb. 2 oz. 16 pwt. by 8.
2. £18 16s. 9d. by 9.
3. 96 bu. 3 pk. 2 qt. by 4.
4. 131 yr. 13 hr. by 5.

Divide

5. 59° 18′ 36″ by 6.
6. 8 T. 5 cwt. 8 lb. by 7.
7. 75 mi. by 9.
8. 37 gal. 3 qt. 1 pt. by 5.

9. Divide 385 yd. 2 ft. 8 in. by 41.

SOLUTION.

41) 385 yd. 2 ft. 8 in. (9 yd.
369
16 yd. remainder.
3
41) 50 (1 ft.
41
9 ft. remainder.
12
41) 116 ($2\frac{34}{41}$ in.
82
34
41

EXPLANATION. — If the divisor is greater than 12 and not composite, it is usually more convenient to obtain the quotient by long division.

385 yd. ÷ 41 = 9 yd. and a remainder of 16 yd. or 48 ft., to which add 2 ft. in the dividend, obtaining 50 ft. as the next partial dividend.

50 ft. ÷ 41 = 1 ft., and a remainder of 9 ft. or 108 in., to which add 8 in. in the dividend, obtaining 116 in. as the next partial dividend.

116 in. ÷ 41 = $2\frac{34}{41}$ in.

Complete answer : 9 yd. 1 ft. $2\frac{34}{41}$ in.

Divide
10. £718 17s. 5d. by 325.
11. 175 yr. 5 mo. 18 da. by 41.

Divide
12. 425 mi. 70 rd. 1 ft. by 19.
13. 342 cd. 39 cu. ft. by 537.

184. Division of one compound number by another.

ILLUSTRATIVE EXAMPLE.

Divide 8 bu. 5 qt. by 2 pk. 3 qt. 1 pt.

SOLUTION.

8 bu. 5 qt. = 522 pt.
2 pk. 3 qt. 1 pt. = 39 pt.
522 pt. ÷ 39 pt. = $13\frac{5}{13}$ times.

EXPLANATION.—Reduce both divisor and dividend to simple denominate numbers of the lowest denomination mentioned in either, that is, to pints, obtaining 522 pt. as the equivalent dividend and 39 pt. as the equivalent divisor. Then divide.

EXAMPLES FOR PRACTICE.

1. Divide £226 9s. 5d. by £17 8s. 5d.
2. Divide 29 bu. 3 pk. 3 qt. by 5 bu. 3 pk. 7 qt.
3. Divide £44 2s. by £1 11s. 6d.
4. Divide 5 A. 140 sq. rd. 5 sq. yd. by 96 sq. in.
5. Divide 3 mi. by 5 yd. 2 ft. 5 in.
6. Divide 2 lb. 2 oz. 5 pwt. 16 gr. by 3 oz. 5 pwt. 17 gr.

REVIEW OF DENOMINATE NUMBERS.

185. 1. Add £45 19s. 6¾d.; £234 4s. 5½d.; £218 4s. 6d.; £45 3s. 8d.; 13s. 7d.; £18 9s. 9¼d.; and £147 17s. 6d.

2. What is the total cost of 15 tons of wheat at $1.20 per bushel and 75 centals of rye at 95¢ per bushel?

3. How many forks averaging 2 oz. 9 pwt. each can be made from 5 lb. 1 oz. 5 pwt. of silver?

4. A merchant retailed 57 gal. 3 qt. 1 pt. of wine at $1.25 per pint. How much did he receive for the quantity?

5. A farmer sold 3 loads of wheat containing respectively 1574, 1643, and 1583 lb., at $1.32 per bushel. How much did he receive?

6. What is the cost of 3 hhd. of sugar containing respectively 6 cwt. 5 lb., 5 cwt. 90 lb., and 5 cwt. 83 lb., at 7¢ per pound?

7. A farm laborer was employed on the morning of March 15, 1884, at 45¢ per day, Sundays and holidays included, and discharged on the evening of Jan. 18, 1885. What was the balance due him if he had previously drawn $75.30?

8. A peddler bought 7 bushels of peanuts at $1.30 per bushel, and retailed them at 8¢ per quart. What was his total gain?

9. A dealer bought 220 long tons of coal at $4.80 per ton, and sold it at $5.10 per short ton. What was his gain?

10. The American Civil War commenced April 11, 1861, and closed April 19, 1865. How long did it continue?

11. At $20 per ounce, what is the value of 24 gold rings, each weighing 10 pwt., and 4 gold bracelets, each weighing 3 oz. 16 pwt.?

12. What quantity of potatoes will 145 A. yield, if the average per acre is 119 bu. 2 pk. 5 qt.?

13. A grocer sold 7 gal. 2 qt. 1 pt. of syrup daily for 25 days at the rate of 5¢ per pint. How much did he receive for the syrup in the given time?

14. A man is engaged to work at $1.80 per day of 10 hours. What should be his week's wages, if on Monday he worked 8 hr. 15 min., on Tuesday 9 hr. 30 min., on Wednesday 5 hr., on Thursday 7 hr. 45 min., on Friday 8 hr. 20 min., and on Saturday 6 hr. 20 min.?

15. What is the total cost of 9875 lb. coal at $4.80 per ton (of 2000 lb.), 12360 lb. at $5.25 per ton, and 7240 lb. at $5.45 per ton?

16. If 2 square miles of land be equally divided among 24 settlers, how much land will each receive?

17. A note drawn May 15, 1885, was not paid until Jan. 5, 1886. How many days did it remain unpaid?

18. At 8¢ per quart, what is the cost of 5 bu. 1 pk. 3 qt. 1 pt. of beans?

19. From a cask containing 42 gallons of water, 1 gal. 3 qt. leak out daily. In what time will the cask be emptied?

20. If the average capacity per barrel is 2 bu. 2 pk. 7 qt., how many barrels will a farmer need to ship 230 bu. potatoes?

21. At $6.25 per barrel, how much should be paid for 4704 pounds of flour?

22. A lady paid £5 for 40 yd. linen. What did she pay per yard?

23. An apothecary paid 22¢ per pound for 140 pounds, Avoir., of drugs, and sold the same at 80¢ per pound, Troy. What was his gain?

24. What is the value in U. S. money of 953 francs (France)? **25.** Of £547 (England)? **26.** Of 35 crowns (Norway)? **27.** Of £78 15s. (England)? **28.** Of 915.35 marks (Germany)? **29.** Of 4187.93 roubles (Russia)? **30.** 79.18 yen (Japan)? **31.** Of 819.35 florins (Austria)? **32.** Of 2193.22 lira (Italy)?

33. Reduce $419.78 to German marks. **34.** $675.40 to French francs. **35.** $517.75 to Russian roubles. **36.** $91.50 to Netherlands guilders. **37.** $729.45 to Spanish peseta. **38.** $1491.60 to Egyptian pounds. **39.** $672.85 to £'s sterling. **40.** $825.90 to Brazilian milreis. **41.** $4981.35 to Mexican dollars.

METRIC SYSTEM.

186. The **Metric System** is a decimal system of weights and measures, which has for its basis a certain unit of length called the *meter*.

REM. 1.—The *meter* is $\frac{1}{10000000}$ of the distance from the equator to either pole. The length of the meter is 39.37079 inches.

REM. 2.—The use of the metric system in the United States is limited to importers of foreign merchandise, the invoices of which are made out upon this system in most of the countries of Europe, in Mexico, the West Indies, the Central and South American States, etc.

187. The **Primary Units** of the metric system are:

KIND OF MEASURE.	PRIMARY UNIT.	PRONUNCIATION.
Measure of Length, - - - -	*Meter*, - - - -	(mee′ter).
Measure of Capacity, - - -	*Liter*, - - - -	(lee′ter).
Measure of Weight, - - - -	*Gram*, - - - -	(gram).
Land Measure, - - - - -	*Ar*, - - - - -	(air).
Ordinary Surface Measure, -	*Square Meter*, -	(square mee′ter).
Ordinary Cubic Measure, - -	*Cubic Meter*, - -	(cubic mee′ter).
Wood Measure, - - - - -	*Ster*, - - - -	(stair).

188. Higher Denominations than the above primary units are denoted by prefixing the following Greek numerals: *deka* (10 primary units); *hekto* (100 primary units); *kilo* (1000 primary units); *myria* (10000 primary units).

189. Lower Denominations than the primary units are denoted by prefixing the following Latin numerals: *deci* ($\frac{1}{10}$ of a primary unit); *centi* ($\frac{1}{100}$ of a primary unit); *milli* ($\frac{1}{1000}$ of a primary unit).

Rem. 1.—The above numeral prefixes indicate the decimal value of the derivative units as compared with the primary units. Thus, 5 millimeters denote .005 of a meter; 18 centigrams denote .18 of a gram; 9 deciliters, .9 of a liter; 25 hektoliters, 2500 liters; 13 kilometers, 13000 meters; 5 dekasters, 50 sters; etc., etc.

Rem. 2.—Metric quantities other than surfaces or solids are written and read, like other decimals, on a scale of 10, each denomination being allotted *one* place. In square measure, however, denominations are expressed on a scale of 100, and in cubic measure on a scale of 1000; hence, each denomination of square measure must have *two* places of figures, and of cubic measure *three* places.

190. The following equivalents are such as have been established by Act of Congress. In the United States, reductions from the metric system to the common system, or the reverse, must be made in accordance therewith, to be legal.

EQUIVALENTS OF LINEAR MEASURE.

1 inch = 2.54 centimeters.
1 foot = .3048 of a meter.
1 yard = .9144 of a meter.
1 rod = 5.029 meters.
1 mile = 1.6093 kilometers.

1 centimeter = .3937 of an inch.
1 decimeter = .328 of a foot.
1 meter = 1.0936 yards.
1 dekameter = 1.9884 rods.
1 kilometer = .62137 of a mile.

EQUIVALENTS OF SQUARE MEASURE.

1 sq. inch = 6.452 sq. centimeters.
1 sq. foot = .0929 of a sq. meter.
1 sq. yard = .8361 of a sq. meter.
1 sq. rod = 25.293 sq. meters.
1 acre = 40.47 ars.
1 sq. mile = 259 hektars.

1 sq. centimeter = .155 of a sq. inch.
1 sq. decimeter = .1076 of a sq. foot.
1 sq. meter = 1.196 sq. yards.
1 ar = 3.954 sq. rods.
1 hektar = 2.471 acres.
1 sq. kilometer = .3861 of a sq. mile.

EQUIVALENTS OF CUBIC MEASURE.

1 cu. inch = 16.387 cu. centimeters.
1 cu. foot = 28.317 cu. decimeters.
1 cu. yard = .7645 of a cu. meter.
1 cord = 3.624 sters.

1 cu. centimeter = .061 of a cu. inch.
1 cu. decimeter = .0353 of a cu. foot.
1 cu. meter = 1.308 cu. yards.
1 ster = .2759 of a cord.

EQUIVALENTS OF CAPACITY.

1 liquid quart = .9463 of a liter.
1 dry quart = 1.101 liters.
1 liquid gallon = .3785 of a dekaliter.
1 peck = .881 of a dekaliter.
1 bushel = .3524 hektoliters.

1 liter = 1.0567 liquid quarts.
1 liter = .908 of a dry quart.
1 dekaliter = 2.6417 liquid gallons.
1 dekaliter = 1.135 pecks.
1 hektoliter = 2.8375 bushels.

EQUIVALENTS OF WEIGHT.

1 ounce, Avoir. = 28.35 grams.
1 ounce, Troy = 31.104 grams.
1 pound, Avoir. = .4536 of a kilogram.
1 pound, Troy = .3732 of a kilogram.
1 grain, Troy = .0648 of a gram.
1 ton (of 2000 lb.) = .9072 of a tonneau.

1 gram = .03527 of an ounce, Avoir.
1 gram = .03215 of an ounce, Troy.
1 gram = 15.432 grains, Troy.
1 kilogram = 2.2046 pounds, Avoir.
1 kilogram = 2.679 pounds, Troy.
1 tonneau = 1.1023 tons (of 2000 lb.).

191. Reductions from the common system to the metric, or the reverse, are effected by the use of the above table of equivalents.

Thus, to find how many sters equal 75 cords of wood, refer to the equivalents of cubic measure to find how many sters equal 1 cord (3.624 sters), and 75 cords must equal 75 times 3.624 sters, or 271.8 sters.

Reduce
1. 385.25 ars to acres.
2. 375 hektoliters to bushels.
3. 153 kilometers to miles.

Reduce
4. 483 dekaliters to gallons.
5. 75.5 kilograms to lb., Avoir.
6. 157.75 hektars to acres.

Reduce

7. 27.5 centimeters to inches.
8. 87 yards of cloth to meters.
9. 187 cords of wood to sters.
10. 157.35 acres of land to ars.

Reduce

11. 173 gallons to dekaliters.
12. 3 ounces of gold to grams.
13. 187 bushels to hektoliters.
14. 845 cu. yd. to cubic meters.

15. What is the cost of 34.5 kilos of sugar at 14¢ per kilo?

16. What is the value in U. S. money of 364.625 meters of cloth, invoiced at 8.40 francs per meter?

17. A merchant imported 10000 dekaliters of wine at 62 fr. per dekaliter, and sold the same at $4.75 per gal. What was his gain?

18. Find the cost of 24 dekaliters of peaches at 40¢ per peck.

19. An importer mixed 18 liters of wine costing 1.75 francs per liter with 15 liters costing 2.15 francs per liter, and sold the entire mixture at $1.75 per gallon. Did he gain or lose, and how much?

20. What is the total cost in U. S. money of the following: 52.75 M. of cloth bought at 3.20 fr. per M.; 28.375 M. at 4.75 fr. per M.; 91.85 dm. at 6.50 fr. per M.; and 325 Dm. at 8.70 fr. per M.?

SURFACE MEASUREMENTS.

192. A **Surface** is the outside of a solid or body. It can therefore contain only *two* of the three dimensions of a body, its depth or thickness not being considered.

A flat surface is called a plane surface, or simply, a *plane*. To discover if any surface is a plane, place the straight edge of a ruler upon it in many directions. If all parts of the edge touch the surface, it is a plane.

193. A **Rectangular Surface** is a plane surface of four sides, each of which is straight, and perpendicular to its adjacent sides.

The opposite sides of a rectangular surface are equal to each other.

194. A **Square** is a rectangular surface having *all* of its sides equal in length.

A square inch is a rectangular surface, *each side* of which is one inch in length; a square foot is a rectangular surface each side of which is one foot; etc.

195. The **Area** of a rectangular surface is the number of square units which it contains.

Rem.—In obtaining areas, the most convenient unit of measure is a square, each side of which is a unit of the lowest denomination in either of the two given dimensions. Hence, to find the area of a rectangular surface 8 ft. long and 5 ft. wide, the measuring unit will be 1 sq. ft. Having selected the measuring unit, the area is found as follows:

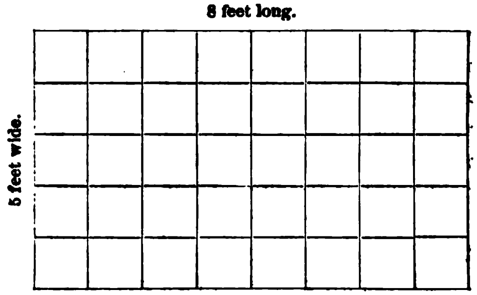


Figure 3.

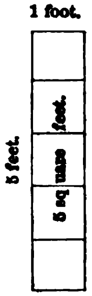


Fig. 2.

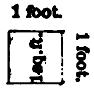


Fig. 1.

5 square feet × 8 = 40 square feet, area of Fig. 3.

Fig. 1 is the measuring square, which is 1 ft. long and 1 ft. wide, and therefore represents 1 sq. ft. In Fig. 2 are 5 such squares, and hence it represents 5 sq. ft. In Fig. 3 are 8 rows of measuring squares, each row containing as many measuring squares as Fig. 2, and it must therefore represent 8 times 5 sq. ft., or 40 sq. ft.

196. To find the area of a rectangular surface.

ILLUSTRATIVE EXAMPLE.

How many square yards are in a sidewalk 48 ft. long and 11 ft. 4 in. wide?

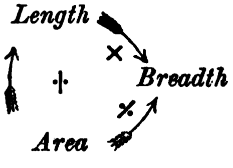
SOLUTION.

11 ft. 4 in. = 11$\frac{1}{3}$ ft.

48 sq. ft. × 11$\frac{1}{3}$ = 544 sq. ft.

544 ÷ 9 = 60$\frac{4}{9}$ sq. yd.

Explanation.—A sidewalk 48 ft. long and 1 ft. wide contains 48 sq. ft. Hence, a sidewalk 48 ft. long and 11$\frac{1}{3}$ ft. wide must contain 11$\frac{1}{3}$ times 48 sq. ft., or 544 sq. ft., which reduced to sq. yd. equal 60$\frac{4}{9}$.



General Formula.

Direction.—If any two of the three terms of this formula are given, perform the operation indicated by the sign between them, and the result will be the third term.

Rem.—Before applying the General Formula, see that the length and breadth, if given, are expressed in the same linear units; and if the area and one dimension are given, that the area is expressed in square units and the given dimension in linear units of the same denomination.

EXAMPLES FOR PRACTICE.

1. How many acres are contained in a rectangular field 847 yards long and 240 yards wide?

2. How many square yards are in the walls of a room 24 ft. long, 18 ft. wide, and 12 ft. high?

3. How many square feet are in the surface of a box 4 ft. long, 3 ft. 9 in. wide, and 2 ft. high?

4. How many square feet are in the floor of a room 27 ft. 6 in. long and 13 ft. 7 in. wide?

5. A street ¼ of a mile long was paved at a cost of 45 cents per square foot. What was the cost of paving if the street was 40 ft. wide?

6. How much will it cost to plaster the ceiling of a room 36 ft. long and 27 ft. wide, at 23 cents per square yard?

7. At $7.25 per 100 sq. ft., how much will it cost to tin the roof of a house 45 ft. long, and which measures 23 ft. from each eave to the ridge?

8. A blackboard 8 ft. 6 in. long and 3 ft. wide was bought at 65 cents per square foot. What was the cost?

9. If a rectangular field containing 30 acres is 40 rods wide, what is its length?

SOLUTION.

30 acres = 4800 sq. rd.
4800 ÷ 40 = 120 rd., length.

EXPLANATION.—A field 40 rd. wide and 1 rd. long contains 40 sq. rd.; therefore to contain 30 acres or 4800 sq. rd., it must be as many times 1 rd. long as 4800 sq. rd. are times 40 sq. rd., or 120 rd. long.

10. If a blackboard contains 36 sq. ft. and is 3 ft. wide, what is its length?

11. If a rectangular field of wheat contains 391 acres, and its length is 85 chains, what is its width?

12. A farmer raised 1575 bushels of wheat in a rectangular field which averaged 25 bushels to the acre. If the field was 112 rods long, what was its width?

13. A rectangular piece of land contains 11520 acres, and its length is 15840 yards. What is its width?

14. If the area of a floor is 48 sq. yd. 5 sq. ft., and its breadth is 7 yd. 2 ft., what is its length?

15. A plasterer whose price was 25¢ per sq. yd., charged $8 for plastering the ceiling of a room 16 ft. wide. What was the length of the room?

16. If a rectangular field which contains 57 A. 155 P. is 175 rods long, what is its width?

17. What is the length of a blackboard which contains 9 sq. yd. 5 sq. ft. 36 sq. in., if its breadth is 3 ft. 9 in.?

197. To find the quantity of material to cover a given surface.

ILLUSTRATIVE EXAMPLE.

Allowing nothing for waste, how many yards of carpet 30 in. wide are required to cover a floor 17 ft. 6 in. long and 16 ft. 6 in. wide?

SOLUTION.

$17\frac{1}{2} \times 16\frac{1}{2} = 288\frac{3}{4}$ sq. ft., area of floor.
$3 \times 2\frac{1}{2} = 7\frac{1}{2}$ sq. ft., area of 1 yd. of carpet.
$288\frac{3}{4} \div 7\frac{1}{2} = 38\frac{1}{2}$ yd., length of carpet.

EXPLANATION.—1 yard of carpet of the given width can cover only $7\frac{1}{2}$ sq. ft. of the floor; hence, to cover the entire surface of the floor, the length of the carpet must be as many times 1 yard as the area of the floor ($288\frac{3}{4}$ sq. ft.) is times the area of 1 yard of carpet ($7\frac{1}{2}$ sq. ft.), or $38\frac{1}{2}$ yards.

RULE.—*Divide the area of the floor by the area of one yard of the given material. The quotient will be the number of yards of material.*

NOTE 1.—To find the number of stones, bricks, shingles, etc., to cover a given surface, *divide the area of the surface to be covered by the* EXPOSED *area of one unit of the given material.*

NOTE 2.—With this class of problems, the area of 1 yard, 1 shingle, 1 brick, etc., is the *concrete factor* of Gen'l Formula II., **55**; the number of yards, shingles, bricks, etc., is the *abstract factor;* and the area of the surface to be covered is the *product.*

EXAMPLES FOR PRACTICE.

Allowing nothing for waste, how many yards of carpet are required

1. For a room 16 ft. long and 14 ft. wide; carpet 30 in. wide?

2. For a room 20 ft. 3 in. by 18 ft. 6 in.; carpet 1 yd. wide?

3. For a room 30 ft. 5 in. by 24 ft. 8 in.; carpet 2 ft. 4 in. wide?

4. What is the cost of carpeting a room 24 ft. 3 in. long and 18 ft. 8 in. wide; carpet 28 in. wide and $1.25 per yard?

5. How many bricks 8 in. long and 4 in. wide are necessary to cover a sidewalk 34 ft. 4 in. long and 14 ft. 3 in. wide?

6. How many flagstones 20 in. long and 16 in. wide will pave a yard 57 ft. 11 in. long and 10 ft. 8 in. wide?

7. Making no allowance for waste, how many shingles 4 in. wide are required to cover the roof of a house 36 ft. long and measuring 20 ft. from each eave to the ridge, if only 5 inches of the length of the shingles are exposed to the weather?

8. How many sods 10 in. long and 8 in. wide are required to turf a plat of ground 28 ft. 4 in. long and 22 ft. 4 in. wide?

9. How many yards of carpet are required to cover the floor of a room 22 ft. 1 in. long and 18 ft. 9 in. wide, if the width of the carpet is 27 in., and the strips are laid crosswise?

SOLUTION.

(22 ft. 1 in.) ÷ 27 in. = $9\frac{22}{27}$ strips, or practically 10 strips.

(18 ft. 9 in.) × 10 = 62 yd. 1 ft. 6 in.

EXPLANATION.—Since the strips are to be laid crosswise (parallel to the less dimension), divide the length of the room (22 ft. 1 in.) by the width of the carpet (27 in.) to find the number of strips (practically 10 strips), the excess of the 10th strip (5 in.) being either turned under in laying, or cut off before laying.

As the carpet is to be laid crosswise, the length of each strip must be the same as the less dimension (18 ft. 9 in.). Hence, multiply the length of each strip (18 ft. 9 in.) by the number of strips (10) to find the number of yards to be purchased.

If the design of the carpet is such as to involve no loss in matching, how many yards must be purchased to cover the floor of a room

10. 18 ft. 6 in. by 15 ft. 3 in.; carpet 32 in. wide, laid lengthwise?

11. 31 ft. 8 in. by 25 ft. 2 in.; carpet 28 in. wide, laid crosswise?

12. I wish to carpet a room 20 ft. long by 18 ft. wide with carpeting 2 ft. 6 in. wide. At $2.25 per yard, how much shall I save by laying the carpet crosswise instead of lengthwise?

MEASUREMENT OF VOLUMES.

198. A **Rectangular Volume** is one which is bounded by six rectangular surfaces (**193**).

All rectangular volumes have three dimensions: *length*, *breadth*, and *thickness*.

199. A **Cube** is a rectangular volume which is bounded by six equal squares, called *faces*. All the edges of a cube are of equal length.

A cubic inch is a cube each edge of which is one inch in length; a cubic foot is a cube each edge of which is one foot in length; etc.

200. The **Contents** or **Volume** of a rectangular solid is the number of cubic units which it contains.

Rem.—In obtaining volumes, the most convenient unit of measure is a cube each edge of which is a unit of the lowest denomination in any of the three given dimensions. Hence, to find the volume of a rectangular solid which is 6 ft. long, 4 ft. wide, and 3 ft. deep, the measuring unit will be 1 cu. ft. After selecting the measuring unit, the required volume is found as follows:

Fig. 4.

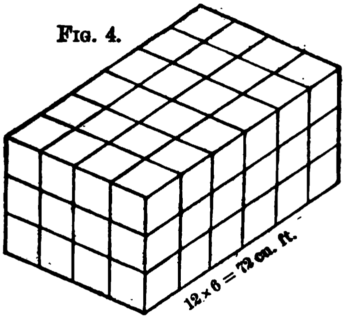


Fig. 3.

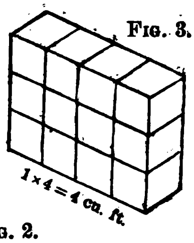


Fig. 2.

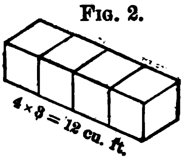
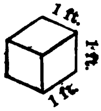


Fig. 1.



1×1×1=1 cu. ft.

4×3×6 = 72 cu. ft., contents of Fig. 4.

Fig. 1 is the measuring cube, which is 1 ft. long, 1 ft. wide, and 1 ft. deep, and therefore represents 1 cu. ft. In Fig. 2, which represents the upper tier of Fig. 3, are 4 such cubes, and hence it represents 4 cu. ft. In Fig. 3, which represents an end section of Fig. 4, are 3 tiers equal to Fig. 2, and hence it represents 3 times 4, or 12 cu. ft. In Fig. 4 are 6 sections equal to Fig. 3, and hence it represents 6 times 12, or 72 cu. ft.

ILLUSTRATIVE EXAMPLE.

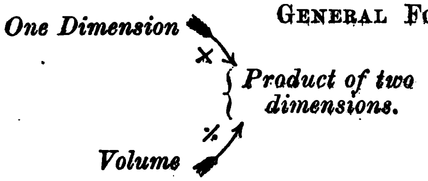
What is the charge for freight upon a box 3 ft. 8 in. long, 3 ft. 6 in. wide, and 2 ft. 3 in. high, at 8¢ per cubic foot?

SOLUTION.

$3\frac{2}{3}$ cu. ft. $\times 3\frac{1}{2} \times 2\frac{1}{4} = 28\frac{7}{8}$ cu. ft.
8¢ $\times 28\frac{7}{8} =$ \$2.31. *Ans.*

EXPLANATION.—The volume of the box is equal to the continued product of its three dimensions (Rem. **200**), or $28\frac{7}{8}$ cu. ft. Hence, multiply the freight per cu. ft. (8¢) by the number of cu. ft. ($28\frac{7}{8}$).

GENERAL FORMULA.



REMARK.—If necessary, reduce the given dimensions to linear units, and the volume to cubic units of the same denomination, before applying the formula.

EXAMPLES FOR PRACTICE.

1. How many cubic feet of air are in a room 18 ft. long, 16 ft. wide, and 10 ft. 6 in. high?

2. How many cubic yards of stone are in a pile 15 ft. long, 12 ft. wide, and 9 ft. high?

3. What is the cost of digging a cellar 30 ft. long, 18 ft. wide, and 12 ft. deep, at 32 cents per cubic yard?

4. How much should be paid for a pile of wood 32 ft. long, 4 ft. wide, and 5 ft. high, at \$6.50 per cord?

5. What is the weight of a square piece of timber 28 ft. long, 15 in. thick, and 15 in. wide, at 45 pounds per cubic foot?

6. Find the cost of removing a rectangular embankment of earth 22 ft. 9 in. long, 8 ft. 6 in wide, and 8 ft. high, at 54¢ per cu. yd.

7. What is the cost of a pile of wood 18 ft. long, 4 ft. wide, and 6 ft. high, at \$6.80 per cord?

8. A rectangular volume 7 ft. long and 5 ft. wide contained 140 cu. ft. What was its height?

SOLUTION.

$140 \div (7 \times 5) = 4$ ft.

EXPLANATION.—A rectangular volume 7 ft. long, 5 ft. wide, and 1 ft. high, contains 35 cu. ft.; hence a rectangular volume 7 ft. long and 5 ft. wide to contain 140 cu. ft. must be as many times 1 ft. high as 140 cu. ft. is times 35 cu. ft., or 4 ft. high.

9. A room 30 ft. long and 25 ft. wide contains 7125 cu. ft. What is its height?

10. What must be the height of a pile of wood 32 ft. long and 6 ft. wide, to contain 9 cords?

11. A contractor agreed to dig a cellar 18 ft. long and 15 ft. wide

200

at 65¢ per cu. yd., and received therefor $45.50. What was the depth of the cellar?

12. What must be the height of wagon-body 10 ft. long and 4 ft. wide, that its contents may be 60 cu. ft.?

13. A man excavated a cellar 18 ft. 6 in. long and 15 ft. 3 in. wide, taking out 64 cu. yd. 11 cu. ft. 1332 cu. in. of earth. How deep was the cellar dug?

14. What is the height of a rectangular embankment 18 ft. 9 in. long, 5 ft. 1 in. wide, and containing 28 cu. yd. 6 cu. ft. 864 cu. in.?

15. I bought a pile of wood 24 ft. long and 4 ft. wide for $28.50. If the price per cord was $4.75, how high was the pile?

BOARD MEASURE.

201. Board Measure is used in measuring boards or planks, scantling, joists, and other sawed timber. The unit of board measure is the *Board Foot.*

202. A **Board Foot** is 1 ft. long, 1 ft. wide, and 1 in. thick; and hence is 1 sq. ft. of the *face* of a board 1 in. thick. A board foot is one-twelfth of a cubic foot.

Thus, a board 8 ft. long, 2 ft. wide, and 1 in. thick, contains 16 board feet. If the board had been 1½ in. thick, it would have been 1½ times 16, or 24 board feet; if 2 in. thick, 2 times 16, or 32 board feet, etc.

203. The **Average Width** of a regularly tapering board is found by taking *one-half the sum* of the width of its two ends, or by measuring the width of the board midway between its two ends.

Thus, if the width of one end of a tapering board is 12 in. and the width of the other end 8 in., the average width is one-half of (8 + 12), or 10 in.

204. To find the number of board feet in a board.

ILLUSTRATIVE EXAMPLE.

How many board feet in a plank 20 ft. long, 9 in. wide, and 2 in. thick?

Solution.—A board which is 1 ft. long and 9 in. wide contains ¾ of a board foot; and if 20 ft. long contains 20 times ¾ of a board foot, or 15 board feet. Hence, a 2-in. *plank* must contain 2 times 15, or 30 board feet.

RULE.—I. *Multiply the length in feet by the width in feet. Or,*

II. *Multiply the length in feet by the width in inches, and divide the product by 12.*

NOTE.—If more than 1 in. thick, multiply the result obtained by the rule by the thickness in inches.

EXAMPLES FOR PRACTICE.

Find the number of board feet in a board or plank which is

	Long.	*Wide.*	*Thick.*		*Long.*	*Wide.*	*Thick.*
1.	18 ft.	7 in.	1 in.	6.	20 ft.	10 in.	2½ in.
2.	20 ft.	8 in.	1 in.	7.	18 ft.	1 ft. 2 in.	3 in.
3.	16 ft.	10 in.	1½ in.	8.	16 ft.	1 ft. 3 in.	2 in.
4.	15 ft.	9 in.	2 in.	9.	15 ft.	1 ft. 6 in.	3 in.
5.	18 ft.	11 in.	2 in.	10.	12 ft.	2 ft. 1 in.	1½ in.

11. What is the cost of 15 joists, each 20 ft. long, 9 in. wide, and 3 in. thick, at $2.75 per hundred feet?

12. What is the cost of seven 2-in. planks, each 18 ft. long, and their widths respectively 5 in., 7 in., 6 in., 9 in., 8 in., 11 in., and 10 in., at $2.50 per hundred feet?

13. What is the cost of a lot of boards 16 ft. long, and the united width of which is 58 ft. 6 in., at $23.50 per thousand feet?

14. How many board feet in a regularly tapering 2-in. plank 16 ft. long, 11 in. wide at one end and 7 in. wide at the other (**203**)?

15. What is the cost of 18 2-in. planks, each 16 ft. long, 8 in. wide at one end and 10 in. wide at the other, at $3.40 per hundred feet?

STONE-WORK.

205. Stone-work is usually estimated in perches of 24¾ cu. ft., of which 22 cu. ft. is allowed for the stone, and the remaining 2¾ cu. ft. for mortar and filling.

REM. 1.—In estimating the *quantity of stone* to be used, allowance should be made for doors, windows, and other openings; but in estimating stone-*work*, no allowance is usually made for openings less than 3 feet wide.

REM. 2.—The length of walls of stone is found by taking the *girth* or gross outside measurement. This practically allows double measurement of the corners, since the corners are more difficult to build.

ILLUSTRATIVE EXAMPLE.

At $3.85 per perch, what is the cost of erecting the stone walls of a church which is 45 ft. long by 36 ft. wide, the walls being 24 ft. high and 2 ft. 3 in. thick, allowing for 6 windows, each 5 ft. wide by 12 ft. high, and for two doors, each 6 ft. wide and 9 ft. high?

SOLUTION.

$(45+36)\times 2 = 162$ ft., girth.
$162\times 24\times 2\frac{1}{4} = 8748$ cu. ft., gross.
$5\times 12\times 2\frac{1}{4}\times 6 = 810$ cu. ft., windows.
$9\times 6\times 2\frac{1}{4}\times 2 = 243$ cu. ft., doors.
$8748-(810+243) = 7695$ cu. ft., net.
$7695\div 24\frac{3}{4} = 310\frac{10}{11}$ perches.
$\$3.85\times 310\frac{10}{11} = \$1197$, cost.

EXPLANATION.—Twice the sum of the length (45 ft.) and the width (36 ft.), or 162 ft., gives the girth or outside measurement of the walls. The continued product of the girth (162 ft.) × the height (24 ft.) × the thickness ($2\frac{1}{4}$ ft.), or 8748 cu. ft., gives the gross solidity, from which should be deducted the solidity of the 6 windows (810 cu. ft.) and of the two doors (243 cu. ft.) to obtain the net solidity (7695 cu. ft.). Divide 7695 by $24\frac{3}{4}$ to obtain the net solidity in perches ($310\frac{10}{11}$). Multiply the price per perch ($3.85) by the number of perches ($810\frac{10}{11}$) to find the cost.

RULE.—*Find the number of cubic feet of masonry by* **200,** *divide the result by 24.75, and the quotient will denote perches.*

EXAMPLES FOR PRACTICE.

1. What is the cost of the stone foundation of a house which is 30 ft. long and 20 ft. wide, the foundation being 6 ft. high and 2 ft. thick, at $7.30 per perch?

2. I contracted with a mason to build the stone foundation of a house 36 ft. long by 30 ft. wide, at $3.25 per perch, and paid him $100.80 for the job. If the foundation was 2 ft. thick, what was its height?

3. At $5.75 per perch, what is the cost of erecting the stone walls of a house 36 ft. long by 24 ft. wide, the walls being 30 ft. high and 2 ft. 6 in. thick, allowing for 8 windows, each 4 ft. wide and 6 ft. high, for 1 door 5 ft. wide and 8 ft. high, and 2 doors, each 4 ft. wide and 7 ft. high?

4. A sea-wall 396 ft. long and 3 ft. thick cost $2268, at $5.25 per perch. What was the height of the wall?

BRICK-WORK.

206. Brick-work is usually estimated by the thousand bricks.

Rem. 1.—In measuring brick-work, allowance should be made for doors, windows, or other openings; and for corners, since, unlike stone-work, only the *net length* of the walls is taken.

Rem. 2.—The net length of the walls is obtained by deducting four times the thickness of the walls from the gross outside measurement.

Rem. 3.—The dimensions of an ordinary brick are 8 in. × 4 in. × 2 in.; of Baltimore and Philadelphia bricks, $8\frac{1}{4}$ in. × $4\frac{1}{8}$ in. × $2\frac{3}{8}$ in.; of Maine bricks, $7\frac{1}{2}$ in. × $3\frac{3}{8}$ in. × $2\frac{3}{8}$ in.; of North River bricks, 8 in. × $3\frac{1}{2}$ in. × $2\frac{1}{4}$ in.; and of Milwaukee bricks, $8\frac{1}{2}$ in. × $4\frac{1}{8}$ in. × $2\frac{3}{8}$ in.

ILLUSTRATIVE EXAMPLE.

How many ordinary bricks are required for the walls of a house 60 ft. deep, 36 ft. high, and 24 ft. front, if 240 square feet of the surface of the walls are allowed for windows and doors and $\frac{1}{4}$ of an inch for mortar, the walls being 3 bricks thick?

SOLUTION.

$8+\frac{1}{4}=8\frac{1}{4}$ in., length of brick and mortar.
$2+\frac{1}{4}=2\frac{1}{4}$ in., thickness of brick and mortar.
$8\frac{1}{4}\times 2\frac{1}{4}=18\frac{9}{16}$ sq. in., exposed surface per brick.
$144\div 18\frac{9}{16}=7\frac{25}{33}$, exposed bricks per sq. ft. of wall.
$7\frac{25}{33}\times 3=23\frac{3}{11}$, total number of bricks per sq. ft. of wall.
$(60+24)\times 2=168$ ft., outside girth of wall.
$(4\times 3)+\frac{2}{4}=12\frac{1}{2}$ in., thickness of walls, including 2 inner joints.
$12\frac{1}{2}\times 4=50$ in., or $4\frac{1}{6}$ ft., deduction for corners.
$168-4\frac{1}{6}=163\frac{5}{6}$ ft., net length of walls.
$(163\frac{5}{6}\times 36)-240=5658$ sq. ft., net surface of walls.
$5658\times 23\frac{3}{11}=131677\frac{2}{11}$ bricks, the answer.

Rule.—I. *Add $\frac{1}{4}$ of an inch to the length and thickness of the brick, and multiply the two results to find the exposed surface of 1 outside brick.*

II. *Divide 144 by the exposed surface of 1 brick, to find the number of outside bricks per square foot of wall.*

III. *The continued product of the number of outside bricks* *per square foot of wall × the number of bricks in the thickness of the walls × the number of square feet in the surface of the walls, will give the total number of bricks.*

EXAMPLES FOR PRACTICE.

1. The walls of a warehouse built of Baltimore bricks, are 3 bricks thick, 60 ft. high, 50 ft. front, and 200 ft. deep. How many bricks does the warehouse contain, if 675 square feet are allowed for doors and windows?

2. How many Milwaukee bricks are required for the walls of a building 40 ft. front, 75 ft. deep, 50 ft. high, and 3 bricks thick, allowing for 20 windows, each $6\frac{1}{4}$ ft. high and 4 ft. wide, and for 4 doors, each 8 feet high and $3\frac{3}{4}$ feet wide?

3. How many North River bricks are required to build a house 40 ft. front, 75 ft. deep, 60 ft. high, 3 bricks thick, and with gables 12 ft. high at the front and rear, allowing 1250 sq. ft. for openings?

REM.—The surface of a gable is found by multiplying the length of the wall upon which the gable is built by *one-half* the height of the gable above the wall.

MEASUREMENT OF CAPACITIES.

207. Measurement of Capacities, as here treated, will embrace the measurement of granaries, bins, boxes, cisterns, tanks, etc.

REM. 1.—*Stricken Measure* means the measuring vessel even full, and stricken off by a rule or striker. Grains, seeds, berries, etc., are sold by stricken measure. The standard bushel when stricken contains 2150.42 cubic inches.

REM. 2.—*Heaped Measure* means the contents of the measuring vessel heaped in the form of a cone. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles are sold by heaped measure. The standard bushel when heaped contains 2747.71 + cubic inches. It is customary to allow 5 stricken bushels for 4 heaped ones.

REM. 3.—The gallon, dry measure ($\frac{1}{4}$ pk.), contains 268.8 cu. in., and the gallon, liquid measure, 231 cu. in.

ILLUSTRATIVE EXAMPLE.

A rectangular bin is 8 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels of rye will it hold?

SOLUTION.—$8 \times 6 \times 4 = 192$ cu. ft.; $1728 \times 192 = 331776$, the contents of the bin in cubic inches. $331776 \div 2150.42 = 154.284+$ stricken bushels.

Rule.—*Divide the contents of the bin in cubic inches by 2150.42 to find bushels stricken measure; or by 2747.71 to find bushels heaped measure.*

Note.—A sufficiently close approximation for practical purposes may be obtained by multiplying the contents of the bin in cubic feet by .8 to find stricken bushels, or by .63 to find heaped bushels.

EXAMPLES FOR PRACTICE.

1. How many bushels of wheat will exactly fill a rectangular bin 9 ft. 6 in. long, 4 ft. 8 in. wide, and 3 ft. 9 in. high?

2. A rectangular bin is 10 ft. long, 5 ft. wide, and 3 ft. 6 in. deep. How many bushels of turnips will it hold?

3. How many bushels of apples can be stored in a rectangular box 4 ft. long, 3 ft. wide, and 2 ft. 6 in. deep?

4. A wagon-body is 10 ft. long, 4 ft. wide, and 18 in. deep. How many bushels of shelled corn will it hold?

5. How many gallons of water will fill a rectangular cistern 8 ft. 6 in. long, 8 ft. 6 in. wide, and 7 ft. deep?

Capacity in cubic inches ÷ 231 = capacity in liquid gallons.

6. How many bushels of oats will fill a vessel which is capable of holding exactly 325 gallons of water?

7. How many hogsheads of water will fill a rectangular tank 25 ft. long, 18 ft. wide, and 16 ft. 9 in. deep?

8. How many gallons of water can be put into a vessel which will hold exactly 18 bushels of wheat?

9. What must be the width of a wagon-body 9 ft. long and 20 in. deep, to contain 40 bushels of potatoes?

Solution.—$2747.71 \times 40 = 109908.4$ cu. in.; $109908.4 \div (108 \times 20) = 50.88\frac{19}{54}$ in. = 4 ft. $2.88\frac{19}{54}$ in., required width.

10. What must be the depth of a rectangular cistern 8 ft. 6 in. long and 7 ft. 9 in. wide, to contain 1800 gallons of water?

11. What must be the length of a wagon-body 4 ft. wide and 18 in. deep, to hold 50 bushels of shelled corn?

12. A grocer bought a barrel of molasses containing 36 gallons at 45 cents per gallon, and retailed it at 50 cents per gallon, using by mistake a set of dry measures. What was his net loss (Rem. 3)?

13. Allowing 80 lb. per bushel, what is the weight of coal which will fill a rectangular bin 6 ft. long, 5 ft. wide, and 4 ft. deep?

14. Allowing 28 bu. to the ton of 2240 lb., how deep must be a rectangular bin 18 ft. long and 5 ft. wide, to hold 20160 lb. coal?

(In the following, apply Note, **207**, for obtaining approximate results.)

15. A merchant has a rectangular bin 12 ft. long, 5 ft. wide, and 4 ft. high, filled with shelled corn, which he retailed at 85 cents per bushel. How much did he receive for the corn?

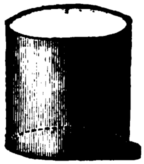
16. A rectangular bin 12 ft. long and 10 ft. wide will hold exactly 378 bushels of apples. What is the depth of the bin?

When the crib or bin is flared at the sides, the average width is found by taking *one-half the sum of the top and bottom widths* (**203**).

Thus, the contents of a flaring crib 16 ft. long, 10 ft. high, 9 ft. wide at the bottom, and 13 ft. wide at the top, are $16 \times 10 \times \frac{1}{2}$ of $\overline{9+13}$, or 1760 cu. ft.

17. A bin is 8 ft. long, 5 ft. deep, 3 ft. wide at the bottom, and 5 ft. wide at the top. How many bushels of rye will it hold?

208. A cylindrical vessel is one whose sides are *uniformly* round and whose ends are equal and parallel circles.



Cylindrical Vessel.

Rem.—It can be proved by geometry that the contents of a cylindrical vessel are .7854 of the contents of a rectangular vessel which has two of its dimensions equal to the diameter of the cylindrical vessel.

18. A cylindrical vessel is 9 ft. deep and 3 ft. in diameter. How many gallons of water can it hold?

Solution.—$3 \times 3 \times 9 = 81$ cu. ft., contents of a rectangular cistern with two dimensions equal to the given diameter. $81 \times .7854 = 63.6174$ cu. ft., contents of cylindrical vessel. $(63.6174 \times 1728) \div 231 = 475.8912$ gallons.

19. How many gallons of water are in a well 5 ft. in diameter if the water is 7 ft. deep?

20. A cylindrical cistern is 8 ft. deep and 5 ft. in diameter. How many gallons of water will it hold when full?

21. A circular reservoir is 90 ft. in diameter and 15 ft. deep. How many gallons of water will it hold when full?

22. A cylindrical vessel 8 ft. high and 5 ft. in diameter is filled with potatoes. What is the value of the potatoes at 75¢ per bushel?

PERCENTAGE.

209. Percentage, as a general term, means any computation *by hundredths*.

The phrase *per cent.*, or its symbol (%), signifies *by the hundred*. Thus, 8% of $900 means $8 of every $100 in $900.

210. The **Base** is the quantity of which one or more hundredths are required to be taken.

211. The **Rate per cent.** denotes *how many* hundredths of the base are to be taken.

When the phrase *per cent.* is not annexed, Rate means *any* part of the base. Thus, ¾, .035, and .05 are all rates; the first being a common fractional rate in *fourths*, the second a decimal rate in *thousandths*, but only the last a decimal rate in *hundredths*, or a rate per cent.

212. The **Percentage** is the result obtained by taking one or more hundredths of the base.

REM.—In popular language, the *Base* is the *whole quantity* under consideration; the *Percentage* is any *part* of the whole quantity, and the *Rate* denotes *what part* the percentage is of the base.

Thus, in the expression 5% of $800 = $40, $800 is the whole quantity here considered, or the *base; $40* is a part of the whole quantity, or the *percentage;* and 5% denotes what part (.05) the percentage ($40) is of the base ($800).

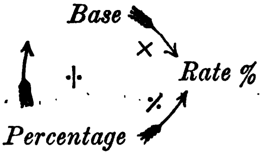
213. Problems in percentage differ in no respect from those in common or decimal fractions. It is true, *new names* of terms are introduced in percentage, but by no means any *new terms*. The definitions and General Formula, **114**, are therefore equally applicable to percentage by a simple change of names; the concrete factor being now called the base; the abstract factor, the rate; and the product, the percentage.

REM. 1.—In "simple percentage," *any* quantity can be taken as the base. That is, if it is required to compare $50 and $100, $50 may be compared with $100 taken as the basis of comparison, in which case ½ or 50% will be the rate; or $100 may be compared with $50 taken as the basis, when 2 times or 200% will be the rate. Hence, in simple percentage, where bases are not usually self-evident, some form of language must necessarily be employed to indicate which of two or more quantities is to be regarded as the base. The word *of*, or the phrases *more than* or *less than* are used for this purpose; one of these forms of expression immediately following the rate, and immediately preceding the base, to indicate these two factors of a given or required percentage. Thus, in the expressions, "$50 is what *per cent. of* $100," or "$50 is what *per cent less than* $100," the indicated base is $100; but in the expressions, "$100 is what *per cent. of* $50," or "$100 is what *per cent. more than* $50," the indicated base is $50.

REM. 2.—After the base has been identified by the preceding Remark, any concrete quantity which is more or less than the base may be regarded as a percentage.

214. The base, rate, and percentage are called the *terms* of percentage. A problem in percentage may require any one of these three terms to be found, if the remaining two terms are given. Any required term can be found by means of the following

GENERAL FORMULA.



DIRECTION.—If any two of the three terms of this formula are given, perform the operation indicated by the sign between them, and the result will be the third term.

215. Rates per cent. are often less convenient to use than their equivalent common fractional rates. The following Table includes the principal fractional rates in common use:

TABLE.

Rate %.	*Com. Frac.*	*Rate %.*	*Com. Frac.*	*Rate %.*	*Com. Frac.*	*Rate %.*	*Com. Frac.*
$2\frac{1}{2}\%$	$= \frac{1}{40}$.	10%	$= \frac{1}{10}$.	$33\frac{1}{3}\%$	$= \frac{1}{3}$.	$62\frac{1}{2}\%$	$= \frac{5}{8}$.
5%	$= \frac{1}{20}$.	$12\frac{1}{2}\%$	$= \frac{1}{8}$.	$37\frac{1}{2}\%$	$= \frac{3}{8}$.	$66\frac{2}{3}\%$	$= \frac{2}{3}$.
$6\frac{1}{4}\%$	$= \frac{1}{16}$.	$16\frac{2}{3}\%$	$= \frac{1}{6}$.	40%	$= \frac{2}{5}$.	75%	$= \frac{3}{4}$.
$8\frac{1}{3}\%$	$= \frac{1}{12}$.	20%	$= \frac{1}{5}$.	50%	$= \frac{1}{2}$.	80%	$= \frac{4}{5}$.
$6\frac{2}{3}\%$	$= \frac{1}{15}$.	25%	$= \frac{1}{4}$.	60%	$= \frac{3}{5}$.	$87\frac{1}{2}\%$	$= \frac{7}{8}$.

216. The base and rate being given, to find the percentage.

ILLUSTRATIVE EXAMPLES.

1. What is 8% of $28.65?

SOLUTION.

\$28.65
.08
\$2.2920

Ans., \$2.29+.

EXPLANATION.—8% or .08 is the rate, and \$28.65 the indicated base. Hence, multiply these two factors, obtaining .08 times \$28.65, or \$2.29+, as the required product, or percentage.

REM.—8% of \$28.65 means .08 times \$28.65.

2. A drover bought 375 head of cattle and afterwards sold 60% of his purchase. How many cattle had he remaining?

SOLUTION.

100%, rate of purchase.
60%, " " sale.
40%, " " remaining cattle.
375 cattle × .40 = 150 cattle.

EXPLANATION.—His purchase (that is, 375 cattle) is the indicated base. Since 100% of the base = the base, if 60% of the base is sold, there must be 100 − 60, or 40% of the base remaining. Hence, .40 times 375 cattle = 150 cattle, the percentage remaining.

3. What is 25% more than \$800?

SOLUTION.

4/4 of \$800 = \$800
¼ of \$800 = 200
5/4 of \$800 = \$1000

EXPLANATION.—The indicated base is \$800. 25% or ¼ more than \$800 means ¼ of \$800 more than \$800. Hence, ¼ of \$800, or \$200, added to \$800 = \$1000, the required percentage.

RULE.—*Multiply the base by the rate % of any required percentage, and the product will be that percentage.*

NOTE 1.—All rates % denote hundredths of some quantity taken as the base, said base being indicated by Rem. 1, **213.** Hence, as 100/100 of any base = the whole of that base, the rate % of all bases must be 100%; and the rate % of any required percentage must be so much of this 100% (that is, such a % of the base) as equals that percentage.

NOTE 2.—A certain % more or less than the base means that *% of the base* more or less than the base, "% of," wherever used, invariably meaning *hundredths times* the indicated base.

EXAMPLES FOR PRACTICE.

1. 8 times \$7 = what sum? **2.** .03 times \$500 = what sum? **3.** 6% of 800 lb. (that is, .06 times 800 lb.) = what weight? **4.** 45% of 786 gallons = how much? **5.** 75% of 3184 ft. = how much? **6.** 72% of \$839.75 = what percentage? **7.** 34% of \$3127.85 = what percentage? **8.** 5% of \$3.75 = how much? **9.** 33⅓% of 948 men = what percentage? **10.** 125% of \$7134.56 = how much? **11.** ¼% of

216

364 miles = what distance? **12.** 40% of £325 17s. 6d. = what percentage?

13. What is 12½% more than 3488 ft.? **14.** Find 16⅔% more than 4584 bu. **15.** Find 20% more than $575. **16.** Find 10% less than $5975. **17.** What is 37½% less than 856 francs? **18.** Find 80% less than 3895 tons.

19. A gentleman is worth $75000, and had 35% of his wealth invested in real estate. What was the value of his real estate?

20. A speculator invested $35400 in stocks and lost 16⅔% of his investment. How much of his investment had he remaining?

21. A grazier owned 580 sheep, but lost 15% of his flock in a snowstorm. How many sheep had he remaining?

22. In a certain school 420 pupils are enrolled, 45% of the enrollment being boys. How many girls are enrolled?

23. A merchant has $18750 on deposit, 25% of which is in coin, and the remainder in bank notes. How much in notes has he on deposit?

24. A merchant bought goods for $587.35, and sold them for 40% more than he paid for them. What did he receive for the goods?

25. A merchant retired from business with a fortune of $82400, invested 37½% of it in government bonds, 20% of it in manufacturing stock, 15% of it in a farm, and deposited the remainder in bank. What was the sum deposited?

26. A farmer sold 530 bushels of wheat at $1.18 per bushel, and 20% more oats than wheat at 70¢ per bushel. What did he receive for both?

217. The base and percentage being given, to find the rate.

ILLUSTRATIVE EXAMPLES.

1. What per cent. of $84 are $12.60?

SOLUTION.

$12.60 ÷ $84 = .15 = 15%

EXPLANATION.—$84 is the indicated base and $12.60 the percentage. Since the percentage is the product of the base and rate (Rule, **216**), divide the given percentage or product ($12.60) by its given factor (the base, $84) and the quotient, carried to hundredths, will be its remaining factor (the rate, 15%).

2. A man's income is $875 per annum and his expenses $1050. What per cent. more than his income does he spend?

SOLUTIO

$1050 — $875 = $175

$175.00 ÷ $875 = .20 = 20%

Explanation.—His income, $875, is the indicated base. Subtract his income ($875) from his expenses ($1050) to find *how much* more than his income are his expenses, obtaining $175 as the percentage of which it is required to find the rate per cent.

Divide the percentage or product ($175) by one of its factors (the base, $875), obtaining 20% as the remaining factor, or the required rate %.

Rule.—*Divide any percentage by its base, and the quotient carried to hundredths will be the rate % of that percentage.*

Note 1.—To obtain a quotient in hundredths, the dividend must contain exactly two decimal places more than the divisor.

Note 2.—To compute the % of a given percentage is simply to find *how many hundredths times* the base will produce that percentage.

EXAMPLES FOR PRACTICE.

1. How many times $8 are $72? **2.** $200 are how many times $50? **3.** How many hundredths times $500 are $50? **4.** What % of $80 (that is, how many hundredths times $80) are $20? **5.** What % of $75 are $25.25? **6.** What % of $875 are $455? **7.** 58 lb. are what % of 725 lb.? **8.** $800 are what % of $500? **9.** What per cent. of 2 years are 146 days?

10. What % more than $750 are $800? **11.** What % more than 528 gal. are 660 gal.? **12.** What % more than $18.60 are $19.53? **13.** What % less than 750 bu. are 615 bu.? **14.** What % less than $345 are $227.70? **15.** What % less than $1.50 are 60¢?

16. A collector charged $25.92 for collecting a bill of $324. What per cent. of the sum collected did he charge for his services?

17. A man's income is $840 per annum, and his expenses $560. What per cent. of his income does he save?

18. The bread made from 720 pounds of flour weighs 972 pounds. What per cent. more than the flour does the bread weigh?

19. A grocer sold 175 pounds of coffee and had 1075 pounds remaining. What % of his original stock of coffee did he sell?

20. A farm contained 42 acres in wheat, 75 acres in corn, 58 acres in oats, 65 acres in grass, 124 acres in woodland, and the remaining 11 acres in orchard. What % of the farm was in corn?

21. If 7 gallons of water are added to 168 gallons of alcohol, what per cent. of the mixture is alcohol?

22. A merchant retired from business with a capital of $35825, invested $12180.50 of it in real estate and the remainder in U. S. bonds. What per cent. of his capital was invested in bonds?

23. A tank contains 175 gallons of water, 50 gallons having previously leaked out. What per cent. of what the tank originally held was lost by leakage?

218. The percentage and rate being given, to find the base.

ILLUSTRATIVE EXAMPLES.

1. An inventor sold 35% of his patent-right for $3065.30. At that rate, what was the estimated value of his whole patent-right?

SOLUTION.

$3065.30 ÷ .35 = $8758

EXPLANATION.—His whole patent-right (required) is the indicated base; and $3065.30, the value of .35 of the patent-right, the percentage. Since the percentage is the product of .35 times the required patent-right, divide this product ($3065.30) by its given factor (the rate, .35) to find its required factor (the patent-right or base, $8758).

2. A is worth $13021, or 45% more than B. How much is B worth?

SOLUTION.

100% + 45% = 145% = 1.45.

$13021.00 ÷ 1.45 = $8980.

EXPLANATION.—B's worth (required) is the indicated base; and $13021, the value of 45% more than B's worth, the percentage. If $13021 is 45% more than the base (B's worth), the rate of $13021 must be 45% more than the rate of the base, or 145%. Since the percentage is the product of 1.45 times the required worth of B, divide this product ($13021) by its given factor (1.45) to find its required factor (B's worth, $8980).

RULE.—*Divide any percentage by its rate, and the quotient will be its base.*

NOTE 1.—The rate % of any percentage is as much greater or less than the rate of the base (100%), as such percentage is greater or less than the base.

NOTE 2.—To find the base is simply to ascertain a given number of hundredths of *what number* will equal a given percentage.

EXAMPLES FOR PRACTICE.

1. 54 bushels = 9 times what quantity? **2.** 42 yards = .07 times what quantity? **3.** $68 = 8% of (that is, .08 times) what base? **4.** 360 lb. = 45% of what quantity? **5.** 33⅓% of what quantity are

210 gallons? **6.** 20% of what distance = 250 miles? **7.** 365 tons are 73% of how much hay?

8. $8915 = 25% more than what sum? **9.** What sum increased by 31% of itself = 19.65 francs? **10.** $1333.26 are 62% more than what sum? **11.** 225 tons are 25% less than what weight of guano? **12.** What sum diminished by 40% of itself = $4800? **13.** 28% less than how many bushels = 2466 bu.?

14. A merchant sold 14% of his stock of prints. How many yards of prints had he in stock if the quantity sold was 1260 yards?

15. A farmer lost 25% of his flock of sheep and had 240 remaining. How many sheep did he lose?

16. A's orchard contains 910 fruit trees, which is 4% more than the number in B's orchard. How many fruit trees in B's orchard?

17. A merchant withdrew $2058 from bank, which was 28% of his deposit. How much had he remaining in bank?

18. A dealer sold 35% of his stock and had $8027.50 worth of goods remaining. What was the value of the goods sold?

19. A debtor paid 24% of a creditor's claim and still owed him $5194.60. What was the sum paid?

20. A owns 15% of a ship; B, 25%; C, 28%; and D, the remainder. What is the value of A's share of the ship, if D's is worth $17232?

21. A man paid $25 for a suit of clothes, $22 for an overcoat, $7.66 for books, $120 for a watch, and had 18% of his money remaining. How much money had he at first?

REVIEW OF PERCENTAGE.

219.—**1.** A merchant lost $5000 of his capital and had $25000 remaining. What per cent. of his capital did he lose?

2. I sold a bill of merchandise, received $805 cash, and took the buyer's note for the remainder which was 12½% of the bill. What was the face of the note?

3. A, B, and C are partners, A investing $5200, B 25% more than A, and C 15% less than B. What is the capital of the firm?

4. A farmer bought 50 acres of land, which was 20% of what he previously owned. How many acres does he now own?

5. A and B unite in buying a house, A investing $2964, or 38% of the purchase money. How much did B invest?

6. A grocer sells 28% more granulated sugar than loaf-sugar. If his yearly sales of loaf-sugar amount to 38275 pounds, what are his monthly sales of granulated sugar?

7. The assets of a bank are $292312.50 and its liabilities $389750. What per cent. of its indebtedness can it pay?

8. A merchant bought 780 yards of goods, sold 25% of the purchase at one sale, and 20% of the remainder at a second sale. How many yards remained unsold?

9. In a city of 52340 inhabitants, 25% were Germans, 30% Irish, and 5% French. What number of each did the city contain?

Some bases are understood; that is, they are not indicated by Rem. 1, **213.** A base understood will become apparent to the learner by placing *of what* directly after the rate %. Thus, in the above example, 25% *of what* were Germans? *Ans.* Of the population. Therefore 52340 inhabitants is the base understood.

10. A bankrupt owed $12840 and agreed to liquidate the debt in equal annual installments of 16⅔%. How much did he owe after paying the fourth installment?

11. If coffee loses 8% in roasting, how much green coffee will be required to produce 828 pounds when roasted?

12. On July 18, 1889, a merchant bought a bill of goods amounting to $736.80 on 4 months' credit, or 5% off if paid within 30 days of date of sale. What sum will pay the bill on Aug. 12, 1889?

13. A grocer sold 20% of an invoice of sugar at one sale, 25% of the invoice at a second sale, 60% of what was left of the invoice at a third sale, and finally had 473 pounds remaining. How many pounds did the invoice contain?

14. A farm contains 350 acres of land, 15% of it having been bought at $28 per acre, 25% at $35 per acre, 18% at $37 per acre, and the remainder at $40 per acre. What was the cost of the farm?

15. A and B engage in partnership, A investing $5976 and B $6474. What % of the capital of the firm did each invest?

16. A owned 40% of a mill and sold 25% of his share for $3575.25. At that rate, what was the value of B's share, who owned 45% of the mill?

17. Immediately before a battle, the garrison of a fort numbered 775 men, and after the battle 713 men. What per cent. was lost?

In problems involving increase or decrease, the quantity increased or diminished, that is, the quantity *before* the increase or decrease, is the base understood.

18. A merchant commenced business with a capital of $8375, and at the expiration of the first year found that his capital was $11725. What was his per cent. of increase?

19. Since 1880, a town has increased 5850 in population and now contains 38350 inhabitants. What is the % of increase since 1880?

20. A man's real estate is worth $7500 and his personal property $8200. If the former increase in value 18% and the latter 5%, what will be the per cent. of increase in the value of both?

21. A invested $32500 in business, and at the end of the first year found that he had gained $5200, which he withdrew, and at the end of the second year that he had lost $2925 during that year. What was his per cent. of net gain for the two years?

22. By purchasing improved machinery, a miller increased the capacity of his mill 25%, or 340 barrels of flour per month. What is the present capacity of the mill?

23. A farmer gathered 15 bushels of potatoes from a planting of 1 bu. 3 pk. What was the per cent. of increase?

24. By an unfortunate speculation, a man had his annual income decreased 10%. If his present income is $2115, what was it before the speculation?

25. A speculator by judicious investments increased his income 24%. If his former income was $1800 per annum, what is it now?

26. Last year a grocer sold 42580 pounds of sugar, and this year 68128 pounds. What % more did he sell this year than last year? What % less did he sell last year than this year?

27. 18% of a man's wealth is in real estate, 24% in bank stock, 26% in railroad bonds, and the remainder in money. What is the value of his bank stock, if his money amounts to $10288?

28. A quantity of gunpowder is compounded of 6255 pounds saltpetre, 834 pounds sulphur, and 1251 pounds charcoal. What % of the gunpowder is each ingredient?

29. A gentleman paid $1701 in settlement of an account, and afterwards discovered that he had paid 26% more than the correct sum. What was the amount of overcharge?

30. A's taxes for 1888 amounted to $386, subject to a discount of 5% if paid before Nov. 1st of that year; to a discount of 3% if paid between Nov. 1st and Dec. 1st; and to an additional charge of 5% if paid on or after Jan. 1st, 1889. What was the net amount of his taxes if paid Oct. 30th, 1888? If paid Nov. 18th, 1888? If paid Dec. 24th, 1888? If paid Jan. 15th, 1889?

31. A man paid $375 for a pair of horses, and 18% more than that sum for a carriage. What was the total cost?

32. The salary of a clerk is $560 per annum, of which he pays $192 per annum for board, $85 for clothes, $59 for incidentals, and the remainder he deposits in bank. What per cent. of his salary does he deposit in bank?

33. A man withdrew 30% of his bank deposit, and invested 12% of what he withdrew in purchasing an organ for $180. How much had he remaining in bank?

34. Two R. R. companies transported 5318 lb. freight at the through rate of 35¢ per cwt. If one company received 45% of the through rate, how much should the other receive?

35. Two railroads, whose lengths were respectively 175 and 125 miles, transported 7500 pounds of merchandise at the through rate of 45 cents per 100 pounds. What sum did each company receive?

36. The charge for a certain class of "through freight" between two cities, 400 miles distant from each other, was 95 cents per 100 pounds, and the quantity transported 8600 pounds. The freight was divided *pro rata* between three railroad companies whose mileages were respectively 120, 180, and 100 miles. What per cent. of the total freight should each company have received? What sum?

37. A bought a horse and buggy for $570, paying 28% more for the horse than for the buggy. What did he pay for each?

The required cost of the buggy is the indicated base, and $570 the percentage. As the given percentage ($570) is the cost of the buggy plus the cost of the horse, its rate must be the rate of the cost of the buggy (100%) plus the rate of the cost of the horse (128%), or 228%.

38. A lady spent $64.50 for jewelry and dress goods, paying 15% more for jewelry than for dress goods. How much did she pay for each?

39. In the manufacture of a certain quality of cloth, 1122 pounds of cotton and wool were mixed, the weight of the cotton being 30% of the weight of the wool. How many pounds of each were used?

40. A and B together have $1680, and A has 25% less money than B. How much money has A?

41. A firm exported 8494 barrels of flour in three shipments. Their second shipment was 25% greater than their first, and their third 32% less than their second. How many barrels were exported in each shipment?

42. A dealer purchased a quantity of oysters, fish, and clams, paying $166.75 for the entire quantity. The cost of the fish was 45% of that of the oysters, and the cost of the clams 25% of that of the oysters and fish together. What did he pay for each?

43. The sales of a firm were increased 25% during the second year, 20% the third year, and 16⅔% the fourth year. What was the amount of sales during the first year, if the fourth year's sales were $60248?

44. The property of a testator was divided among his wife, son, and daughter. His wife received 35% of the property, and the remainder, $16250, was divided between the son and daughter, the daughter receiving 33⅓% less than the son. How much did each receive?

45. The population of a certain city decreased 10% in 1887, and 8% in 1888. If the population on Jan. 1, 1889, was 26910, what was the population on Jan. 1, 1887?

46. An importer paid £243 15s. for an invoice of goods, and £73 2s. 6d. for freight, custom-house charges, etc. What per cent. of the invoice price were the charges?

PROFIT AND LOSS.

220. Profit and **Loss** are commercial terms which denote the gain or loss in business transactions.

Rem. 1.—The *gross cost* of merchandise is its prime cost plus all expenses incidental to the purchase, such as freight, commission, transit insurance, drayage, etc.; and its *net selling price* is the gross selling price minus any trade discount allowed, or any expense incidental to the sale, such as free delivery charges, commission on sales, etc.

Rem. 2.—In estimating the profit or loss on merchandise, its *gross cost* is taken as the *base* or standard of comparison. If its net selling price is greater than its gross cost, there is a profit; if less, a loss; and the amount of profit or loss will depend upon the difference between its gross cost and net selling price.

221. Computations in Profit and Loss are made in accordance with the principles of percentage, the cost being regarded as the base; and the profit, loss, and selling price as percentages. Hence, General Formula, **214**, may be adapted to Profit and Loss as follows:

GENERAL FORMULA.

$$\begin{tikzcd}
\textit{Cost} = & \textit{Base} \arrow[dr, "\times"] & \\
& & \textit{Rate}\ \% \\
\left.\begin{array}{c}\textit{Profit,}\\ \textit{Loss, or}\\ \textit{Sell'g Price}\end{array}\right\} = & \textit{Percentage} \arrow[uu, "\div"'] \arrow[ur, "\%"] &
\end{tikzcd}$$

The direction for the use of this formula is the same as that of Genl. Form., **214.**

REM. 1.—Since the selling price equals the cost plus the profit or minus the loss, the rate of selling price must equal the rate of cost (100%) plus the rate % of profit or minus the rate % of loss.

REM. 2.—All rates %, whether of profit, loss, or selling price, are understood to be expressed on the cost; thus, 5% profit means a profit of .05 times the cost.

EXAMPLES FOR PRACTICE.

1. What is the gain on goods which cost $5876.40 and are sold at 15% ($\frac{1}{10}+\frac{1}{2}$ of $\frac{1}{10}$) profit? **2.** Which cost $375.75 and are sold at 8% profit? **3.** Which cost $735 and are sold at 26% (25%+1%) profit? **4.** Which cost $17.25 and are sold at 40% profit?

Cost × rate of profit = profit.

5. What is the loss on goods which cost $150 and are sold at 25% loss? **6.** Which cost $5896.26 and are sold at 33⅓% loss? **7.** Which cost $175 and are sold at 20% loss. **8.** Which cost $715.32 and are sold at 12½% loss.

Cost × rate of loss = loss.

9. At what price must goods be sold to gain 20%, if they cost $67.15? **10.** To gain 10%, if they cost $6.50? **11.** To gain 25%, if they cost $3.24? **12.** To gain 16⅔%, if they cost 90¢? **13.** To gain 12½%, if they cost $4.64? **14.** To gain 33⅓%, if they cost $4.35. **15.** To gain 37½% ($\frac{1}{4}+\frac{1}{2}$ of $\frac{1}{4}$), if they cost $5.20? **16.** To gain 15% ($\frac{1}{10}+\frac{1}{2}$ of $\frac{1}{10}$), if they cost $41.60? **17.** To gain 6¼%, if they cost $113.92? **18.** To gain 18¾% ($\frac{1}{8}+\frac{1}{2}$ of $\frac{1}{8}$), if they cost $27.35? **19.** To gain 30%, if they cost $17.50. **20.** To gain 40%, if they cost $18.75? **21.** To gain 35% ($\frac{1}{4}+\frac{1}{10}$), if they cost $73.40?

Cost + (cost × rate of gain) = selling price.
Cost × (100% + rate % of gain) = selling price.

22. At what price must goods be sold to lose 12%, if they cost $43.50? **23.** To lose 20%, if they cost $315? **24.** To lose 6¼%, if they cost $1204.80?

Cost—(cost × rate of loss) = selling price.
Cost × (100% —rate % of loss) = selling price.

25. What is the % of gain on goods which cost $3745 and are sold at a profit of $749 ? **26.** Which cost $5136 and are sold at a profit of $1712 ? **27.** Which cost $8934 and are sold at a profit of $2233.50 ?

Gain÷cost = rate of gain.

28. What is the % of loss on goods which cost $7134 and are sold at a loss of $1189 ? **29.** Which cost $5824 and are sold at a loss of $1.82 ? **30.** Which cost $7832 and are sold at a loss of $48.95 ?

Loss÷cost = rate of loss.

31. What is the % of gain or loss on goods which cost $5172 and are sold for $5301.30 ? **32.** Which cost $3072 and are sold for $2560 ? **33.** Which cost $850 and are sold for $765 ? **34.** Which cost $8292 and are sold for $11056 ? **35.** Which cost $21 and are sold for $23.97 ?

(Selling price—cost)÷cost = rate of gain.
(Cost—selling price)÷cost = rate of loss.

36. What is the cost of goods which are sold at $1346.40 profit, if the rate of gain is 15% ? **37.** Which are sold at $75 profit, if the rate of gain is 20% ? **38.** Which are sold at $33 profit, if the rate of gain is 4% ?

Gain÷rate of gain = cost.

39. What is the cost of goods which are sold at a loss of $129, if the rate of loss is 33⅓% ? **40.** Which are sold at a loss of $51.75, if the rate of loss is ¼% ? **41.** Which are sold at a loss of $885, if the rate of loss is 15%?

Loss÷rate of loss = cost.

42. What is the cost, if goods are sold for $406.25, or at a gain of 25%? **43.** If sold for $73.85, or at a gain of 16⅔% ? **44.** If sold for $81.42, or at a loss of 40%? **45.** If sold for $7583.40, or at a gain of 10% ? **46.** If sold for $38.24, or at a loss of 20%?

Selling price÷100% + rate of gain = cost.
Selling price÷100% —rate of loss = cost.

47. A farmer bought 35 acres of land for $1750 and sold it at 20% profit. What was his gain per acre ?

48. I bought a house for $5600, paid $400 additional for repairs, and then sold it at 12% profit. What was my gain ?

49. What is the per cent. of profit on a piano which cost $350 and is sold for $420 ?

50. A bill of goods was sold for $13 less than cost, by which a loss of 5% was sustained. What was the cost of the goods?

51. What is the loss on 153 tons of iron bought at $45 per ton and sold at 20% below cost?

52. What is the rate per cent. of profit on merchandise which cost $573 and is sold for $647.49?

53. A dealer sold a quantity of lumber for $3172.50 and lost 37½%. What did he pay for the lumber?

54. A merchant bought 80 bushels of oats at 56 cents per bushel, paid $1.40 for sundry expenses, and sold out at 15% profit. How much did he receive for the oats?

55. A stationer bought 9 reams of paper at $2.40 per ream, and retailed it at 1 cent per sheet. What was his per cent. of gain?

56. A quantity of wheat which cost $1.14 per bushel was sold at a loss of 20%. How many bushels were sold if the total loss was $30.78?

57. A set of jewelry was sold for $126.30. What was the cost if the seller gained 25%?

58. A speculator bought three houses at $1800 each, and sold them at a profit respectively of 8%, 12%, and 15%. What did he receive for the three houses?

59. A grain dealer bought 735 bushels of wheat at $1.14 per bushel, and sold the entire quantity for $977.55. What was his per cent. of gain?

60. A grocer bought 174 gallons of molasses at 55 cents per gallon, paid 87 cents drayage to get it to his store, and retailed it at a profit of 33⅓%. What was his selling price per gallon?

61. A drover bought 15 horses at $125 per head, sold two of them at $127.75 per head, 8 at $140 per head, and the remainder at $150 per head. If his expenses in taking them to market amounted to $5 per head, what was his per cent. of gain?

62. A man sold a house for $9775 and gained $1275. What was his rate per cent. of profit?

63. A merchant lost $9.96 by selling an article for $73.04. What was his rate per cent. of loss?

64. A music dealer sold a piano for $413 and gained 18%. What was his profit?

65. A grocer bought 536 gallons of vinegar for $150.08, sold 175 gallons at 30 cents per gallon, 124 gallons at 35 cents per gallon, and the remainder for $91.70. What was his rate per cent. of gain?

66. Oats purchased at 45⅝ cents per bushel were sold for 54¾ cents per bushel. What was the rate per cent. of gain ?

67. A grocer sold a quantity of sugar for $330, and thereby lost 12%. How much did he lose ?

68. A gentleman gained 24% by selling land for $195.60 more than he paid for it. What did he obtain for the land ?

69. A jeweler purchased a pound of gold at $16.58⅓ per ounce, added 2 ounces, Troy, of alloy at 50 cents per ounce, and converted the whole into rings which he sold at $1 per pennyweight. What was his per cent. of gain ?

70. What did I receive for 3872 pounds of sugar bought at 5¾ cents per pound, and sold at 16⅔% profit ?

71. A merchant sold a quantity of rye at 15% profit and gained $26.70. What did he receive for the rye ?

72. A farmer bought two horses at $175 each, sold one at a profit of 20%, and the other at a loss of 5%. What was his net gain ?

73. A man sold two lots for $1800 each, gaining 25% on one lot and losing 25% on the other. Did he gain or lose, and how much ?

74. A merchant sold a bill of goods for $89.30, and thereby lost 6%. How much did he lose ?

75. A house was sold for $5616 at a gain of 8%. What was the gain ?

76. If a house increase in value 20% per annum for three successive years and is then worth $6912, what was its value at the beginning of the first year ?

77. A dealer sells goods, wholesale, at 10% profit; and, retail, at 20% advance on wholesale rates. How much greater is his profit on goods sold at $1.98 retail than if sold at wholesale rates ?

78. A grocer sold 50% of an invoice of coffee at 40% gain, and the remainder at 20% gain. What was his per cent. of gain on the whole ?

79. A merchant sold 40% of a barrel of molasses at 10% gain, and the remainder at cost. What was his gain per cent. on the whole ?

80. A merchant sold 30% of an invoice of grain at 20% profit, and the remainder at 10% loss. If his net loss was $3.15, what was the total amount of sales ?

81. A bill of goods cost $520, and freight 5% additional. If 15% of the goods were sold at 20% profit, 60% of the remainder at 33⅓% profit, and what was left of the two preceding sales at 25% profit, what was the total gain ?

82. A dealer sold 20% of an invoice of tea at a loss of 25%, and the remainder at a gain of 12½%. What was the cost of the tea if the total sales amounted to $840?

83. A fruit dealer bought 530 crates of peaches at 95 cents per crate, sold 150 crates at a gain of 20%, 75 crates at a gain of 12%, 50 crates at cost, 200 crates at a loss of 5%, 25 crates at a loss of 50%, and threw away the remainder as worthless. What was his net gain or loss?

84. I bought a quantity of cloth at $2 per yard. Allowing 5% for bad debts, what must I ask per yard to net a profit of 20%?

85. A cabinet-maker sold a set of furniture for $18.75 more than cost, and gained 30%. What would have been his rate per cent. of gain or loss if he had sold the furniture for $87.50?

86. A clothier bought 50 yards of broadcloth at $3.60 per yard and 76 yards of cassimere at $2.50 per yard. He sold the cassimere at a loss of 10%. What should he ask per yard for the broadcloth to net a profit of 20% on the cost of both?

87. The profits of a wholesale house in 1880 were 20% of its capital; in 1881, 25%; in 1882, 16⅔%, and in 1883 it suffered a loss of 4%. At the close of each year the firm reinvested the entire capital and gain in the business, and on Jan. 1, 1884, the capital of the firm was $75264. What was the capital of the firm on Jan. 1, 1881?

88. A grocer purchased a quantity of vinegar of which 20% was lost by leakage. Expecting to obtain cost for the entire quantity, he sold the remainder at 20% advance on its cost. What was his per cent. of gain or loss, if any?

89. A merchant bought a lot of notions, 25% of the cost of which he paid in cash, and on the balance due, amounting to $741, he was allowed a credit of 60 days. If he sold the notions at a uniform profit of 12½%, what did he receive for them?

90. A man bought a house for $5600, paid $900 for improvements, and then sold it at a profit of 12%, receiving 30% of the selling price in cash, and accepting a note secured by mortgage for the balance. What was the face of the note?

91. How much should be asked for coffee which costs 18 cents per pound that a gain of 5% may be realized, allowing for a loss of 10% in roasting?

92. A man sold a house at 20% profit, and with the proceeds purchased another which he sold at 30% profit, realizing a total gain of $3136 on both. What did he pay for each?

TRADE DISCOUNTS.

222. Trade Discounts are allowances or abatements made by manufacturers and dealers upon their catalogue or list prices.

Rem. 1.—It is customary in some branches of business for manufacturers or dealers to list or catalogue their goods or wares at a fixed price for each article, and allow an abatement or discount on all orders of a certain amount, a second discount on larger orders, a third discount on still larger orders, etc. In some cases. trade discounts are designed as the fee or compensation to agents or other middlemen for handling a line of goods, without regard to the quantities involved.

Rem. 2.—Since the price of an article will vary with the supply and demand, instead of publishing a new catalogue or list price with each fluctuation, manufacturers or jobbers simply *change the rate* of discount with each variation of price.

Rem. 3.—Merchandise is frequently sold at "time prices," upon which certain rates of discount are allowed if paid at an earlier date. Thus, an invoice of goods may be sold for $325, payable in 60 days, and subject to a discount of 1% if paid in 30 days, or 2% if paid in cash or within 10 days. Even when no discount *is offered* upon credit sales, sellers are generally willing to allow bank discount for the interval, if "time payments" are made before maturity.

223. Computations in Trade Discount are made in accordance with the principles of percentage, the gross selling price (that is, the invoice, list, or marked price) being taken as the base; and the discount and net selling price as percentages. Hence,

General Formula.

$$\begin{tikzcd}
\textit{Gross Selling Price} = & \textit{Base} \arrow[dr, "\times"] & \\
& & \textit{Rate \%} \\
\left.\begin{array}{l}\textit{Trade Discount, or}\\ \textit{Net Selling Price.}\end{array}\right\} = & \textit{Percentage} \arrow[uu, "\div"] \arrow[ur, "\%"'] &
\end{tikzcd}$$

The direction for the use of this formula is the same as that of General Formula, **214.**

Rem. 1.—As the net selling price is the gross selling price minus the discount, so the rate of net selling price must be the rate of gross selling price (100%) minus the rate % of discount.

Rem. 2.—If two or more rates of discount are allowed, only the first is reckoned upon the gross selling price, each succeeding rate of discount being computed upon the proceeds of the preceding discounts.

224. To find the net selling price.

ILLUSTRATIVE EXAMPLE.

What is the net price of hardware, invoiced at $460, and subject to a discount of 20%, 10%, and 5%?

SOLUTION.

$460,	gross selling price,
92,	20% or $\frac{1}{5}$ off.
$368,	proceeds of 1st disct.
36.80,	10% or $\frac{1}{10}$ off.
$331.20,	proceeds of 2d disct.
16.56,	5% or $\frac{1}{20}$ off.
$314.64,	net selling price.

EXPLANATION.—Find the first discount ($\frac{1}{5}$ of $460), and subtract it ($92) from the gross price ($460), obtaining $368 as the proceeds of the first discount.

Next find the second discount ($\frac{1}{10}$ of $368) and subtract it ($36.80) from the base of the second discount ($368), obtaining $331.20 as the proceeds of the second discount.

Next find the third discount ($\frac{1}{20}$ of ($331.20) and subtract it ($16.56) from the base of the third discount ($331.20), obtaining $314.64 as the proceeds of the third discount, or the required net price.

NOTE.—Each discount and intermediate proceeds should be carried to mills, to secure accuracy in the cents' order of the final result.

EXAMPLES FOR PRACTICE.

Find the net selling price.

Gross price.	*Discount off.*	*Gross price.*	*Discount off.*
1. $418	25%, 20%, 10%.	6. $617.40	15%, 5%, 3%.
2. $619.45	10%, 10%, 5%.	7. $585.75	20%, 10%, 8%.
3. $496.82	20%, 10%, 2%.	8. $781.25	40%, 25%, 4%.
4. $718.25	30%, 20%, 5%.	9. $910.32	50%, 30%, 2%.
5. $347.60	10%, 5%, 1%.	10. $843.50	20%, 18%, 7%.

11. What is the net cost to the purchaser of hardware invoiced at $815, and subject to a discount of 20%, 10%, and 5%?

12. The gross amount of a bill of goods is $750.35, and the rates of discount are 10%, 10%, and 5%. What is the net cost to the purchaser?

13. An invoice of crockery amounting to $1575.50 was sold Jan. 18, 1889, payable in 90 days, and subject to a discount of 20% and 10%, with an additional discount of 1% if paid in 60 days, 2% if paid in 30 days, or 3% if paid within 10 days of date of purchase. How much will pay the bill on Feb. 20, 1889?

14. The gross amount of a bill of tinware is \$275.30, of which \$75 is sold at a discount of 25% and 10%; \$150 at 20% and 5%; and the remainder at 10% and 5%. What is the net amount of the bill?

225. To reduce a discount series to a single discount.

ILLUSTRATIVE EXAMPLE.

What single discount on the gross price is equivalent to a discount series thereon of 25%, 20%, and 10%?

SOLUTION.

100% = gross price.
25% = 1st disct.
75% = 1st proceeds.
[$\frac{1}{5}$ of 75] 15% = 2d disct.
60% = 2d proceeds.
[$\frac{1}{10}$ of 60] 6% = 3d disct.
54% = final proceeds.
100% — 54% = 46%, *Ans.*

EXPLANATION.—Since the gross price is the base, its rate is 100%.

Deduct the rate of first discount (25%) from the rate of the gross price (100%), obtaining 75% as the rate of 1st proceeds.

Then reduce the rate of second discount (20%) which is expressed on the 1st proceeds to an equivalent rate expressed on the gross price (20% of 75% of the gross price = 15% of the gross price) and subtract, obtaining 60% as the rate of 2d proceeds.

Continuing, it is found that 54% of the gross price is the rate of final proceeds. Hence, 100% of the gross price minus 54% of the gross price, or 46% of the gross price, equals the required rate of single discount.

NOTE.—The net result of a series of discounts is the same regardless of the order of their arrangement. Thus, 25%, 20%, and 10% off an invoice is the same as 20%, 25%, and 10%; or 10%, 20%, and 25% off the same invoice; but the net result will be different even if the *sum* of each discount series is the same, if the rates themselves differ. Thus, 20% and 10% off an invoice will not produce the same result as 25% and 5% off.

Reduce the following discount series to equivalent single discounts:

1. 10% and 5%.
2. 20% " 10%.
3. 15% " 10%.
4. 30%, 10%, and 5%.
5. 20%, 10%, " 5%.
6. 10%, 5%, " 3%.

7. What is the difference between a discount series of 10%, 5%, and 2%, and a single discount of 17%?

8. If the rates of discount are 20%, 10%, and 1%, what % of the gross price will be the net price?

ADDITIONAL PROBLEMS IN DISCOUNT.

226. 1. What must be asked for goods that a discount of 20% and 10% may be allowed, and net $2.88 to the seller?

Net price ($2.88) ÷ rate of net price (72%) = gross price ($4).

At what price must goods be marked

2. To net $2.72, after allowing a discount of 15%?
3. To net $12, after allowing a discount of 10% and 10%?
4. To net $3.60, after allowing a discount of 25%, 20%, and 10%?
5. To net $30, after allowing a discount of 10%, 10%, and 10%?
6. To net $14.58, after allowing a discount of 40%, 10%, and 10%?

7. At what price must goods be marked to net 10% profit after allowing a discount of 10% and 5%, if they cost $4.75?

Cost ($4.75) + profit ($.475) = net selling price ($5.225).
Net price ($5.225) ÷ rate of net price (85½%) = marked price ($6.11 +).

At what price must goods be marked to net

8. 25% profit after allowing 25%, 20%, and 10% off; cost $216?
9. 20% profit after allowing 10%, 5%, and 3% off, if they cost $96?
10. 12½% profit after allowing 20%, 10%, and 2% off; cost $171.20?
11. 10% loss after allowing 10% and 5% off, if they cost $15.30?
12. 20% loss after allowing 25%, 10%, and 5% off; cost $15.75?

13. If goods which cost $120 are marked $150, what per cent. of discount must be allowed on marked price to net cost?

Discount ($150 − $120 = $30) ÷ marked price ($150) = rate of discount (20%).

What per cent. of discount must be allowed on marked price

14. To net cost, if the goods cost $5 and are marked $6?
15. To net cost, if the goods cost $3.24 and are marked $4.05?
16. To net cost, if the goods cost $8.50 and are marked $12.75?
17. To net 20% profit, if the goods cost $7.50 and are marked $12?
18. To net 10% loss, if the goods cost $8 and are marked $9?

19. Merchandise was bought at a certain gross price with 20% and 10% off, and sold at the same gross price with 10% and 5% off. What was the per cent. of profit?

20. If goods are sold at their marked price the rate of profit will be 20%. What will be the rate of profit if the same goods are sold at 10% discount on their marked price?

(21. Hardware.)

All accounts remaining unsettled 90 days subject to sight draft.

Baltimore, *June 15, 1889.*

Mr. Fairman A. Sadler,

Bought of S. G. B. Cook & Co.

Terms: 60 days, net; 30 days, 3 per cent. off; 10 days, 5 per cent. off.

1	Doz. Table Knives 4003 - - - - - - - -	$10	00		
	25% & 10%			*	**
½	" Butcher " ea. $\overset{3.00}{\text{8-in.}}$ $\overset{4.50}{\text{10-in.}}$ - - - - -			*	**
1	" Sticking " G. B. 6-in. - - - - - -			2	10
½	" Pocket " ea. $\overset{3.00}{261}$ $\overset{5.00}{310}$ $\overset{4.50}{318}$ - - -			*	**
3	" Brandford Locks $\overset{3.65}{01800}$ - - - - - - -	**	**		
3	" " Knobs $\overset{2.50}{202}$ - - - - - - -	*	**		
		**	**		
	60% & 10%			*	**
1	" Kipp's A. E. Farrier's Hammers - - -	6	00		
	30%			*	**
¼	" Ives Braces $\overset{36.00}{182}$ - - - - - - - - -	*	**		
	60% & 5%			*	**
½	" Oneida Traps ea. $\overset{3.50}{\$1}$ $\overset{6.00}{\$1\frac{1}{2}}$ - - - - - -	*	**		**
2	" " " $\overset{9.00}{\$2}$ - - - - - - - -	**	**		
⅙	" " " $\overset{14.50}{\$4}$ - - - - - - - -	*	**		
		**	**		
	60%, 10%, & 5%			*	**
1	" Cotton Hooks ea. $\overset{1.85}{\$1}$ $\overset{3.35}{\$4}$ - - - - - -	*	**		
	25%			*	**
1½	" Bolts 481 $\overset{6.00}{\text{6-in.}}$ - - - - - - - - - -	*	**		
1	" " " 8-in. - - - - - - - - - -	7	50		
1	" " 490 ea. $\overset{5.00}{\text{6-in.}}$ $\overset{6.50}{\text{8-in.}}$ - - - - - -	**	**		
		**	**		
	60%, 10%, & 10%			*	**
	Pkg. and Drayage,				75
				**	**

What sum will pay the above bill on June 20, 1889?

COMMISSION AND BROKERAGE.

227. Commission or **Brokerage** is the sum charged by an agent for transacting business for his principal; as, for buying or selling property, for collecting or investing money, etc.

Rem. 1.—Commission or brokerage is usually estimated at a per cent. of the gross proceeds of a sale or of the prime cost of a purchase; but on some kinds of merchandise it is often computed at a certain price per unit; as, grain per bushel, flour per barrel, cotton per bale, etc.

Rem. 2.—If sales are made on credit, a certain per cent. of the sum credited is usually charged by the agent for guaranteeing its final payment. This additional charge is called *guaranty*.

Rem. 3.—The agent who buys or sells merchandise for another is called a *Commission Merchant* or *Broker*. A *commission merchant* is usually placed in possession of the goods bought or sold, and acts in his own name; a *broker* does not take possession, but simply effects contracts to buy or sell in the name of his principal.

Rem. 4.—The term *broker* is also applied to those whose business it is to buy and sell stocks, bonds, bills of exchange, promissory notes, specie, etc., etc.

Rem. 5.—A *Consignment* is a shipment of merchandise by one party to another; the party who ships the merchandise being called the *Consignor* or *Shipper*, and the party to whom the merchandise is shipped being called the Consignee.

228. The **Gross Proceeds** of a sale or collection is the total amount received by the agent for his principal *before* deducting the commission or other charges.

Rem. 1.—After deducting the commission and all other charges from the gross proceeds, the remainder is called the *net proceeds*.

Rem. 2.—An *account sales* is a written statement rendered by an agent to his principal, specifying the goods sold and prices obtained, detailing the several charges or expenses, and exhibiting the net proceeds or balance due the principal.

229. The **Prime Cost** of a purchase is the net sum paid by an agent, and does not include his commission or other charges.

Rem. 1.—After adding the commission and other charges to the prime cost, the sum is called the *gross cost*.

Rem. 2.—An *account purchase* is a written statement rendered by an agent to his principal, specifying the property bought and prices paid, detailing the various charges or expenses, and exhibiting the gross cost of the purchase.

227—229

230. Computations in Commission and Brokerage are made in accordance with the principles of percentage, the gross proceeds (if a sale) or the prime cost (if a purchase) being regarded as the base; and all other terms than gross proceeds or prime cost (such as commission, guaranty, net proceeds, gross cost, etc.), as percentages. Hence,

GENERAL FORMULA.

$$\begin{tikzcd}
\left.\begin{array}{r}\textit{Gross Proceeds of a sale,}\\ \textit{Prime Cost of a purchase.}\end{array}\right\} = & \textit{Base} \arrow[dr, "\times"] & \\
& & \textit{Rate \%} \\
\left.\begin{array}{r}\textit{Commission,}\\ \textit{Guaranty,}\\ \textit{Net Proceeds of a sale,}\\ \textit{Gross Cost of a purchase.}\end{array}\right\} = & \textit{Percentage} \arrow[uu, "\div"] \arrow[ur, "\div"'] &
\end{tikzcd}$$

REM. 1.—Since the net proceeds is the gross proceeds minus all the expenses, the rate of net proceeds must be the rate of gross proceeds (100%) minus the rate of all the expenses; and since the gross cost is the prime cost plus all the expenses, the rate of gross cost is the rate of prime cost (100%) plus the rate of all the expenses.

REM. 2.—If non-percentage charges, as freight, drayage, storage, etc., are considered in a sale or purchase, the net proceeds or gross cost will also be non-percentages; and they must be changed to percentages by adding the non-percentage charges to the net proceeds or subtracting them from the gross cost, before the above formula can be applied.

EXAMPLES FOR PRACTICE.

1. What is the commission if the gross proceeds of a sale are $180 and the rate of commission for selling is $2\frac{1}{2}\%$?

Gross proceeds ($180) × rate of commission ($2\frac{1}{2}\%$) = commission ($4.50).

What is the commission

2. If the gross proceeds are $53.50, and the rate of commission 2%?
3. If the prime cost is $12360, and the rate of commission $\frac{3}{11}\%$?
4. If the gross proceeds are $1240, and the rate of commission $\frac{5}{8}\%$?
5. If the gross proceeds are $236, and the rate of commission $1\frac{1}{4}\%$?
6. If the prime cost is $7500, and the rate of commission $\frac{3}{4}\%$?

What are the net proceeds of a sale if

7. The gross proceeds are $750, the rate of commission 3%, and the other expenses $5.25?

8. The gross proceeds are $1275, the rate of commission 4%, and the other expenses $18.30?

9. The gross proceeds are $18000, the rate of brokerage $\frac{3}{8}$%, and the other expenses $175.30?

10. The gross proceeds are $700, the rate of commission 2½%, and the other expenses $12.75?

What is the gross cost of the purchase if

11. The prime cost is $15.50, the rate of commission 4%, and the other expenses $1.30?

12. The prime cost is $7320, the rate of commission 2¼%, and the other expenses $150.25?

13. The prime cost is $2480, the rate of brokerage $\frac{5}{16}$%, and the other expenses $25?

14. The prime cost is $73.20, the rate of commission 2¾%, and the other expenses $5.25?

15. What is the rate per cent. of commission if the prime cost of merchandise is $480, and the commission for buying $4.20?

Commission ($4.20) ÷ prime cost ($480) = rate of commission ($\frac{7}{8}$%).

What is the rate per cent. of commission if

16. The gross proceeds are $960 and the commission $15.60?

17. The prime cost is $3264 and the commission for buying $3.06?

18. The gross proceeds are $3200 and the commission for selling $6?

19. The prime cost is $460 and the commission for buying $23?

20. The prime cost of the purchase is $275 and the gross cost $286?

21. The gross proceeds are $105 and the net proceeds $102.90?

22. The gross proceeds are $380 and the net proceeds $379.05?

23. The prime cost is $124 and the gross cost $128.34?

24. What are the gross proceeds of the sale if the commission is $132 and the rate of commission 4%?

Commission ($132) ÷ rate of commission (4%) = gross proceeds ($3300).

What are the gross proceeds or prime cost if

25. The commission for selling is $50.30 and the rate 2%?

26. The commission for buying is $75.30 and the rate 5%?

27. The commission for selling is $36.60 and the rate 1½%?

28. The brokerage for buying is $29.25 and the rate ¾%?

29. The net proceeds are $736.96 and the rate of commission 2%?

Net proceeds ($736.96) ÷ rate of net proceeds (98%) = gross proceeds ($752).

30. The net proceeds are $77.90 and the rate of commission 5%?

31. The net proceeds are $344.75 and the rate of commission 1½%?

32. The net proceeds are $495.36 and the rate of commission 3¼%?

33. The gross cost is $138.69 and the rate of commission 3½%?

Gross cost ($138.69) ÷ rate of gross cost (103½%) = prime cost ($134).

34. The gross cost is $185.40 and the rate of commission 3%?

35. The gross cost is $564.20 and the rate of brokerage ¼%?

36. The gross cost is $1606 and the rate of brokerage ⅜%?

37. The net proceeds are $51.90, the rate of commission 5%, and the other charges $1.30?

($51.90 + $1.30) ÷ .95 = $56, gross proceeds of sale.

38. The net proceeds are $145.50, the rate of commission 2%, and the other charges $2.50?

39. The net proceeds are $704.20, the rate of commission 1¾%, and the other charges $3.20?

40. The net proceeds are $774.25, the rate of commission 2½%, and the other charges $5.75?

41. The gross cost is $668.60, the rate of commission 4%, and the other charges $8.20?

($668.60 − $8.20) ÷ 1.04 = $635, prime cost of purchase.

42. The gross cost is $33.50, the rate of commission 3⅛%, and the other charges $.50?

43. The gross cost is $141.56, the rate of commission 2¼%, and the other charges $2.50?

44. The gross cost is $256.15, the rate of commission 1⅞%, and the other charges $3.50?

45. A commission merchant sold a consignment of cotton for $736 and charged 1½% commission. What was his commission?

46. A grain broker charged $20 for effecting the sale of 16000 bu. corn at 80¢ per bushel. What was his rate per cent. of brokerage?

47. An agent charged $13.40 for selling a consignment of merchandise. What were the gross proceeds of the sale if his rate of commission for selling was 2½%?

48. A commission merchant received $544.35 to invest in wool after deducting all expenses. How much did he pay for the wool, if his rate of commission for buying was 2%, and the sum of the remaining charges $3.75?

49. A fruit buyer received an order to purchase 324 barrels of apples. The purchase was made at $2.25 per barrel, and in settlement the buyer drew on his principal for $750.87. What was his rate % of commission?

50. An attorney collected 30% of a note the face of which was $75, and charged 10% commission for collecting. What were the net proceeds due the principal?

51. An agent invested $7500 in cotton, charged $1\frac{1}{8}$% commission for buying, $125 for freight prepaid, $15 for drayage, and drew on his principal for the gross cost. What was the face of the draft?

52. If an agent's commission was $21.30 for selling 284 yards of cloth at $3 per yard, what was his rate per cent. of commission?

53. A commission merchant remitted to his principal $158 as the net proceeds of a consignment of eggs. What were the gross proceeds if the rate of commission was 3%, of guaranty 2%, and the sum of the other charges $6.54?

54. An agent sold 42 barrels of potatoes at $3.25 cash per barrel, and 58 barrels at $3.50 per barrel, payable in 30 days. His charge for selling was 3%, for guaranty 2%, and for cooperage $3. What were the net proceeds due the principal?

55. What is the cost of $17\frac{15}{}$ bushels of clover seed at $7.25 per bushel, commission $2\frac{1}{2}$% additional?

56. A commission merchant received a consignment of 420 barrels of flour, on which he advanced to the consignor $200 cash, paid $42 freight, $5 cooperage, $12.60 storage, and $10.50 cartage. He sold 80 barrels at $7.25 per barrel, 150 barrels at $7.75 per barrel, 50 barrels at $8.25 per barrel, and the remainder at $8.50 per barrel. What was the balance due the consignor, the commission for selling being 2%?

57. The gross cost of a purchase is $1221, which includes the commission $6, and other charges amounting to $15. What is the rate per cent. of commission?

58. A commission merchant remitted $1323 as the net proceeds of a consignment, after deducting $27 as commission. What was his rate per cent. of commission?

59. An agent purchased 7200 bushels of wheat at $1.34 per bushel, charged $\frac{9}{16}$% commission for buying, $108 for storage, and $120 for cartage. What was the total cost to the principal?

60. A commission merchant sold 350 barrels of flour at $7.50 per barrel, paid 50 cents per load of 10 barrels for drayage, and $3 for cooperage. He purchased for the consignor 1570 bushels of wheat at $1.20 per bushel, paid 1½ cents per bushel for storage, and 1 cent per bushel for drayage. What is the balance due the consignor, if the rate of commission was 2% for selling and the same for buying?

61. A commission merchant sold 80 bales of cotton, averaging 400 pounds per bale, at 12½ cents per pound, charged 2½% for selling, 3% for guaranty, and $24.36 for other expenses. He invested the proceeds in coffee at 14 cents a pound, and charged 2% commission for buying. How many pounds of coffee did he buy?

62. An agent received $300 to invest in apples after deducting all expenses. How many barrels did he buy at $3.50 per barrel, his charges being 4% commission for buying, 2½ cents per barrel for drayage, 12 cents per barrel for freight, $2.15 for cooperage, and $3.50 for advertising? What was the unexpended balance of the remittance?

It is customary in the commission business to find the number of *whole* barrels, by first deducting from the remittance all charges which are not estimated per barrel, and dividing the remainder by the prime cost per barrel plus the commission and remaining charges per barrel. If the division is not exact, treat the remainder as an unexpended balance to be returned to the principal or placed to his credit.

63. An agent received $96 to invest in onions after deducting his commission of 5% and drayage of 25 cents per load of ten barrels. How many barrels of onions did he buy at $1.60 per barrel and what was the unexpended balance of the remittance?

After finding the number of whole barrels as in Ex. 62, divide by 10 to find the number of loads, considering the fraction of a load as a full load. Deduct from the unexpended balance the increased drayage per barrel on the fraction of a load.

64. A commission merchant received $4456.40 to invest in wheat after deducting his commission of 2%, and drayage of 50 cents per load of 36 bushels. How much wheat did he purchase at $1.20 per bushel?

Gross cost of all the bushels ÷ gross cost of 1 bushel = number of bushels.

65. At what price shall an agent be ordered to buy potatoes at 2% commission that, after paying 7 cents per barrel for transportation, they can be sold at $1 76 per barrel and net 10% profit?

66. I remitted $578 to my agent in Chicago with instructions to invest in wheat after deducting his commission of 2%, and consign the purchase to my agent in New Orleans. What was my net gain, if my agent in New Orleans sold the wheat for $625, charged 3% for selling, and $12.35 for sundry other expenses?

67. What is the brokerage for effecting the sale of 7000 bushels of wheat at $1.25 per bushel if the rate of brokerage is $\frac{1}{8}$¢ per bushel?

(68. Account Sales.)

BALTIMORE, *Aug. 15, 1889.*

SOLD *for account of* J. T. JANNEY

By G. W. HUNTER & Co.

1889.							
Aug.	8	75 bbls. Queen City Flour - - -	$7.25	***	**		
"	"	50 " Snow Flake - - - - -	7.30	***	**		
"	9	130 " Patapsco - - - - - -	7.00	***	**		
"	10	200 " Howard St. - - - - -	6.25	****	**		
"	15	80 " Occoquan Family - - -	5.25	***	**	****	**
		Charges:					
"	6	Freight, 535 bbls. @ 15¢ - - - - - - -		**	**		
"	"	Drayage, 535 bbls. @ 5¢ - - - - - - -		**	**		
"	10	Storage, 535 bbls. @ 3¢ - - - - - - -		**	**		
"	15	Inspection, 535 bbls. @ 1¢ - - - - -		*	**		
		Commission, 3% - - - - - - - - - -		**	**	***	**
		Net Proceeds to your Credit - - - -				****	**

(69. Account Purchase.)

ST. LOUIS, MO., *March 12, 1889.*

BOUGHT *for account and risk of* S. MEEMS & Co.

By SUTRO & WILLIAMS.

15	bbls. Early Rose Potatoes - - -	$3.25	**	**		
8	" Goodrich " - - -	2.75	**	**		
4	" Burbank " - - -	2.63	**	**		
35	bu. Seed Corn - - - - - - - - -	1.25	**	**		
75	" " Oats - - - - - - - - -	.75	**	**		
83	" " Wheat - - - - - - - -	1.45	**	**		
72	" Buckwheat - - - - - - - -	1.15	**	**		
5	" Clover Seed - - - - - - - -	7.35	**	**	***	**
	Charges:					
	Drayage - - - - - - - - - - - - - -		1	50		
	Commission, 2% - - - - - - - - - - -		*	**	*	**
	Debited your account - - - - - - -				***	**

INTEREST.

231. **Interest** is a charge for the use of money.

232. The **Principal** is the sum for the use of which interest is charged.

233. The **Rate of Interest** is the per cent. of the principal which is charged for its use one year.

234. The **Time** is the period during which the principal bears interest.

235. **Simple Interest** is the sum charged for the use of the original principal, or of so much of the original principal as may at any time remain unpaid.

236. **Ordinary Interest** is interest computed on the basis of 360 days to a year.

By reason of its great convenience, this method of computing interest is generally employed by business men.

237. **Accurate Interest** is interest computed on the basis of 365 days to a common year or of 366 days to a leap year.

238. The **Amount** is the *sum* of the principal and its accrued interest.

239. **Legal Interest** is interest which is computed at the rate % per annum established by law.

1. When no particular rate is specified in a contract, the legal rate is understood. In several States, when *agreed to* by both borrower and lender, more than the legal rate is allowed, ranging from 7% to 12%; and in a few States, the higher rate is unlimited.

2. To charge a higher rate of interest than is allowed by law is called *usury*, for which penalties are imposed by the laws of nearly all the States.

3. Debts of all kinds draw interest from the time they become due; but not before unless so specified.

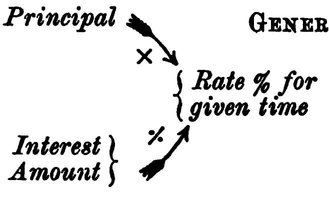
240. The legal and maximum rates of interest allowed by law in the different States and Territories of the United States are as follows:

TABLE OF INTEREST RATES.

States and Territories.	Rate Per Annum.		States and Territories.	Rate Per Annum.	
	Legal.	Maximum.		Legal.	Maximum.
Alabama	8%	8%	Missouri	6%	10%
Alaska	....	.....	Montana	10%	any %
Arizona Ter.	10%	any %	Nebraska	7%	10%
Arkansas	6%	10%	Nevada	7%	any %
California	7%	any %	New Hampshire	6%	6%
Colorado	10%	any %	New Jersey	6%	6%
Connecticut	6%	any %	New Mexico Ter.	6%	any %
Delaware	6%	6%	New York	6%	6%
District of Columbia	6%	10%	North Carolina	6%	8%
Florida	8%	any %	North Dakota	....	.....
Georgia	7%	8%	Ohio	6%	8%
Idaho Ter.	10%	24%	Oregon	8%	10%
Illinois	6%	8%	Pennsylvania	6%	6%
Indiana	6%	8%	Rhode Island	6%	any %
Indian Ter.	6%	any %	South Carolina	7%	7%
Iowa	6%	10%	South Dakota	....	.....
Kansas	7%	12%	Tennessee	6%	6%
Kentucky	6%	6%	Texas	8%	12%
Louisiana	5%	8%	Utah	10%	any %
Maine	6%	any %	Vermont	6%	6%
Maryland	6%	6%	Virginia	6%	6%
Massachusetts	6%	any %	Washington	10%	any %
Michigan	7%	10%	West Virginia	6%	.....
Minnesota	7%	10%	Wisconsin	7%	10%
Mississippi	6%	10%	Wyoming	12%	any %

Rem.—The legal rate of interest for England and France is 5%, and for the Dominion of Canada 6%.

241. Computations in Interest may be made in accordance with the principles of percentage, the principal being regarded as the base; and the interest and amount as percentages. Hence,



GENERAL FORMULA.

The direction for the use of this formula is the same as that of General Formula, **214.**

REM. 1.—The rate of interest for the given time has the same relation to the rate per annum as the given time has to one year. Hence, if the rate per annum is 8%, the corresponding rate for two years will be twice 8%, or 16% ; for six months will be ½ of 8%, or 4%; and for 18 days will be $\frac{18}{360}$ of 8%, which by cancellation $\left(\frac{\overset{.18}{\cancel{18}}}{\underset{45}{\cancel{360}}} \text{ of } \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{100}}}\right)$ may be changed to the following form: .18+45 = .004. Hence, to find the rate of ordinary interest for any number of days, *divide the days regarded as hundredths by the quotient* (obtained mentally) *of the rate per annum into 360.*

REM. 2.—Since the amount equals the principal plus the interest for the given time, the rate of amount equals the rate of the principal (100%) plus the rate of interest for the given time.

"Product Rule" at any % per annum.

242. At 6% for one year (360 days), the corresponding rate will be ⅙ of 6%, or 1% of the principal for ⅙ of 360 days, or 60 days; at 5% per annum, it will be ⅕ of 5%, or 1% of the principal for ⅕ of 360 days, or 72 days; at 4% per annum, it will be ¼ of 4%, or 1% of the principal for ¼ of 360 days, or 90 days; and, generally, at any rate % per annum, it will be one-hundredth of the principal for as many days as the rate per annum is contained times in 360.

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $486.82 for 47 days at 5% per annum?

SOLUTION.

```
   $4.8682
        47
   -------
    340774
   194728
   -------
8)228.8054
  --------
 9)28.60
  ------
   $3.18—
```

EXPLANATION.—At 5% for one year (360 da.) the corresponding rate will be .01 of the principal for ⅕ of 360 da., or 72 da.; and it will be $\frac{47}{72}$ of .01 of the principal for $\frac{47}{72}$ of 72 da., or 47 da. Hence, move the decimal point of the principal *two* places to the left ($4.8682), multiply by the number of days (47), and divide the product by 72, or successively by its factors 8 and 9, obtaining $3.18— as the required interest.

2. What is the interest of $751.45 for 23 days at 6% per annum?

SOLUTION.

$7.5145
23
225435
150290
6)17.2.8335
$2.88+

EXPLANATION.—At 6% for one year (360 da.) the rate will be .01 of the principal for $\frac{1}{6}$ of 360 da., or 60 da.; and it will be $\frac{23}{60}$ of .01 of the principal for $\frac{23}{60}$ of 60 da., or 23 da. Hence, proceed as in Ex. 1, but divide the product by 60, obtaining $2.88+ as the required interest.

To divide by 60, move the decimal point of the product one additional place to the left and divide by 6 (43).

3. What is the interest of $625.75 for 5 mo. 13 da. at 8% per annum? 4. Of $518.60 for 39 days at 7% per annum? 5. Of $842.35 for 1 yr. 3 mo. 19 da. at 9% per annum?

SOLUTIONS.

(3)

5 mo. (150 da.) 13 da. = 163 da.
$6.2575
163
187725
375450
62575
5)1019.9725
9)203.994
$22.67 — , *Ans.*

(4)

$5.1860
39
466740
155580
6)20.2.2540
$3.371 —, int. @ 6%
($\frac{1}{6}$ of $3.371 =) .562 —, " " 1%
$3.93 +, " " 7%

(5)

1 yr. (360 da.) 3 mo. (90 da.) 19 da. = 469 days.
$8.4235
469
758115
505410
336940
4)395.0.6215
$98.77 —, *Ans.*

RULE.—*Multiply the principal by the number of days, point off from the right of the product two more decimal places than are contained in the principal, and if the rate per annum be*

1%, divide by 360 (= 60 × 6)			6%, divide by 60
2%, " " 180 (= 60 × 3)			8%, " " 45 (= 5 × 9)
3%, " " 120			9%, " " 40
4%, " " 90			10%, " " 36 (= 6 × 6)
4½%, " " 80			12%, " " 30
5%, " " 72 (= 9 × 8)			

NOTE 1.—Where any of the above divisors end with a cipher, discard the cipher and move the decimal point of the product one additional place to the left.

NOTE 2.—At 3½% per annum, find the interest at 4% and deduct one-eighth; at 5½%, find the interest at 6% and deduct one-twelfth; at 7%, find the interest at 6%, and add one-sixth.

NOTE 3.—When it is seen that a factor is common to both multiplier and divisor, it will materially abridge the solution to cancel such factor (Prin. 2, **74**). Thus, to find the interest for 48 days at 6%, multiply by ⅙ of 48, or 8, and divide the product by ⅙ of 60, or 10; to find the interest for 54 days at 5%, multiply by ⅑ of 54, or 6, and divide the product by ⅑ of 72, or 8; etc., etc.

NOTE 4.—To find the multiplier when the time is expressed in years, months, and days, regard each year as equivalent to 360 days, and each month to 30 days.

EXAMPLES FOR PRACTICE.

Find the interest at 6% per annum.

	Principal.	*Time.*		*Principal.*	*Time.*		*Principal.*	*Time.*
1.	$348	38 da.	**5.**	$182.10	45 da.	**9.**	$128.35	1 mo. 23 da.
2.	$846	219 "	**6.**	$8531.40	31 "	**10.**	$135.75	2 " 17 "
3.	$250.20	55 "	**7.**	$5187.24	141 "	**11.**	$357.19	3 " 5 "
4.	$918.30	62 "	**8.**	$7152	59 "	**12.**	$718.25	10 " 12 "

What is the interest of

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.*
13.	$672.85	9%	71 da.	**23.**	$198.75	7%	341 da.
14.	$289.40	4%	145 "	**24.**	$2138.40	3½%	192 "
15.	$314.62	6%	67 "	**25.**	$1873.65	5½%	86 "
16.	$718.55	12%	193 "	**26.**	$428.50	8%	5 mo. 18 da.
17.	$641.83	5%	215 "	**27.**	$5130.75	9%	3 " 27 "
18.	$925.10	10%	75 "	**28.**	$698.10	6%	6 " 13 "
19.	$482.70	6%	36 "	**29.**	$427.40	5%	8 " 25 "
20.	$658.25	4½%	117 "	**30.**	$583.25	4%	4 " 9 "
21.	$2341.80	3%	59 "	**31.**	$692.18	6%	1 yr. 9 mo. 20 da.
22.	$5987.35	8%	286 "	**32.**	495.15	4½%	1 " 7 " 14 "

242

243. To find the interest at 6% by the 6-day rule.

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $650 for 24 days at 6% per annum?

SOLUTION.

$$\begin{array}{r|l} \$ & 650,\ \text{int. for}\ \ 6\ \text{da.} \\ \hline \$2 & 600,\ \text{``}\ \ \text{``}\ \ 24\ \text{``} \end{array}$$

EXPLANATION.—If .01 of the principal is the interest for 60 da. at 6% (**242**), 1/10 of .01, or .001 of the principal will be the interest for 1/10 of 60 da. or 6 da. Hence, using a perpendicular separatrix, cut off three figures from the right of the principal, obtaining $.65 as the interest for 6 da.; and 4 times this result, or $2.60, will be the interest for 4 times 6 da. or 24 da.

2. What is the interest of $1812.75 for 51 days at 6% per annum?

SOLUTION.

$$\begin{array}{r|lcl} \$1 & 812.75 & = & \text{int. for}\ \ 6\ \text{da.} \\ \hline 14 & 502 & = & \text{``}\ \ \text{``}\ \ 48\ \text{``} \\ & 906 & = & \text{``}\ \ \text{``}\ \ 3\ \text{``} \\ \hline \$15 & 41-, & = & \text{``}\ \ \text{``}\ \ 51\ \text{``} \end{array}$$

EXPLANATION.—Using a perpendicular separatrix, cut off three figures, obtaining $1.81275 as the interest for 6 da.; 8 times this result, or $14.502, will be the interest for 48 da.; one-half the interest for 6 da., or .906, will be the interest for 3 da.; and the sum of the interest for 48 da. and the interest for 3 da., or $15.41 — will be the interest for 51 da.

3. What is the interest of $416.30 for 47 days at 6% per annum?

SOLUTION.

$$\begin{array}{r|lcl} \$ & 416.30 & = & \text{int. for}\ \ 6\ \text{da.} \\ \hline 3 & 330+ & = & \text{``}\ \ \text{``}\ \ 48\ \text{``} \\ & 069+ & = & \text{``}\ \ \text{``}\ \ 1\ \text{``} \\ \hline \$3 & 26+ & = & \text{``}\ \ \text{``}\ \ 47\ \text{``} \end{array}$$

EXPLANATION.—If $.4163 is the interest for 6 da., 8 times $.4163, or $3.330+ will be the interest for 8 times 6 da., or 48 da. (1 day more than the given time). Subtract the interest for 1 da. (1/6 of $.4163 = $.069) from the interest for 48 da. ($3.330+), obtaining $3.26+ as the interest for 48—1, or 47 da.

RULE.—I. *Draw a perpendicular line three places to the left of the decimal point of the principal. The result will be the interest of the principal for 6 days, the figures to the left of the perpendicular line being dollars.*

II.—*Take as many times the interest for 6 days as the given days are times 6 days, or such a part of the interest for 6 days as the given days are a part of 6 days.*

NOTE 1.—Consider the interest for 1 day as 1/6 of the interest for 6 days; for 2 days as 1/3 of the interest for 6 days; for 3 days as 1/2 of the interest for 6 days; for 4 days, find the interest for 2 days more than the given time and deduct 1/3 of the interest for 6 days from the result; for 5 days, find the interest for 1 day more than the given time and deduct 1/6 of the interest for 6 days from the result.

243

EXAMPLES FOR PRACTICE.

Find the interest by the above rule, and prove results by **242.**

	Principal.	*Time.*
1.	$862.65	38 da.
2.	$258	27 "
3.	$532.50	19 "
4.	$937.58	40 "
5.	$3275.25	32 "
6.	$1250.75	52 da.
7.	$47.85	29 "
8.	$285.30	1 mo. 14 da.
9.	$563.27	1 " 23 "
10.	$827.50	1 " 16 "

244. To find the interest at 6% by the "60-day rule."

ILLUSTRATIVE EXAMPLES.

1. What is the interest of $534 for 80 days at 6%?

SOLUTION.

$5	34, int. for 60 da.
1	78, " " 20 "
$7	12, " " 80 "

EXPLANATION.—Using a perpendicular separatrix, cut off *two* figures from the right of the principal, obtaining $5.34 as .01 of the principal, or the interest for 60 days at 6% per annum (**242**), and $\frac{1}{3}$ of the interest for 60 da. will be the interest for 20 da. Add the two results to find the interest for 60+20, or 80 da.

2. What is the interest of $691.25 for 327 days at 6%?

SOLUTION.

$6	91.25, int. for 60 da.
34	562+, " " 300 "
2	765, " " 24 "
	346—, " " 3 "
$37	67+, " " 327 "

EXPLANATION.—If $6.9125 is the interest for 60 da., 5 times that sum, or $34.562+, must be the interest for 5 times 60 da., or 300 da.; .1 of $6.9125 must be the interest for $\frac{1}{10}$ of 60 da., or 6 da., and .4 times $6.9125, or $2.765, the interest for .4 times 60 da., or 24 da.; and $\frac{1}{20}$ of $6.9125, or $.346—, the interest for $\frac{1}{20}$ of 60 da., or 3 da. Add the three results, obtaining $37.67+ as the interest for 300+24+3, or 327 da.

3. What is the interest of $837.40 for 1 yr. 7 mo. 23 da. at 6%?

SOLUTION.

$8	37.40, int. for 2 mo.
74	366, " " 18 mo.
6	699+, " " 48 da.
	698—, " " 5 "
$82	76+, " 1 yr. 7 mo. 23 da.

EXPLANATION.—Draw a separatrix as before, obtaining $8.374 as the interest for 60 da., or 2 mo., and 9 times this sum will be the interest for 18 mo., leaving the interest for 1 mo. 23 da., or 53 da., still to find. .8 times $8.374, or $6.699+ is the interest for .8 times 60 da., or 48 da.; and $\frac{1}{12}$ of $8.374, or $.698—, the interest for $\frac{1}{12}$ of 60 da., or 5 da. Add the three results.

RULE I.—*Draw a perpendicular separatrix two places to the left of the decimal point in the principal. The result will be the interest for 60 days or 2 months.*

II. *Take as many times the above result as the given days are times 60 days, or as the given months are times 2 months; and such a part of the above result as the remaining time, expressed in days, is a part of 60 days.*

NOTE 1.—The several multipliers and aliquot parts are best obtained as follows: Divide the given time in days by 60, or in months by 2, to find the integral multiplier; divide any possible remainder from the first division, expressed in days, by 6, to find the tenths' multiplier; and *any* remainder from the second division will be an aliquot part of 60 days; making three statements of partial interest the maximum in any problem.

NOTE 2.—If after finding the integral multiplier by Note 1, the remainder should be an aliquot part of 60, omit the tenths' multiplier, and at once take such aliquot part of the interest for 60 days. The following are aliquot parts of 60: 30 = $\frac{1}{2}$; 20 = $\frac{1}{3}$; 15 = $\frac{1}{4}$; 12 = $\frac{1}{5}$; 10 = $\frac{1}{6}$; 6 = $\frac{1}{10}$; 5 = $\frac{1}{12}$; 4 = $\frac{1}{15}$; 3 = $\frac{1}{20}$; 2 = $\frac{1}{30}$; 1 = $\frac{1}{60}$.

NOTE 3.—If after finding the integral multiplier by Note 1, the remainder should *lack* an aliquot part of being 60, increase the integral multiplier by 1, and deduct from the resulting product such aliquot part of the interest for 60 days.

NOTE 4.—If the given time contains years, reduce the years to months and add thereto the given months. If, in dividing the months by 2 to find the integral multiplier, there should be a remainder of 1 month, regard it as 30 days, add thereto the given days, and divide the result by 6, to find the tenths' multiplier.

NOTE 5.—To multiply by any number of tenths, proceed as if multiplying by an integer, and place the figures of the product *one place to the right* of the figures multiplied.

EXAMPLES FOR PRACTICE.

Using the above Rule, and proving results by **242**, find the interest.

	Principal.	*Time.*		*Principal.*	*Time.*		*Principal.*	*Time.*
1.	$750	137 da.	**7.**	$7136.20	250 da.	**13.**	$4500	192 da.
2.	$175.50	173 "	**8.**	$7500.50	117 "	**14.**	$825.30	144 "
3.	$5282	121 "	**9.**	$596.20	136 "	**15.**	$612.20	156 "
4.	$723	161 "	**10.**	$317.40	112 "	**16.**	$718.25	312 "
5.	$1835.80	174 "	**11.**	$4181.25	85 "	**17.**	$875.75	189 "
6.	$812.80	311 "	**12.**	$950	76 "	**18.**	$1532.38	320 "

244

	Principal.	Time.		Principal.	Time.
19.	$7500......	3 mo. 15 da.	24.	$325.45...	1 yr. 6 mo. 18 da.
20.	$8125.42....	9 " 18 "	25.	$896.30...	2 " 3 " 6 "
21.	$918.30.....	11 " 25 "	26.	$7125.30..	1 " 7 " 15 "
22.	$4126.50....	8 " 7 "	27.	$85.15....	1 " 2 " 19 "
23.	$816.10.....	5 " 3 "	28.	$815.20...	2 " 3 " 11 "

Finding the exact time in days (**181**), what is the interest at 6%

29. Of $875.10 from June 1, 1889, to Aug. 3, 1889?
30. Of $74.50 from Jan. 3, 1888, to May 4, 1888?
31. Of $512.75 from Dec. 15, 1888, to Jan. 18, 1889?
32. Of $5826.45 from Aug. 15, 1888, to June 3, 1889?
33. Of $925.38 from May 13, 1889, to Dec. 24, 1889?
34. Of $1917.25 from Jan. 25, 1888, to Aug. 8, 1888?
35. Of $5863.50 from Feb. 28, 1887, to Oct. 18, 1887?
36. Of $928.80 from June 2, 1888, to Aug. 19, 1888?
37. Of $759.40 from May 5, 1889, to Sept. 19, 1889?
38. Of $2359.65 from Jan. 12, 1887, to May 15, 1887?
39. Of $8211.75 from Mar. 7, 1889, to Aug. 31, 1889?
40. Of $506.15 from Dec. 15, 1888, to Mar. 8, 1889?
41. Of $1275.25 from Feb. 15, 1887, to Sept. 8, 1887?
42. Of $5918.70 from Oct. 21, 1888, to Feb. 23, 1889?
43. Of $2612.50 from Jan. 12, 1889, to Aug. 7, 1889?
44. Of $4105.09 from May 30, 1887, to Nov. 18, 1887?
45. Of $917.25 from Apr. 20, 1888, to Dec. 12, 1888?
46. Of $250.70 from Jan. 25, 1889, to Nov. 15, 1889?
47. Of $1215.30 from Mar. 7, 1889, to Aug. 3, 1889?
48. Of $720.19 from Nov. 15, 1886, to Feb. 4, 1887?

Finding the time by comp. subtraction (**180**), what is the interest

49. Of $478.32 from May 3, 1886, to July 6, 1888?
50. Of $1375.70 from Oct. 8, 1883, to Jan. 30, 1889?
51. Of $51.30 from Feb. 13, 1885, to July 9, 1887?
52. Of $1286.58 from Sept. 25, 1886, to Mar. 3, 1889?
53. Of $5.36 from Nov. 10, 1882, to May 8, 1889?
54. Of $138.50 from Jan. 18, 1888, to Jan. 15, 1889?
55. Of $8270.75 from Mar. 12, 1884, to Feb. 28, 1888?
56. Of $415.60 from Aug. 8, 1886, to May 19, 1888?
57. Of $12.80 from July 30, 1884, to Aug. 31, 1887?
58. Of $280.70 from May 1, 1883, to Apr. 12, 1889?

245. To find the interest at any % by the "60-day rule."

ILLUSTRATIVE EXAMPLE.

What is the interest of $673.70 for 96 days at 8%?

SOLUTION.

$6	73.70,	int. for 60 da. at 6%.
4	042,	" " 36 " " "
$10	779,	" " 96 " " 6%.
3	593,	" " 96 " " 2%.
$14	37+,	" " 96 " " 8%.

EXPLANATION.—First find the interest at 6% for 96 da. by the 60-day rule, obtaining $10.779.

Since 8% is ⅓ more than 6%, the required interest at 8% must be ⅓ more than the obtained interest at 6%. Hence, increase the interest at 6% ($10.779) by ⅓ of itself ($3.593), obtaining $14.37+ as the required interest at 8%.

RULE.—*Find the interest at 6% for the given time, and increase or diminish the result by such a part of itself as the given rate per annum is greater or less than 6%.*

NOTE.—The interest at 6% being found, to obtain the interest at 1%, divide by 6; at 1½%, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract ⅓ of the interest from itself; at 4½%, subtract ¼ of the interest from itself; at 5%, subtract ⅙ of the interest from itself; at 5½%, subtract 1/12 of the interest from itself; at 6½%, add 1/12 of the interest to itself; at 7%, add ⅙ of the interest to itself; at 7½%, add ¼ of the interest to itself; at 8%, add ⅓ of the interest to itself; at 9%, add ½ of the interest to itself; at 10%, add ⅔ of the interest to itself; at 12%, multiply by 2; *and at any per cent., multiply the interest at 6% by the given rate per annum, and divide the product by 6.*

EXAMPLES FOR PRACTICE.

Find the interest by the above rule, and prove results by **242.**

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	mo.	da.
1.	$105	8%	157 da.	10.	$895.80	7%		3	5
2.	$450	4%	79 "	11.	$375.20	10%		2	15
3.	$575.30	3%	85 "	12.	$4105.30	8%		10	24
4.	$350.90	5%	48 "	13.	$592.25	5%		5	17
5.	$7500	7%	139 "	14.	$318.60	3%		9	8
6.	$519.75	4½%	217 "	15.	$295.45	12%		4	25
7.	$836.28	9%	175 "	16.	$182.90	4½%		7	13
8.	$417.40	3½%	183 "	17.	$9500	9%	1	9	12
9.	$325.80	10%	341 "	18.	$182.40	4%	1	3	18

245

Finding the exact time in days, what is the interest

19. Of $375.45 from May 9, 1889, to July 7, 1889, at 8%?
20. Of $1240.35 from Sept. 15, 1888, to Feb. 5, 1889, at 5%?
21. Of $526.62 from June 8, 1888, to Dec. 30, 1888, at 7%?
22. Of $857.25 from Feb. 12, 1888, to Nov. 9, 1888, at 4%?
23. Of $7285.15 from July 4, 1886, to Jan. 3, 1887, at 4½%?
24. Of $612.75 from Jan. 1, 1889, to Mar. 5, 1889, at 9%?
25. Of $2196.30 from Mar. 23, 1887, to Sept. 3, 1887, at 5%?
26. Of $826.65 from Oct. 19, 1888, to Apr. 23, 1889, at 10%?
27. Of $1475.50 from Jan. 25, 1888, to July 20, 1888, at 4½%?
28. Of $3825.75 from Apr. 12, 1887, to Feb. 13, 1888, at 4%?

Finding the time by compound subtraction, what is the interest

29. Of $2536.75 from May 19, 1883, to July 15, 1888, at 3%?
30. Of $746.20 from Apr. 5, 1886, to Jan. 9, 1890, at 7%?
31. Of $428.50 from Oct. 25, 1882, to May 21, 1889, at 12%?
32. Of $815.30 from July 15, 1889, to Oct. 19, 1889, at 7%?
33. Of $9183.70 from May 13, 1880, to Aug. 25, 1889, at 5%?

246. Direct method of finding the interest at any rate per annum.

ILLUSTRATIVE EXAMPLE.

What is the interest of $416.20 for 108 days at 8%?

$4	16.20,	int.	for	45	da.
8	324,	"	"	90	"
1	387+,	"	"	15	"
	277+,	"	"	3	"
$9	99—,	"	"	108	"

Explanation.—.01 of $416.20, or $4.162, is the interest for ⅛ of 360 da., or 45 da. at 8% per annum (**242**). Hence, twice $4.162, or $8.324, is the interest for twice 45 da., or 90 da.; one-third of $4.162, or $1.387+, the interest for one-third of 45 da., or 15 da.; and one-fifth of $1.387, or $.277+, the interest for one-fifth of 15 da., or 3 da. Add the three results.

Rule.—I. *Draw a perpendicular line two places to the left of the decimal point of the principal, and the result will be the interest of the principal for as many days as the given rate is contained times in 360.*

II. *Take as many times, or such a part of, the above result as the given days are times or part of the days required to produce such result.*

NOTE 1.—Cutting off two integral places from the right of the principal, that is, taking .01 of the principal, will give the interest

At 1% per an. for	360 da.	At 4½% per an. for	80 da.	At 9% per an. for	40 da.
" 2% " " "	180 "	" 5% " " "	72 "	" 10% " " "	36 "
" 3% " " "	120 "	" 6% " " "	60 "	" 12% " " "	30 "
" 4% " " "	90 "	" 8% " " "	45 "		

NOTE 2.—To find the integral multiplier, divide the given number of days by the days required to produce .01 of the principal, as given in Note 1; next separate the days remaining from this division into aliquot parts of the days required to produce .01 of the principal.

NOTE 3.—If the time be expressed in months and days, reduce to days by Note 4, **242**, before applying the Rule.

EXAMPLES FOR PRACTICE.

Find the interest by the above rule, and prove results by **242.**

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.*
1.	$126	5%	39 da.	6.	$2758.25	4½%	200 da.
2.	$720	10%	51 "	7.	$317.95	4%	165 "
3.	$1800.50	8%	140 "	8.	$418.30	5%	96 "
4.	$475.08	9%	65 "	9.	$3275.30	8%	4 mo. 8 da.
5.	$1950.15	3%	118 "	10.	$475.65	5%	8 mo. 12 da.

TO TEACHERS.—The examples of **245** will afford additional practice by the "direct method," if deemed necessary.

247. Interchangeability of principal and time.

Whatever may be the rate per annum, it is evident from Rule, **242**, that in finding the interest of $75 for 40 days, the factors and divisor will be the same as in finding the interest of $40 for 75 days. The principal and time expressed in days are therefore interchangeable; that is, the principal can be regarded as days, and the days as dollars of principal. Much labor can often be saved by this interchange.

REM. 1.—If, at 6% per annum, to cut off *three* integral places from the right of the principal will be the interest for 6 days (**243**); to cut off two integral places, the interest for 60 days (**244**); then, by analogy, to cut off *one* integral place must be the interest for 600 days; and to cut off *no* integral place, that is, the principal itself, must be the interest for 6000 days.

ILLUSTRATIVE EXAMPLES.

1. At 6% per annum, what is the interest of $1500 for 219 days? 2. Of $400 for 317 days? 3. Of $200 for 51 days?

```
          (1)                        (2)                       (3)
$219 |     = int. 6000 da.   $31 | 7     = int. 600 da.   5 | 1=int. 600 da.
-----|-----                   10 | 567—, int. 200 da.    ---|---------------
  54 | 75, int. 1500 da.    -----|-------------------     1 | 70, int. 200 da.
                             $21 |133,    int. 400 da.
```

EXAMPLES FOR PRACTICE.

Find the interest by the above method:

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.*
1.	$5000	6%	287 da.	7.	$105	8%	157 da.
2.	$1000	6%	129 "	8.	$450	4%	213 "
3.	$150	6%	173 "	9.	$900	5%	260 "
4.	$800	6%	79 "	10.	$4000	7%	161 "
5.	$2000	6%	41 "	11.	$800	4½%	291 "
6.	$7000	6%	5 mo. 17 "	12.	$720	5%	175 "

248. To find the amount.

ILLUSTRATIVE EXAMPLE.

What is the amount of $416.35, if placed at interest for 78 days at 6% per annum?

SOLUTION.

```
 $4|16.35 = int. for 60 da.
  1|249   =  "    "  18 "
416|35    = principal.
---|------
$421|76+  = amount.
```

RULE.—*Find the interest by the most convenient of the preceding rules; and add the principal thereto.*

EXAMPLES FOR PRACTICE.

Find the amount.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	mo.	da.
1.	$890.40	5%	29 da.	5.	$515.50	9%		1	19
2.	$315.30	4%	72 "	6.	$734.40	8%		7	28
3.	$3750.30	7%	33 "	7.	$600	5%		8	15
4.	$840	6%	168 "	8.	$815.15	4%	1	5	8

249. To find the accurate interest (237).

ILLUSTRATIVE EXAMPLE.

What is the accurate interest of $325 for 24 days at 6%?

SOLUTION.

$$\begin{array}{r l} \$325 & \\ .06 & \\ \hline 19.50, & \text{int. for 1 yr.} \\ \times 24 & \\ \hline 365)\$468.00(\$1.28+ & \end{array}$$

EXPLANATION.—First find the interest for 1 yr., obtaining $19.50. Since the accurate number of days in a common year is 365, the accurate interest for 24 days must be $\frac{24}{365}$ of $19.50, or $1.28+.

RULE.—*Multiply the principal by the rate per annum to find the interest for one year.*

Multiply the interest for one year by the number of days and divide the product by 365.

NOTE.—If the time is greater than one year, multiply the interest for one year by the number of years, and add the product to the interest for the odd days in the fractional part of year as obtained by the above rule.

250. 365 days, the basis for computing accurate interest, is $\frac{73}{72}$ of 360 days, the basis for computing ordinary interest. Hence, inversely, ordinary interest is $\frac{73}{72}$ of accurate interest, that is, *ordinary interest is $\frac{1}{73}$ more than accurate interest.* The most convenient method of computing accurate interest is to calculate the ordinary interest first, and then deduct $\frac{1}{73}$ of itself, as follows:

Ex. What is the accurate interest of $715.35 for 219 days at 6%?

SOLUTION.

$7	15.35, ord. int. for 60 da.
21	460+, " " " 180 "
4	292+, " " " 36 "
	357+, " " " 3 "
$26	109, " " " 219 "
	261 = $\frac{1}{100}$ of ord. interest.
	65 = 261 ÷ 4.
	26 = $\frac{1}{1000}$ of ord. interest.
	5 = 26 ÷ 5.
$25	752, acc. int. for 219 da.

NOTE.—To diminish ordinary interest by $\frac{1}{73}$ of itself: Under the ordinary interest write $\frac{1}{100}$ of itself and divide by 4; under this quotient write $\frac{1}{1000}$ of the ordinary interest and divide by 5. That is, *move the ordinary interest two places to the right and divide by 4, then move it one more place to the right and divide by 5.* Subtract the sum of these four results from the ordinary interest.

EXAMPLES FOR PRACTICE.

Find the accurate interest.

	Principal.	*Rate.*	*Time.*		*Principal.*	*Rate.*	*Time.* yr.	da.
1.	$187.30	5%	15 da.	7.	$8135.25	3%		93
2.	$1837.75	4%	115 "	8.	$782.50	5%		113
3.	$853.25	6%	18 "	9.	$4150.35	7%		25
4.	$75.50	8%	93 "	10.	$913.25	3½%	1	320
5.	$375.40	3½%	146 "	11.	$2150.75	7%	5	271
6.	$982.75	4½%	75 "	12.	$538.50	6%	2	118

What is the accurate interest

13. Of $758.20 from Mar. 13, 1889, to Aug. 28, 1889, at 6%?
14. Of $1290.75 from Jan. 10, 1887, to Oct. 7, 1887, at 7%?
15. Of $75.30 from May 13, 1884, to Jan. 15, 1889, at 3½%?
16. Of $945.38 from Aug. 5, 1885, to May 12, 1889, at 8%?
17. Of $3200 from Oct. 12, 1886, to May 31, 1889, at 4%?
18. Of $625.80 from Nov. 28, 1885, to Sept. 16, 1887, at 5%?

INTEREST PROBLEMS.

251. To find the rate per annum, when the principal, interest, and time are given.

ILLUSTRATIVE EXAMPLE.

At what rate will $342 produce $4.56 interest in 96 days?

SOLUTION.

$3	42,	int. for 60 da. @ 6%.
2	052,	" " 36 " " 6%.
÷6) 5	472,	" " 96 " " 6%.
	912,	" " 96 " " 1%.

$4.560÷$.912 = 5 times 1%, or 5%.

EXPLANATION.—The interest of $342 for 96 da. at 1% is $.912.

Since the given interest ($4.56) is 5 times the interest at 1%, the rate of the given interest must be 5 times 1%, or 5%.

RULE.—*Find the interest of the given principal for the given time at 1% per annum, and divide the result into the given interest.*

NOTE.—If the amount is given, diminish it by the principal to find the interest.

EXAMPLES FOR PRACTICE.

At what rate per annum will

1. $648 yield $7.29 interest in 81 days?
2. $3500 produce $105 interest in 4 mo. 15 da.?
3. $8730 amount to $8974.44 in 8 mo. 12 da.?
4. $296 in 45 days produce $3.70 interest?
5. $500 in 9 mo. 6 da. yield $23 interest?
6. $3800 amount to $3874.10 in 78 days?
7. $810 yield $17.64 interest from May 3, '89, to Nov. 15, '89?
8. $750 amount to $776.50 from Mar. 1, '88, to Aug. 7, '88?
9. $380 amount to $396.91 from Apr. 7, '87, to Oct. 2, '87?

252. To find the time, when the principal, interest, and rate are given.

ILLUSTRATIVE EXAMPLE.

At 4% per annum, in what time will $315 produce $20.79 interest?

SOLUTION.

$|315, int. for 9 da. @ 4%.
|035, " " 1 " " 4%.
20.790 ÷ .035 = 594 da.
1 ord. int. year = 360 "
234 "
7 ord. int. months = 210 "
24 "

Ans. 1 yr. 7 mo. 24 da.

EXPLANATION.—The ordinary interest of $315 for 1 day at 4% is $.035.

Since the given interest ($20.79) is 594 times the interest for 1 day, the time of the given interest must be 594 times one day, or 594 days, which equal 1 ordinary interest year (360 da.), 7 ordinary interest months (210 da.), and 24 days.

RULE.—*Find the interest of the given principal at the given rate for 1 day, and divide the result into the given interest to find the number of days.*

NOTE 1.—To obtain a result in years, months, and days, count each 360 days of the answer as a year; each 30 days of the remainder as a month, and the final remainder as days.

NOTE 2.—To avoid fractions, instead of following the above rule multiply the given interest by the quotient of the rate per annum into 360, and divide the result by $\frac{1}{100}$ of the principal. Thus, in the Ill. Ex. (20.79 × 90)+3.15 = 594 da.

EXAMPLES FOR PRACTICE.

In what time will

1. \$395 produce \$82.16 interest, if loaned at 6% per annum?
2. \$1940 amount to \$2273.68, if loaned at 8% per annum?
3. \$936 yield \$69.68 interest, if loaned at 8% per annum?
4. \$765 amount to \$765.51, if loaned at 4% per annum?
5. \$856 produce \$2.14 interest, if loaned at 5% per annum?
6. \$7200 amount to \$7239.20, if loaned at 7% per annum?
7. Any principal double itself at 3%? At 4%? At 5%? At 6%?
8. Any principal treble itself at 8%? Quadruple itself at 7%?
9. On Aug. 23, 1888, I loaned \$530 at 6% per annum, and when the debt became due I received \$536.36 in full of principal and interest for exact time in days. At what date was the debt paid?
10. A note of \$850 amounted to \$855.95 on Oct. 13, 1889. At what date was the note drawn, if it drew interest at 4% per annum, and the time was computed by compound subtraction?

253. To find the principal, when the interest, time, and rate are given.

ILLUSTRATIVE EXAMPLE.

What sum loaned from Apr. 20, 1889, to Dec. 19, 1889, at 8% per annum, will produce \$8.10 interest?

SOLUTION.

Apr. 20 to Dec. 19 = 243 da.
\$2.43 ÷ 45 = \$.054, int. of \$1.
8.100 ÷ .054 = \$150, *Ans.*

EXPLANATION.—Dividing the days regarded as hundredths (2.43) by the quotient of the rate per annum into 360 (360 ÷ 8 = 45) produces .054 as the rate of interest for 243 days at 8% per annum (Rem. 1, **241**); or, regarded concretely, \$.054 is the interest of \$1 for 243 days at 8% per annum. Hence, since the given interest (\$8.10) is 150 times the interest of \$1 (\$.054) the principal of the given interest must be 150 times \$1.

RULE.—*Divide the given interest by the interest of \$1 for the given time and at the given rate.*

NOTE 1.—To find the interest of \$1, divide the days regarded as cents by the quotient of the rate per annum into 360. If the time be given in years, months, and days, reduce to days by Note 4, **242.**

NOTE 2.—To avoid fractions, instead of applying the above rule, multiply the given interest by the quotient of the rate per annum into 360, and divide the result by the time in days regarded as cents. Thus, in the Ill. Ex., (\$8.10 × 45) ÷ 2.43 = \$150.

EXAMPLES FOR PRACTICE.

What principal will yield

1. $21.85 interest in 69 days at 5%?
2. $8.82 interest in 84 days at 6%?
3. $22.50 interest in 135 days at 4%?
4. $30.94 interest in 6 mo. 28 da. at 9%?
5. $40.60 interest in 1 yr. 5 mo. 12 da. at 8%?
6. $67.20 interest in 2 yr. 9 mo. 18 da. at 4%?
7. $19.60 interest in 1 yr. 2 mo. 21 da. at 5%?
8. What sum loaned from Sept. 18, 1887, to Mar. 4, 1888, at 6% per annum, will produce $24.22 interest (find exact time in days)?
9. A lender received $182 interest on a sum loaned at 8% on May 30, 1888, and paid Jan. 19, 1889. What was the sum loaned (find the time by compound subtraction)?
10. What sum will produce $514.40 interest in 2 years, 10 months, and 20 days, at 4½% per annum?

254. To find the principal, when the amount, time, and rate are given.

ILLUSTRATIVE EXAMPLE.

What sum of money loaned from Jan. 8, 1889, to May 12, 1889, at 9% per annum, will amount to $3134.24?

SOLUTION.

Jan. 8 to May 12 = 124 days.
$1.24 ÷ 40 = $.031, int. of $1.
$1 + $.031 = $1.031, amt. of $1.
3134.240 ÷ 1.031 = $3040.

EXPLANATION.—Find the interest of $1 at 9% for 124 da. by Note 1, **253**, and annex to $1, obtaining $1.031 as the amount of $1.

Since the given amount ($3134.24) is 3040 times the amount of $1 ($1.031), the principal to produce the required amount must be 3040 times $1, or $3040.

RULE.—*Divide the given amount by the amount of $1 for the given time and at the given rate.*

NOTE 1.—To obtain the amount of $1, find the interest of $1 by Note 1, **253**; and annex the result to $1.

NOTE 2.—To avoid fractions, multiply the amount by the quotient of the rate per annum into 360; and divide the result by the same quotient increased by the days regarded as hundredths. Thus, in the Ill. Ex., ($3134.24 × 40) ÷ (40 + 1.24) = $3040.

EXAMPLES FOR PRACTICE.

What principal will amount to

1. $176.13 in 135 days, if loaned at 8%?
2. $419.80 in 297 days, if loaned at 6%?
3. $677.10 in 120 days, if loaned at 5%?
4. $7420.74 in 3 mo. 19 da., if loaned at 4%?
5. $9051.84 in 1 mo. 11 da., if loaned at 9%?
6. $1302.75 in 1 yr. 10 mo. 25 da., if loaned at 4½%?

7. What principal loaned Sept. 5, 1888, at 6% per annum will amount to $4513.60 on Mar. 2, 1889 (find exact time in days)?

8. On Mar. 18, 1889, I borrowed a sum of money at 5% per annum, and on Aug. 3, 1889, I paid $488.80 in full of principal and interest. What was the sum borrowed (find time by compound subtraction)?

REVIEW OF INTEREST.

255. GENERAL DIRECTIONS.—1. *If interest is required, multiply the principal by the rate of interest for the given time.*

To find the rate of interest for the given time, refer to Rem. 1, **241.** The various processes in which a perpendicular separatrix is used, are the most convenient for executing the above direction.

2. *If interest is given, divide it by the interest of 1 unit of the required term.*

1. A note of $582.75, at 5%, dated May 2, 1888, was paid Feb. 14, 1889. What was the amount of the payment (exact time in days)?

2. A partner was allowed 6% per annum on all sums invested with his firm, and charged 6% per annum on all sums withdrawn. At the beginning of the year he had $8000 invested; on May 13, he invested $1500 additional; and on Aug. 9, he withdrew $800. What was the interest balance to his credit at the close of the year (find the exact time in days)?

3. A note of $850 amounted to $855.10 on July 9, 1888. When was the note drawn, if interest was computed for the exact time in days at 4% per annum?

4. On June 17, 1889, I loaned $812 at 5% per annum, and when

the debt became due I received $818.09 in full of principal and ordinary interest computed for exact days. At what time was the debt paid?

5. On Mar. 3, 1889, a merchant bought goods for $780.16 on 60 days' credit, but did not pay for them until June 15, 1889. What was the amount then due, computing interest at 8% for the exact time in days?

6. What sum loaned at 9% per annum on April 13, 1888, will amount to $977.16 on June 11, 1888 (time obtained by compound subtraction)?

7. A note of $525.30, bearing accurate interest at 5%, was given Jan. 19, 1885. What sum will discharge the note on July 15, 1887, no previous payments having been made?

8. A house which cost $5400 rents for $30 per month. What is the net rate per annum of interest received on the investment, if the average annual expenses are $144?

9. What is the difference between the accurate interest and the ordinary interest of $8500 from May 12, 1889, to Oct. 19, 1889, at 8% per annum, computing the exact time in days in both cases?

10. A house which cost $8400 is rented for $700 per annum. What is the net annual rate % of interest on the investment, the average annual expenses being $125 for taxes, $21 for insurance, $75 for ground rent, and $59 for repairs?

11. On Aug. 15, 1885, I borrowed $7350, and on Mar. 7, 1887, I paid $7810.60 in full of principal and ordinary interest, computing the time for the fraction of a year in exact days. At what rate per annum was the sum loaned?

12. How much must I pay for the use of $8350, borrowed at 5% per annum on Jan. 18, 1887, and returned Dec. 12, 1887 (find the time by compound subtraction)?

13. An attorney collected a claim of $450 with ordinary interest thereon for the exact time from Aug. 29, 1885, to Dec. 14, 1885, at 7%. What were the net proceeds due the creditor, if the attorney's rate of commission was 10%?

14. I bought a house and lot on speculation for $15325; and 5 months and 11 days from the date of purchase I sold the property for $16200. If money was worth 8% per annum, how much more did the transaction yield than by lending the purchase money at ordinary interest?

15. If a man borrow $3450 in Maryland at the legal rate and loan it in California at the legal rate, what will be his gain in 11 mo. 9 da.?

16. What annual rate of interest on the net sales is equivalent to a discount of 5% on the gross sales, if the term of credit is 60 days?

17. What trade discount will equal 8% per annum interest on the net sales, if the term of credit is 90 days?

COMPOUND INTEREST.

256. Compound Interest is interest which for the first interest period is computed upon the principal alone; and for subsequent periods, upon the principal increased by its accrued interest.

Rem.—Interest may be compounded annually, semi-annually, quarterly, or at still shorter intervals, if previously agreed to by the parties interested. In most States, the collection of compound interest at the maximum legal rate cannot be enforced; but if the debtor consents to pay it, to receive compound interest at the maximum legal rate does not constitute technical usury.

ILLUSTRATIVE EXAMPLE.

What is the interest of $3285.23 for 3 years, if compounded annually at 7% per annum?

SOLUTION.

$3285.23,	original principal.
229.966+,	1 yr's int. at 7%.
$3515.196,	principal 2d yr.
246.064−,	1 yr's int. at 7%.
$3761.259,	principal 3d yr.
263.288+,	1 yr's int. at 7 %.
$4024.547,	amt. at close of 3d yr.
3285.23,	original principal.
$739.32−,	comp. int. for 3 yrs.

Explanation.—It is customary in business to carry results no further than *three* decimal places.

Hence, commencing with the tenths' order of each principal, multiply it by the rate % (hundredths), and write the first figure of each product in the third decimal place (thousandths), as in the solution.

To secure accuracy, multiply the rejected part of each principal by the rate % to find how much to carry to the retained product.

Rule.—I. *Find the interest of the principal for the first interest period, add it to the principal, and the sum will be the second principal.*

II. *Find the interest of the second principal for the second period of time, add it to the second principal, and the sum will be the third principal.*

III. *Proceed in this manner as many different times as there are periods in the given time. The last result will be the amount due.*

IV. *Subtract the original principal from the amount due, and the remainder will be the compound interest.*

NOTE.—If the given time contains a fraction of an interest period, find by the rule the amount of the principal to the end of the last *complete* period; then compute the interest of this amount for the fraction of a period, and add it thereto to find the compound amount for the full time.

EXAMPLES FOR PRACTICE.

Find the compound interest.

	Principal.	*Rate per annum.*	*Time.*	*Interest payable.*
1.	$8500	5%	3 yr.	annually.
2.	$6200	8%	2 “	semi-annually.
3.	$3500	4%	4 “	annually.
4.	$7200	6%	3 “	semi-annually.
5.	$5300	5%	2 “ 5 mo.	semi-annually.
6.	$9000	7%	2 “	quarterly.
7.	$8200	8%	1 “ 8 mo.	quarterly.
8.	$725	6%	1 “ 7 “	quarterly.
9.	$1575	10%	1 “ 5 “ 18 da.	quarterly.
10.	$1836	4%	1 “ 2 “ 12 “	quarterly.

[In the following examples, find the time by comp. subtraction.]

11. A man deposited $150 in a savings-bank, and received 5% per annum interest, payable quarterly, and compounded if not withdrawn. What amount was to his credit at the close of the second year?

12. What is the amount due Feb. 15, 1889, upon a note of $375.15, dated May 7, 1885, and drawing interest at 6% per annum, compounded semi-annually, no payment having been previously made?

13. What sum will, on May 19, 1888, discharge a note of $2750, dated Aug. 15, 1886, and drawing interest at 8% per annum, compounded quarterly, no previous payments having been made?

14. A note of $3500, dated May 5, 1884, drawing interest at 6% per annum, compounded semi-annually if not paid, had the first 5 interest payments paid when due. What was the amount due upon the note Dec. 25, 1888, if no subsequent payments were made?

257. The labor of computing compound interest may be abridged by the use of the following

COMPOUND INTEREST TABLE,

Showing the amount of $1 at compound interest for any number of years, from 1 year to 55 years inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs.
1	1.0100 000	1.0150 000	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	2
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	11
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	12
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345	38
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9693	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50
51	1.6610 781	2.1368 211	2.7454 1979	3.5230 3644	4.5154 2320	5.7803 9930	7.3909 5068	51
52	1.6776 889	2.1688 734	2.8003 2819	3.6111 1235	4.6508 8590	5.9827 1327	7.6865 8871	52
53	1.6944 658	2.2014 065	2.8563 3475	3.7013 9016	4.7904 1247	6.1921 0824	7.9940 5226	53
54	1.7114 105	2.2344 276	2.9134 6144	3.7939 2491	4.9341 2485	6.4088 3202	8.3138 1435	54
55	1.7285 246	2.2679 439	2.9717 3067	3.8887 7303	5.0821 4859	6.6331 4114	8.6463 6092	55

NOTE 1.—To find the amount to which any given principal, at compound interest, will increase at any rate per annum, and for any number of years given in the preceding tables:

Multiply the given principal by the amount of $1 of principal for the given time and rate per annum, as shown in the Table.

NOTE 2.—To find the compound interest which any given principal will produce at any rate per annum, and for any number of years given in the preceding tables:

Subtract $1 from the amount of $1 for the given time and rate per annum, as shown in the table; the result will be the compound interest of $1 of principal. Multiply the compound interest of $1 of principal by the given principal.

257

COMPOUND INTEREST TABLE,

Showing the amount of $1 at compound interest for any number of years from 1 year to 55 years inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5306	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7081	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 133	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992	42
43	6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692	43
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338	48
49	8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529	50
51	9.4391 0490	12.0407 698	19.5253 635	31.5190 168	50.6537 415	81.0496 969	129.1299 382	51
52	9.8638 6463	12.6428 083	20.6968 853	33.7253 480	54.7060 408	88.3441 696	142.0429 320	52
53	10.3077 3853	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 252	53
54	10.7715 8677	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477	54
55	11.2563 0817	14.6356 309	24.6503 216	41.3150 015	68.9138 561	114.4082 616	189.0591 425	55

NOTE 3.—The amount of $1 for more than 55 years is equal to the product of the amount of $1 for any two or more numbers of years in the Table whose sum is equal to the given time. Thus, the amount of $1 at compound interest for 98 years at 8 per cent. per annum, is equal to the product of the amount of $1, at 8 per cent., for 50 years ($46.9016125) multiplied by the amount of $1, at 8 per cent., for 48 years ($40.2105731), that is, $46.9016125×40.2105731, or $1885.9407179+.

NOTE 4.—When the interest is to be compounded semi-annually, find, from the Table, the amount or interest at *one-half* the rate per annum for twice the number of years. Thus, the compound interest of $1 for 10 years at 5 per cent. per annum, *payable semi-annually*, is the same as the compound interest of $1 for 20 years at 2½ per cent. per annum, *payable annually;* and the compound interest of any sum for 8 years at 4 per cent. per annum, *compounded quarterly*, is the same as the compound interest of the same sum for 32 years, at 1 per cent. per annum, *compounded annually.*

258. Application of the compound interest tables.

EXAMPLES FOR PRACTICE.

1. What is the interest of $8000 for 28 years, if compounded annually at 6% per annum ?

SOLUTION.

$5.1116867, compound amount of $1 of principal for 28 years at 6%.
1.   , the principal, above compounded, which is to be deducted.
$4.1116867, compound interest of $1 of principal for 28 years at 6%.
8000, given principal.
$32893.4936000, comp. int. of $8000 of principal for 28 years at 6%.

Find from the table the

2. Interest of $9600 for 17 years if compounded annually at 7%.
3. Amount of $7500 for 51 years if compounded annually at 5%.
4. Interest of $3200 for 36 yr. 3 mo. if compounded yearly at 6%.
5. Interest of $875 for 15 years if compounded half-yearly at 4%.
6. Amount of $425 for 23 years if compounded half-yearly at 8%.
7. Interest of $8200 for 9 yr. 5 mo., compounded half-yearly at 6%.
8. Interest of $9300 for 8 years if compounded quarterly at 4%.
9. Amount of $1500 for 18 years if compounded quarterly at 6%.

TRUE DISCOUNT.

259. The **Present Worth** of a debt due at a future time is its value *now*. Hence, it is a sum which, if put at interest to-day, will amount to the debt when that debt becomes legally due.

260. Discount is the sum deducted for the payment of a debt before it is legally due.

261. True Discount is interest to be deducted which has been computed upon the present worth taken as the principal.

REM.—*True Discount* is the difference between the present worth of a debt and its worth at legal maturity.

262. To find the present worth of a debt is an application of **254**, the debt corresponding to the *amount ;* its present worth, to the *principal;* the rate per annum which money is worth, to the *rate;* and the intervening time to the maturity of the debt, to the *time.*

263. To find the present worth of a debt.

ILLUSTRATIVE EXAMPLE.

I owe $757.05, which is legally due in 4 mo. 15 da. without interest. What is the present worth of this debt if the current rate of interest is 8%? What is the true discount if paid to-day?

SOLUTION.

4 mo. 15 da. = 135 da.
$1.35 ÷ 45 = $ .03, true disct. of $1 at 8%.
$1 + $.03 = $1.03, worth of $1 in 4 mo. 15 da.
$757.05 ÷ $1.03 = 735 times $1, or $735.
$757.05 — $735 = $22.05, true discount.

EXPLANATION.—$1 placed at interest to-day at 8% will amount to $1.03 in 4 mo. 15 da.

If the present worth of $1.03 of my debt is $1, the present worth of $757.05, or the whole of my debt, must be as many times $1 as $1.03 is contained times in $757.05, or $735.

APPROXIMATE SOLUTION.—Find the interest of the debt ($757.05) for 4 mo. 15 da. at 8%, obtaining $22.712—; then the interest of this result ($22.712) for 4 mo. 15 da. at 8%, obtaining $.681; then the interest of this last result ($.681+) for 4 mo. 15 da. at 8%, obtaining $.020+. The sum of the debt ($757.05) + the second interest ($.681) diminished by the sum of the first interest ($22.711) + the third interest ($.02) = the present worth ($735). When it is seen that the third interest calculation will produce an immaterial result, it need not be computed (see Approximate Solution, **276**). Labor can be saved by this method with awkward rates like 5¼%, 7%, etc.

By reckoning interest on the debt, which is the true discount more than the correct principal, the result ($22.712) will be the interest of the true discount more than the true discount (**261**). Hence, were this result substracted from the debt, the remainder would be less than the present worth. The second and third interest calculations form the necessary corrections to get the present worth.

RULE.—*Divide the debt by the amount of $1 for the intervening time at the prevailing rate of interest, to find the present worth. Subtract the present worth from the debt to find the true discount.*

NOTE 1.—To find the amount of $1, refer to Note 1, **254**. When it is seen that the amount of $1 will form an awkward divisor, either use the approximate solution of the Ill. Ex., or the solution of Note 2, **254.** Thus, in the Ill. Ex. (\$757.05 × 45) ÷ (45 + 1.35) = \$735.

NOTE 2.—To obtain the present worth of a debt drawing interest, first find what will be its amount at maturity, and then apply the rule.

EXAMPLES FOR PRACTICE.

Find the present worth and true discount

1. Of \$834.08 if due in 18 days and money is worth 5%.
2. Of \$8320.01 if due in 73 days and money is worth 6%.
3. Of \$306.20 if due in 93 days and money is worth 8%.
4. Of \$305.55 if due in 7 mo. 15 da. and money is worth 8%.
5. Of \$276.30 if due in 5 mo. 18 da. and money is worth 5%.

6. I bought 900 barrels of flour at \$8 per barrel, one-half of which was to be paid in 6 months, and the remainder in 9 months. I paid the present worth of the bill at the date of purchase, the current rate of interest being 6%. How much less than the sum due at maturity did I pay for the flour?

7. A merchant offers a lot of goods for \$587.25 and 90 days' credit. What will be an equivalent cash offer, money being worth 5%?

8. I can buy flour at \$8.10 per barrel and 4 months' credit, at \$7.95 and 2 months' credit, or at \$7.86 for cash. What is the cost of 350 barrels bought on the most advantageous of these terms, money being worth 8% per annum?

9. A merchant paid the present worth of a debt of \$803.60, 18 days before it was due. What was the true discount allowed, money then being worth 9%?

10. The invoice price of merchandise purchased on 60 days' time was \$840, on which I was allowed a trade discount of 20% and 10%. I paid the present worth of the debt one week after the date of purchase, the rate of interest then current being 8%. How much less than the invoice price did I pay for the merchandise?

11. A bill of \$5300, payable in 4 months, was settled 3 months after the date of purchase by paying its present worth. What was the true discount at 6% per annum?

12. If a debt of \$3200 is payable in 6 months with interest at 6% per annum, what will be its present worth three months before maturity, money then being worth 8% per annum?

13. On Jan. 18, 1888, I bought a quantity of merchandise for \$8000, on 3 months' credit. On Feb. 1, 1888, I made a payment of

263

$2500; on Mar. 4, 1888, I made a second payment of $1800; and on Mar. 15, 1888, I paid the remainder of the debt at its present worth. How much did I pay Mar. 15, 1888, if the current rate of interest was 8%?

COMMERCIAL PAPER.

264. Commercial Paper embraces notes, checks, drafts, bills of exchange, letters of credit, etc., etc.

265. A **Note** (often called a promissory note) is a written promise by one party to pay to another a specified sum of money on demand, or at a designated time.

266. The **Maker** of a note is the party who makes the promise over his own signature; the **Payee** of a note is the party to whom, or to whose order, the note is made payable; the **Holder** of a note is the party who owns it; and the **Face** of a note is the sum of money (not including the interest which it may accrue) which the maker promises to pay the payee or holder of the note.

267. A **Negotiable Note** is one which is made payable to the bearer, or to the order of the payee. A negotiable note can be sold or transferred.

Rem. 1.—When a note is made payable to *bearer*, any person who is the lawful holder of the note is the payee.

Rem. 2.—When a note is made payable only to the party named therein, it is said to be *non-negotiable.*

268. An **Indorsement** is matter written on the back of a note or other commercial paper.

Rem. 1.—An indorsement is usually the signature of the payee, guarantor, or lawful holder, upon the back of a negotiable note or other negotiable paper; and its purpose is to enable the payee or lawful holder to sell or transfer his title or ownership to another.

Rem. 2.—The person who signs his name upon the back of a note is called an *indorser*, and the person in whose favor the indorsement has been made is called the *indorsee.*

Rem. 3.—One effect of an indorsement is to make the indorser liable to the indorsee or any subsequent holder of the note for its payment at maturity, if the maker of the note or prior indorsers fail to pay it at maturity.

Rem. 4.—An *indorsement in blank* is simply the signature of the indorser written upon the back of the negotiable paper, by which it is made payable without further indorsement to *any* person who may subsequently become its holder. An *indorsement in full* is one in which the indorser states over his signature to *whose order* the negotiable paper is payable. A *conditional indorsement* is one which is subject to certain specified conditions, without the performance of which the indorser cannot be held liable. A *restrictive indorsement* is one which restricts the payment of the negotiable paper to some *particular person* so that he cannot transfer it, even if he wished. A *qualified indorsement* is one in which the indorser relieves himself from responsibility for payment by writing over his signature the words "*without recourse.*" The purpose of a qualified indorsement is to enable the indorser to sell or transfer his title to negotiable paper without subjecting himself to liability for its payment if protested.

SPECIMEN FORMS OF INDORSEMENT.

(*In Blank.*)	(*In Full.*)	(*Conditional.*)	(*Restrictive.*)	(*Qualified.*)
W. S. Hill.	Pay to R. S. Jones, or order. W. S. Hill.	Pay to R. S. Jones, or order, unless, before maturity, I give you notice not to pay. W. S. Hill.	Pay to R. S. Jones only. W. S. Hill.	Pay to R. S. Jones, or order, without recourse. W. S. Hill.

269. A Protest of a negotiable note is a formal statement by a notary public, at the request of the holder, that said note was presented to the maker thereof for payment, and by him refused.

Rem. 1.—In case of non-payment of a note by the maker, each indorser can be held liable only after a formal protest has been made, and proper notice given to him. Each indorser may, in turn, be held liable to every subsequent indorsee, and may look for indemnity to every antecedent indorser.

Rem. 2.—A note, when due, must be presented at the place at which it is made payable. When no particular place is specified, it is payable at the maker's place of business *during business hours;* or if his place of business is not known, at his place of residence *during family hours.*

270. Days of Grace are the *three days* beyond the time specified in the note, which are usually allowed for its payment unless the note contains the words "without grace."

271. The **Maturity** of a note is the time at which the note becomes legally due, and includes the "days of grace."

Rem. 1.—Demand for the payment of a note should be made at maturity; and upon failure of the maker to pay the note, it should at once be protested, notice of which protest must be sent to the indorsers on the day of maturity or the next, otherwise they cannot be held responsible.

Rem. 2.—If a note mature on Sunday or any legal holiday, in most States it must be paid on the day previous; and if a legal holiday occur on Monday, notes maturing on that day must be paid on the preceding Saturday. In New York, however, notes maturing on Sunday are payable the *following* business day.

Rem. 3.—It is lawful to compute interest or discount for *all* 3 of the days of grace, though the debtor may lose the benefit of the last day of grace because it happens to fall on a Sunday or a legal holiday.

Rem. 4.—Calendar months are understood if the time is expressed in months. By a calendar month is meant the length of time from any date of one month to the same date of the succeeding month, whether the included days of the calendar month be 28, 29, 30, or 31.

Rem. 5.—If, in estimating the time of a note by calendar months, it should be found to fall due on the 29th, 30th, or 31st of a month which contains less than that number of days, the note would be nominally due on the last day of such month, and legally due on the 3d day of the succeeding month.

272. To find the date of maturity when the time is expressed in days.

ILLUSTRATIVE EXAMPLE.

A promissory note dated Sept. 9, 1889, was made payable 60 days after date. What was the date of legal maturity?

SOLUTION.

60+3 = 63 days, time to run.
21 " remaining in Sept.
42 " after September.
31 " in October.
Nov. 11, 1889, maturity.

Explanation.—Subtract the number of days which the note still has to run in Sept. (30—9 = 21) from the full time the note has to run (60 da. + 3 da. grace = 63), obtaining 42 days as the time which the note has to run in the months succeeding September. Subtract the full number of days in the next succeeding month to Sept. (Oct. = 31 da.), obtaining 11 da. which the note has to run in the next succeeding month (Nov.).

Mental Process.—Reduce the time to uniform calendar months of 30 da. each (63 da. = 2 mo. 3 da.); add the result to the date of the note (Sept. 9 + 2 mo.

3 da. = Nov. 12); make correction by deducting 1 day for the included 31-day month of Oct. (Nov. 12—1 = Nov. 11).

EXAMPLES FOR PRACTICE.

Find the date of legal maturity.

	Date of note.	*Nominal time.*		*Date of note.*	*Nominal time.*
1.	Feb. 18, 1889....	60 days.	6.	Jan. 19, 1888....	90 days.
2.	Mar. 31, 1888....	30 “	7.	Apr. 4, 1889....	60 “
3.	Aug. 5, 1889....	90 “	8.	June 25, 1888....	4 months.
4.	Dec. 31, 1887....	50 “	9.	May 13, 1887....	3 “
5.	Sept. 30, 1888....	40 “	10.	Oct. 31, 1889....	2 “

273. To find the number of days to legal maturity.

ILLUSTRATIVE EXAMPLE.

A note dated Mar. 12, 1888, was made payable 3 months after date. What was the number of days to legal maturity?

SOLUTION.

Mar. 12 + 3 mo. 3 da. = June 15.

Days remaining in March	= 19.
“ in April	= 30.
“ “ May	= 31.
“ included in June	= 15.
“ from Mar. 12 to June 15	= 95.

EXPLANATION.—Mar. 12 + 3 mo. = June 12, nominal date of maturity. Add 3 days for grace, obtaining June 15 as the legal date of maturity. Find the number of days between Mar. 12 and June 15 by Rule, **181,** obtaining 95.

MENTAL PROCESS.—Multiply the number of calendar months by 30 (3 × 30 = 90); to the product add 3 days for grace (90 + 3 = 93); and increase the result 1 day for each of the included 31-day months of Mar. and May (93 + 2 = 95).

EXAMPLES FOR PRACTICE.

Find the number of days to legal maturity.

	Date of note.	*Nominal time.*		*Date of note.*	*Nominal time.*
1.	Sept. 12, 1888....	4 months.	6.	July 4, 1889.....	5 months.
2.	Aug. 30, 1889....	2 “	7.	Feb. 15, 1888.....	2 “
3.	Jan. 8, 1888....	3 “	8.	Oct. 12, 1887.....	5 “
4.	Dec. 15, 1887....	4 “	9.	May 8, 1888.....	3 “
5.	Apr. 3, 1889....	3 “	10.	July 2, 1889.....	4 “

274. To find the number of days between two dates.

ILLUSTRATIVE EXAMPLE.

A promissory note dated May 15, 1889, was nominally due Aug. 2, 1889. What was the number of days to legal maturity?

SOLUTION.

Days remaining in May,	16
" in June,	30
" in July,	31
" included in Aug.,	2
" of grace,	3
" note has to run,	82

Explanation.—Add 3 days of grace to the result obtained by applying Rule, **181.**

Mental Process.—May 15 to Aug. 15 = 3 mo. or 90 da.; therefore to Aug. 2, it will be 13 days less than 90, or 77 da.; to which add 1 day for each of the 31-day months of May and July, obtaining 79; and 3 days for grace, obtaining 82.

EXAMPLES FOR PRACTICE.

Find the number of days to legal maturity.

	Date of note.	*Nominally due.*
1.	May 3, 1889,	Sept. 15, 1889.
2.	Aug. 19, 1888,	Dec. 5, 1888.
3.	July 8, 1889,	Oct. 12, 1889.
4.	Apr. 5, 1887,	July 8, 1887.
5.	Sept. 9, 1888,	Jan. 12, 1889.
6.	Jan. 8, 1888,	Apr. 15, 1888.
7.	Dec. 31, 1887,	Mar, 3, 1888.
8.	Feb. 12, 1889,	July 8, 1889.
9.	Sept. 8, 1888,	Dec. 12, 1888.
10.	May 30, 1889,	Sept. 28, 1889.

BANK DISCOUNT.

275. A **Bank** is an incorporated institution for the issue, custody, or loaning of money, for discounting commercial paper, and for facilitating the transmission of money from one place to another by means of checks, drafts, or bills of exchange.

Rem.—Many banks perform only a part of the functions enumerated above.

276. Bank Discount is a deduction made from the sum due upon a note at maturity for cashing or buying such note *before maturity*. This deduction is the simple interest of the sum due at maturity, computed for the number of days between the date of discount and the date of maturity.

Rem. 1.— Bank discount is computed on the face of a note which does not draw interest, and upon the face plus its accrued interest at maturity of a note which draws interest; since, in the former case, the face, and in the latter, the face plus its accrued interest, will be the sums respectively due at maturity.

REM. 2.—In allowing interest upon interest-bearing notes when the time of payment is expressed in months, many banks allow only 30 days to each expressed month.

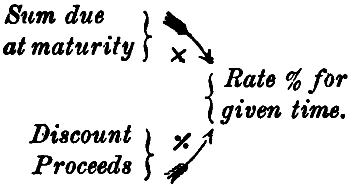
277. The **Term of Discount** is the time from the date of discount of a note to the date of its maturity.

REM.—Ordinarily, in computing the interval between two dates, the first day of the interval is omitted; but in the banks of Philadelphia, Baltimore, and some other cities, it is customary to include both the day of discount and the day of maturity, which is equivalent to charging discount for one day more than the actual time.

278. The **Proceeds** of a note is the sum received for it at the date of discount, and is equal to the *sum due* at maturity minus the *discount*.

279. Computations in Bank Discount may be made in accordance with the principles of percentage, the sum due at maturity being regarded as the base; the bank discount and proceeds as percentages. Hence,

GENERAL FORMULA.



The direction for the use of this formula is the same as that of Gen'l Form., **214**.

REM 1.—The rate of discount for the given time is found by dividing the term of discount, in days, regarded as hundredths, by the quotient of the rate per annum into 360.

REM. 2.—Since the sum due at maturity — the discount for the given time = the proceeds, so the rate of the sum due at maturity (100%) — the rate of discount for the given time (Rem. 1) = the rate of proceeds.

280. To find the bank discount or proceeds of a note.

ILLUSTRATIVE EXAMPLE.

A note of $1200, dated June 13, 1889, and payable 4 months after date, was discounted at bank, July 2, 1889. What were the date of maturity, the term of discount, the bank discount, and the proceeds of the note, the rate of bank discount being 6% ?

SOLUTION.

June 13 + (4 mo. 3 da.) = Oct. 16, date of maturity (Rem. 4, **271**).
July 2 to Oct. 16 = 106 days, term of discount (**274**).
Interest of $1200 for 106 da. at 6% = $21.20, bank discount (**244**).
$1200 — $21.20 = $1178.80, proceeds (**278**).

RULE.—I. *Compute the interest of the sum due at maturity for the term of discount at the given rate per annum; the result will be the bank discount.*

II. *Subtract the bank discount from the sum due at maturity to find the proceeds.*

EXAMPLES FOR PRACTICE.

Find the date of maturity, term of discount, bank discount, and proceeds.

	Face of note.	*Date of note.*	*Time.*	*Date of disc't.*	*Rate of disc't.*
1.	$1500	Jan. 5, 1889	60 days	Feb. 1, 1889	6%
2.	$950	Sept. 30, 1889	90 days	Nov. 8, 1889	7%
3.	$325	Mar. 9, 1888	30 days	Mar. 12, 1888	6%
4.	$150	Nov. 8, 1889	3 months	Jan. 1, 1890	8%
5.	$500	Feb. 5, 1888	60 days	Feb. 28, 1888	6%
6.	$1200	Oct. 31, 1887	4 months	Jan. 8, 1888	5%
7.	$275	Apr. 3, 1889	2 months	Apr. 15, 1889	6%
8.	$2800	Aug. 15, 1888	90 days	Sept. 6, 1888	7%
9.	$5200	May 6, 1888	6 months	Sept. 18, 1888	6%

To enable the learner to discriminate between the various forms of notes in general use, we append a specimen of each.

10. DEMAND NOTE.

$218 $\frac{25}{100}$. BALTIMORE, MD., *Jan. 30, 1889.*

On demand, for value received, I promise to pay SAMUEL WOLVERTON, or bearer, Two Hundred and Eighteen $\frac{25}{100}$ Dollars, with interest.

R. C. CLAY.

REM.—The above note is payable whenever presented (on demand); is negotiable (payable to bearer); and bears interest from the date of the note at the legal **280** rate of Maryland. If "or bearer" had been omitted, it would have been non-negotiable, since it would then have been payable to Samuel Wolverton, only.

What would have been the sum due upon the above note, if it had been presented for payment Feb. 19, 1889?

11. TIME NOTE (*without interest*).

$3000. DAYTON, OHIO, *June 3, 1888.*

Three months after date, for value received, I promise to pay GEO. C. ROUND, or order, Three Thousand Dollars.

JOSEPH S. SINCLAIR.

Rem.—The above note is payable 3 mo. 3 da. from June 3, or on Sept. 6, 1888. Since it is payable to the "order of" Geo. C. Round, it is negotiable with Geo. C. Round's indorsement. No interest is mentioned in the note, hence it cannot draw interest until maturity, after which, if it is not then paid, it will draw interest at the legal rate in Ohio.

Find the term of discount, the discount, and the proceeds, if the preceding note had been discounted at bank July 15, 1888, at 8%.

12. TIME NOTE (*with interest*).

790\frac{30}{100}$. NEW YORK, *May 3, 1889.*

Ninety days after date, I promise to pay JOHN ARMSTRONG, or bearer, Seven Hundred and Ninety $\frac{30}{100}$ Dollars, with interest at 6%. Value received.

W. A. MARBURY.

Rem.—The above note is payable 93 days after May 3, or on Aug. 4, 1889. Since it is payable to John Armstrong, *or bearer*, it is negotiable without John Armstrong's indorsement. It draws interest from May 3, until paid.

What would have been the discount and the proceeds, if the above note had been discounted June 24, 1889, at 8%?

13. JOINT NOTE.

$538. CHICAGO, *Sept. 9, 1889.*

Four months after date, we promise to pay to the order of SAMUEL GUNN & CO., Five Hundred and Thirty-eight Dollars, for value received.

JAMES E. JONES.
ROGER A. PALMER.

Rem.—The above is called a *joint* note because it is signed by more than one person, who are jointly responsible for its payment; that is, each maker is only *individually* responsible for the payment of his proportionate part (one-half) of the face of the note.

What are the date of maturity, the term of discount, the discount, and the proceeds of the above note, if discounted at bank Nov. 13, 1889, at 6% ?

14. JOINT AND SEVERAL NOTE.

489\frac{42}{100}$. BOSTON, *Dec. 8, 1888.*

One month after date, we jointly and severally promise to pay to the order of HENRY BLACK, Four Hundred and Eighty-nine $\frac{42}{100}$ Dollars. Value received.

J. E. PRYOR.
W. O. HOLMES.

REM.—The effect of a *joint note* is to hold its makers only jointly responsible for its payment; but that of a *joint and several note* is to hold either maker *individually* responsible for the payment of the full amount of the note if the other defaults.

What are the date of maturity, the term of discount, the discount, and the proceeds of the preceding note, if discounted at bank Dec. 12, 1888, at 8% ?

15. NOTE PAYABLE AT A BANK.

$500. PHILADELPHIA, *May 2, 1889.*

Ninety days after date, I promise to pay to J. M. GAULDEN, or order, Five Hundred Dollars, without defalcation, at the First National Bank. Value received.

GEO. W. KELLER.

REM.—In Pennsylvania the phrase "without defalcation" is required in a note to make it negotiable.

What were the discount and the proceeds of the above note, if discounted at the bank June 18, 1889, at 8%?

16. NOTE PAYABLE TO ONE'S OWN ORDER.

$600. NEW ORLEANS, *Oct. 15, 1888.*

Sixty days after date, I promise to pay to the order of myself, Six Hundred Dollars. Value received.

W. GARIG.

REM.—Business men frequently draw notes payable *to their own order*, so that with their indorsement in blank (Rem. 4, **263**), they may be transferable without the further indorsement of the holder.

What are the proceeds of the above note if discounted at bank Nov. 2, 1888, at 9% ?

17. A note of $1800, dated June 16, 1889, bearing 6% interest, and

280

payable 4 months after date, was discounted at bank June 29, 1889, at 6%. What were the bank discount and the proceeds?

18. Henry Gould & Co. offered at bank the following, which were passed and discounted Sept. 6, 1889, at 6%:

Note dated July 30, 1889, at 4 months, for $850.25.
" " Aug. 15, 1889, " 60 days, " 325.50.
" " Aug. 31, 1889, " 3 months, " 575.80.

What was the discount on each note, and the amount of proceeds passed to their credit?

19. At 5%, what are the proceeds of a 90-day note of $580.75, dated May 23, 1888, and discounted June 1, 1888?

20. M. E. Whitney's draft of $1500 drawn on J. E. Morse, favor of Wm. J. Robb, dated Oct. 31, 1888, payable 90 days after date, and accepted Nov. 3, 1888, was discounted for E. C. Williamson, Dec. 7, 1888. If the rate of discount was 5½%, what were the date of maturity, term of discount, discount, and proceeds?

21. A draft dated July 10, 1889, drawn by Fairman A. Sadler, Milwaukee, on Allen S. Will, New York, payable sixty days after sight, and accepted July 16, 1889, was discounted Aug. 3, 1889. If the face of the draft was $1123.40, and the rate of discount 5%, what were the date of maturity, term of discount, discount, and proceeds?

22. A bank discounted July 2, 1889, for Wm. H. Patrick, a note of A. M. Sellers for $1250, dated June 9, 1889, and payable 4 months after date. Rate of discount 6%. Find the date of maturity, unexpired time, discount, and proceeds.

23. A merchant discounted Aug. 19, 1888, at 7%, a note of J. O. Moul, indorsed by Wm. A. Heitmueller, for $439.50, dated July 7, 1888, and payable 90 days after date. Find the date of maturity, unexpired time, discount, and proceeds.

24. Discounted Nov. 5, 1889, for J. H. W. Marriott, the following notes:

No. 1. Dated Oct. 28, 1889, payable 4 months after date, $1000.
" 2. " " 29, " " 4 " " " 700.
" 3. " " 30, " " 4 " " " 560.
" 4. " " 31, " " 4 " " " 63.20.

Find the date of maturity, term of discount, discount, and proceeds of each, if the rate of discount was 4½%.

281. To find the face of a note.

ILLUSTRATIVE EXAMPLE.

For what sum must I draw a 60-day note that, when discounted at bank at 8% per annum, the proceeds will be $887.40?

SOLUTION.

60 + 3 = 63 da., term of discount.
$.63 ÷ 45 = $.014, discount of $1 (Note 1, **253**).
$1 − $.014 = $.986, proceeds of $1.
$887.400 ÷ $.986 = 900 times ($1 or $900) face.

EXPLANATION.—Since the given proceeds ($887.40) are 900 times $.986, the proceeds of a note whose face is $1, the face of a note which will yield the given proceeds must be 900 times $1, or $900.

SECOND SOLUTION.—Find the interest of the proceeds ($887.40) for 63 da. at 8%, obtaining $12.424 −; then the interest of this result ($12.424) for 63 da. at 8%, obtaining $.173 +. The sum of the given proceeds ($887.40) + the interest of the proceeds ($12.424) + the interest of this interest ($.173) = face of note ($899.997, practically $900).

By computing interest on the proceeds, which is the bank discount less than the correct basis, the result ($12.424) will lack the interest of the bank discount of being the correct interest. Hence, assuming $12.424 to be a close approximation to the bank discount, make the necessary correction by computing interest upon this interest for the same time and at the same rate. To get the correct answer, interest should be computed upon the last obtained interest as long as a *material result* can be produced.

RULE.—*Divide the given proceeds by the proceeds of $1 at the given rate and for the given time with grace.*

NOTE 1.—To obtain the proceeds of $1, find the interest of $1 by Note 1, **253**, and deduct the result from $1.

NOTE 2.—To avoid fractions, multiply the proceeds by the quotient of the rate per annum into 360, and divide the result by the same quotient diminished by the days regarded as hundredths. Thus, in the Ill. Ex., ($887.40 × 45) ÷ (45 − .63) = $900.

EXAMPLES FOR PRACTICE.

Find the face of note.

	Proceeds.	*Term of disc't.*	*Rate of disc't.*
1.	$673.88	54 days	6%.
2.	$364.56	90 "	8%.
3.	$394.40	72 "	7%.
4.	$237.20	84 "	5%.

Find the face of note.

	Proceeds.	*Term of disc't.*	*Rate of disc't.*
5.	$534.06	66 days	6%.
6.	$1164	135 "	4%.
7.	$392	4 months	6%.
8.	$715.40	3 "	4%.

9. I owed a firm $750, and, in payment thereof, drew a 90-day note, the proceeds of which, when discounted at 6%, equaled the debt. What was the face of the note ?

10. What must be the face of a note nominally payable 4 months after date that, when discounted at 6%, the proceeds will exactly pay a debt of $1000 ?

11. I owed $6000, and, to pay it, I had a note of $3500, legally maturing in 18 days, discounted by a bank at 8%, and drew a second note, nominally payable in 2 months, for such a sum that, when discounted at the same rate, the proceeds of both notes enabled me to pay the debt. What was the face of the second note ?

282. Comparison of bank and true discount.

Worth at maturity ......Base............ *Present Worth.*

$$\left.\begin{array}{r}\textit{Bank Discount,}\\ \text{(Proceeds) }\textit{Present Worth.}\end{array}\right\}\ldots\text{Percentages}\ldots\left\{\begin{array}{l}\textit{True Discount,}\\ \textit{Worth at maturity.}\end{array}\right.$$

Explanation.—In the above comparison, it is seen that in bank discount the worth of a debt at maturity is the base ; and its present worth, that is, its worth on the day of prepayment, a percentage. By contrasting these terms of bank discount on the left with similarly named terms of true discount on the right, it will be further noticed that the base of one discount becomes a percentage of the other; and, consequently, that the two discounts differ only as to the base or principal upon which they are computed : bank discount being computed on the worth of a debt at maturity, and true discount on its present worth. Hence, the difference between bank discount and true discount is the interest of the true discount for the given time and at the given rate.

1. What is the difference between the bank discount and true discount of a note of $1800, dated June 25, 1889, and nominally payable 3 months after date, if discounted July 18, 1889, at 6%?

2. I bought $587.40 worth of goods on 60 days' credit. What sum will pay for the goods on the day of purchase, allowing bank discount at 5%? Allowing true discount at 5%?

Debts not secured by notes are not allowed days of grace. Time notes or drafts are allowed days of grace irrespective of the kind of discount allowed.

3. A debt of $953.25, legally due September 5, 1889, was credited with a cash payment of $375 on July 28, 1889. What was the balance due at maturity, allowing bank discount for the prepayment at 8%? Allowing true discount at 8% ?

PARTIAL PAYMENTS.

283. A **Partial Payment** is a payment of only a part of a debt and its accrued interest.

REM.—A partial payment of a note is usually acknowledged by indorsing on its back the date and amount of the payment.

284. When partial payments have been made on an interest-bearing note, the balance unpaid may be found by the following method, adopted by the Supreme Court of the United States, and hence called the United States rule.

285. United States rule for partial payments.

ILLUSTRATIVE EXAMPLE.

On a note of $3000, dated March 12, 1885, and drawing simple interest at 6% per annum, are indorsed the following payments: August 18, 1885, $325. April 9, 1886, $75. July 3, 1886, $400. Dec. 4, 1886, $150. How much was due February 25, 1887?

SOLUTION.

Face of note or original principal	$3000
Interest from Mar. 12 to Aug. 18 (5 mo. 6 da.)	78
Amount due at the time of the first payment (Aug. 18)	$3078
First payment to be deducted	325
Debt unpaid, forming a new principal from Aug. 18	$2753
Interest of new principal from Aug. 18 to Apr. 9 (7 mo. 21 da.)	105.99
Since the 2d payment is less than this interest, compute the	
Interest of same principal from Apr. 9 to July 3 (2 mo. 24 da.)	38.54
Amount due at the time of the third payment (July 3)	$2897.53
Second and third payments to be deducted ($75 + $400 =)	475
Debt unpaid, forming a new principal from July 8	$2422.53
Interest of above principal from July 3 to Dec. 4 (5 mo. 1 da.)	60.97
Amount due at the time of the fourth payment (Dec. 4)	$2483.50
Fourth payment to be deducted	150
Debt unpaid, forming a new principal from Dec. 4	$2333.50
Interest of above principal from Dec. 4 to Feb. 25 (2 mo. 21 da)	31.50
Amount due at the date of settlement (Feb. 25, 1887)	$2365.00

Rem.—It is usually apparent whether a payment is less than the accrued interest, as with the second payment of the solution; in which case, interest could have been more conveniently computed in *one operation* from the date of the preceding payment (Aug. 18) to the date of the third payment (July 3), or for 10 mo. 15 da., obtaining \$144.53 +, or the same interest as in the solution was obtained in *two* operations (\$105.99 + \$38.54 = \$144.53).

Rule.—I. *Find the interest of the principal from the date of the note to the date of the first partial payment. If the first payment equals or exceeds the interest thus found, subtract the first payment from the sum of the principal and its accrued interest, and consider the remainder as a new principal.*

II. *If the first payment is less than the interest thus found, find the interest of the same principal to the date of the next or of such subsequent partial payment at which the sum of the payments will equal or exceed the interest due at such date. Subtract the sum of the payments to that date from the sum of the principal and its accrued interest to that date, and consider the remainder as a new principal.*

III. *Similarly find the interest of the new principal to the date of the next partial payment. If the next partial payment is greater than the interest thus found, proceed as in* I; *but if less, proceed as in* II. *So continue to the date of settlement.*

Note 1.—It will be observed in the foregoing rule (1) that if a balance were struck for a partial payment which is in excess of the accrued interest it would have the effect of reducing the principal since, after discharging such accrued interest, there will remain a surplus to be so applied; (2) but that if a balance were struck for a payment less than the accrued interest it would have the effect of increasing the principal, since such payment is not sufficient to discharge the accrued interest, *which must first be paid;* (3) that no new principal should exceed a preceding principal, since that would, in effect, be compounding interest on such excess, which it is the purpose of the U. S. Rule to prevent.

Note 2.—In applying the U. S. Rule to partial payments, find the time by compound subtraction (**180**).

EXAMPLES FOR PRACTICE.

(1.)

\$750. Washington, D. C., *July 15, 1886.*

Three years after date, for value received, I promise to pay M. F. Sinclair, or order, Seven Hundred and Fifty Dollars, with interest at 6% per annum. J. F. O'Bannon.

285

What was the balance due July 18, 1889, upon the preceding note, if the following partial payments had been previously made: Oct. 18, 1887, $175; Jan. 12, 1889, $200?

(2.)

$1200. New Orleans, *Aug. 8, 1884.*

Three years after date I promise to pay T. C. Will, or order, Twelve Hundred Dollars, with interest at 8%. Value received.

E. Bensel.

The above note contained indorsements as follows: Sept. 13, 1885, $75; Oct. 12, 1886, $80; Jan. 1, 1887, $25. What was the balance due Sept. 12, 1887?

(3.)

$890. Mobile, *Mar. 18, 1888.*

Two years after date I promise to pay to W. A. Frank & Co., or order, Eight Hundred and Ninety Dollars, value received, with interest at 6%.

R. G. Walker.

The following payments were indorsed on the above note:

May 13, 1888, $35.
Aug. 5, 1888, $10.
Feb. 3, 1889, $75.
Dec. 19, 1889, $150.

What was due Mar. 21, 1890?

(4.)

$3500. Boston, *May 15, 1885.*

Three years after date, for value received, I promise to pay R. L. Thomas, or order, Three Thousand Five Hundred Dollars, with interest at 6%.

F. S. Holmes.

The following indorsements were made on the above note:

Sept. 12, 1885, $350.
May 9, 1886, $75.
Oct. 8, 1886, $250.
Jan. 18, 1887, $400.
Aug. 5, 1887, $90.
Dec. 12, 1887, $350.

What was the amount due July 3, 1888?

5. On a mortgage for $5200, dated June 16, 1884, there were paid Aug. 12, 1884, $300; June 15, 1885, $125; Aug. 19, 1886, $1200; May 4, 1887, $50; and Dec. 24, 1888, $2000. What was the balance due Feb. 12, 1889, at 6%?

6. The following payments were made on a mortgage of $2500, dated Aug. 12, 1884, drawing interest at 5% and lapsing 5 years from

date: July 8, 1885, $100; Sept. 5, 1886, $500; Jan. 19, 1888, $112; Apr. 1, 1889, $900. What was the balance due Aug. 30, 1889?

286. Mercantile rule for partial payments.

ILLUSTRATIVE EXAMPLE.

By the merchants' rule, find the balance due July 18, 1889, on a note of $500, at 6%, dated Sept. 15, 1888, and upon which were the following indorsements: Oct. 12, 1888, $75; Dec. 18, 1888, $128; Mar. 9, 1889, $250.

SOLUTION.

Face of note, dated Sept. 15, 1888		$500.00
Interest of $500 from Sept. 15 to July 18 (306 da.) at 6%		25.50
Sum of principal and accrued interest at date of settlement		$525.50
First payment made Oct. 12, 1888	$75.	
Interest of $75, from Oct. 12 to July 18 (279 da.)	3.49	
Second payment, made Dec. 18, 1888	128.	
Interest of $128 from Dec. 18 to July 18 (212 da.)	4.52	
Third payment, made Mar. 9, 1889	250.	
Interest of $250 from Mar. 9 to July 18 (131 da.)	5.46	
Sum of payments and accrued interest at settlement		466.47
Balance due at date of settlement		$59.03

Explanation.—By mercantile usage, if the time of settlement is one year or less from the date of a note, or other interest-bearing debt, the balance due is found by increasing the debt by its accrued interest to the date of settlement, and diminishing the result by the sum of the partial payments and the interest accrued upon each payment from the time it was made to the date of settlement.

Rem.—In applying the merchants' rule to partial payments, the prevailing custom is to find the exact time in days.

EXAMPLES FOR PRACTICE.

1. On a note of $800, at 6%, and dated Mar. 4, 1887, are the following indorsements: Apr. 12, 1887, $75; July 9, 1887, $150; Sept. 5, 1887, $90; Dec. 8, 1887, $200. What was due Jan. 1, 1888?

2. On a note of $1500, at 5%, dated July 5, 1888, were indorsed the following payments: Sept. 19, 1888, $300; Nov. 5, 1888, $200; Jan. 18, 1889, $175; Mar. 3, 1889, $350. What was due June 15, 1889?

3. What was the balance due Aug. 3, 1888, on a note of $500, at

7%, dated Dec. 15, 1887, and upon which the following payments had been made: Jan. 30, 1888, $150; Mar. 31, 1888, $80; May 2, 1888, $160; July 4, 1888, $75?

4. On Sept. 9, 1888, I gave my note of $2000, at 6%, upon which I subsequently made the following payments: Nov. 20, 1888, $325; Jan. 13, 1889, $75; May 25, 1889, $472; and July 21, 1889, $200. What did I owe upon the note Aug. 17, 1889?

5. What was the balance due Mar. 15, 1889, upon a note of $1725.40, at 5%, dated Apr. 19, 1888, and upon which the following payments were indorsed: June 7, 1888, $150; July 1, 1888, $136; Aug. 12, 1888, $75; Sept. 23, 1888, $125; Nov. 18, 1888, $85; Dec. 20, 1888, $225; Mar. 5, 1889, $178?

6. What was the balance due Sept. 8, 1889, upon a note of $5600, at 6%, dated Oct. 3, 1888, if it contained the following indorsements: Dec. 15, 1888, $500; Feb. 28, 1889, $800; June 13, 1889, $450; Aug. 7, 1889, $1200?

7. On a note of $1800, at 6%, dated Nov. 13, 1886, were indorsed the following payments: Jan. 16, 1887, $100; Mar. 12, 1887, $50; July 3, 1887, $250. What was the balance due exactly one year after the date of the note?

8. If the balance due Nov. 13, 1887, as found in the preceding example, had not been paid at that time, what would have been the balance due exactly one year after that date, if, within the interval, the following additional payments were indorsed upon the note: Dec. 8, 1887, $400; Aug. 15, 1888, $300?

Rem.—On a note running longer than one year, it is the custom of merchants to find the balance due at the end of the first *complete* year from the date of the note, as in Ex. 7. If this balance is not then paid, they regard it as a new principal, and in a similar manner find the balance due at the end of the second year from the date of the note, as in Ex. 8; and so continue until the maker of the note is prepared to settle "in full," when the last balance will be struck to the date of final settlement, as in Ex. 9, following.

9. If the balance due Nov. 13, 1888, as found in Ex. 7, is not paid until Jan. 1, 1889, what sum will cancel the note, no partial payments having been made within the interval?

10. What was the balance due June 13, 1889, upon a note of $1325, at 5%, dated May 6, 1886, and containing the following indorsements: Aug. 5, 1886, $90; Oct. 8, 1887, $325; Jan. 1, 1888, $150; July 6, 1888, $215; Oct. 8, 1888, $45; and May 13, 1889, $286?

286

STOCKS AND BONDS.

287. A **Corporation** consists of several persons who are authorized by law to transact business as a single individual.

REM.—The special act which legalizes the existence of a corporation and specifies its powers and obligations, is called its *Charter.*

288. The **Capital Stock** of a corporation is so much of its capital as is represented by its shares.

REM. 1.—A *Share* is one of the equal parts into which the capital stock is divided. It usually represents $100 of the capital stock, though shares sometimes represent as low as $1 and as high as $1000 of the capital stock. Shares of $50 and $25 are respectively called *half-stock* and *quarter-stock.* The owner of one or more shares of a corporation is called a stockholder of that corporation.

REM. 2.—A *Stock Certificate* is an instrument issued by a corporation to its stockholders, certifying that they are the owners of a stated number of shares of its capital stock.

289. The **Par Value** of a share is the value specified upon its "face," and represents the value of the proportional part of the capital stock for which it was originally issued.

290. The **Market Value** of a share is the sum for which it can be sold.

REM. 1.—When the market value of a share is *greater* than its par value, it is said to be *above par* or *at a premium;* and when its market value is *less* than its par value, it is said to be *below par* or *at a discount.*

REM. 2.—The *Gross Earnings* of a corporation embrace the *total receipts* from its business before any expense has been deducted.

REM. 3.—The *Net Earnings* of a corporation are its profits; that is, what is left of its gross earnings after all expenses have been paid, including the interest upon its debts, if any.

REM. 4.—A *Dividend* is so much of the net earnings of a corporation as is divided among its stockholders.

REM. 5.—An *Assessment* is a sum levied upon the stockholders of a corporation to make up its losses or deficiencies.

REM. 6.—Dividends and assessments are declared at a certain % of the par value of the stock, or at so much per share.

291. **The Preferred Stock** of a corporation is so much of its capital stock as has advantages over the remainder, or common stock, when dividends are declared.

Rem.—To prevent the management of an embarrassed corporation from passing out of the hands of its officers into those of a *receiver* appointed by a court, some of its stockholders or friends may extend pecuniary aid, for which new stock is issued, called preferred stock, because it is entitled to receive a dividend of a specified per cent. out of the net earnings of the corporation before any dividend can be declared upon the remainder of the capital stock.

292. **Watered Stock** is stock which has been inflated by distributing among the stockholders extra shares for which no additional payment is required.

Rem.—When the charter of a corporation forbids it to declare a dividend exceeding a certain rate % of the par value of its capital stock, or if its stockholders wish to keep the public in ignorance of its prosperity, its capital stock can be watered by issuing additional shares, so that a low rate of dividend upon the watered stock will produce as much income as a high rate of dividend upon the original capital stock.

293. A **Bond** is an obligation to pay a certain sum of money at a specified time, with interest at a fixed rate % payable at regular intervals.

Rem. 1.—Bonds are issued for money borrowed by the General Government, States, counties, cities, and business corporations.

Rem. 2.—A *coupon bond* is one with certificates attached, representing the successive installments of interest. Each coupon is detached and presented for payment when the interest for the period mentioned therein becomes due.

Rem. 3.—A *registered bond* is one which is payable to the owner, whose name is registered in the books of the corporation or government which issued it. Registered bonds can be transferred only by a properly acknowledged assignment.

Rem. 4.—Bonds usually derive their names from the rate of interest which they bear and the date at which they become due; as, U. S. 4½'s of '91; Pacific 6's of '95; Baltimore City 5's of 1916.

294. **Quotations** usually denote the price at which $100 par value of stocks or bonds can be bought or sold.

Rem. 1.—Mining securities are usually quoted at so much per share, without regard to their par value. Thus, mining stock, the par value of which is $5 per share, may be quoted at 3, which denotes that its market value is $3 per share. In some cities, *all* stocks whose par value is less than $100 are thus quoted.

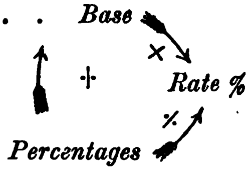
Rem. 2.—*Bonds* of all denominations are quoted at the market value of each $100 of their face value. This is also the prevailing practice with stocks, except mining stocks. Quotations thus expressed are theoretically *rates per cent.* Half-stock worth $57 per share would thus be quoted at 114, the price of 2 shares or of $100 par.

295. Computations in Stocks and Bonds may be made in accordance with the principles of percentage; the par value being taken as the base; and all other concrete terms, as dividend, assessment, premium, discount, brokerage, income, market value, prime or gross cost when buying, net or gross proceeds when selling, being regarded as percentages. Hence,

General Formula.

Par Value of Stocks or Bonds, ... *Base*

Dividend, Assessment, Premium,
Discount, Interest, Market Value,
Prime or Gross Cost, when buying,
Net or Gross Proceeds, when selling, } *Percentages*



Rem. 1.—It should be remembered that the quotation is theoretically the rate % of market value (Rem 2, 294). Hence, since securities are bought and sold at their market value, the quotation regarded as hundredths must be the rate per cent. of prime cost if securities are bought, or of gross proceeds if securities are sold.

Rem. 2.—If securities are bought through a broker, the rate % of gross cost equals the rate % of prime cost (quotation) plus the rate % of brokerage; and if sold through a broker, the rate % of net proceeds equals the rate % of gross proceeds (quotation) minus the rate % of brokerage.

Rem. 3.—In stock computations, it is usually more convenient to operate upon shares (one-hundredth of the par value), and to regard all rates per cent. as *dollars per share.* Thus, a premium of 15% = $15 per share; a quotation of 115 = $115 per share; a brokerage of ¼% = $¼ per share; etc.

EXAMPLES FOR PRACTICE.

1. What is the dividend on $7500 par value of stock, if the rate of dividend is 4%?

Par value ($7500) × rate of dividend (.04) = dividend ($300).

Number of shares (75) × dividend per share ($4) = dividend ($300).

What is the

2. Dividend on $3200 par of stock, if the rate of dividend is 3⅛% ?
3. Dividend on 87 shares of stock, if the rate of dividend is 12½%?
4. Assessment on $10500 par of stock ; rate of assessment 6¼% ?
5. Assessment on 123 shares of stock ; rate of assessment 7⅛% ?
6. Premium on $1800 par of bonds, if the rate of premium is 18%?
7. Discount on 73 shares of stock, if the rate of discount is 8⅝%?

8. What is the market value of $12500 par of stocks, if the rate of premium is 9⅛% ?

Par value ($12500) × rate of market value (1.09⅛) = market value ($13640.63—).
No. of shares (125) × market value per share ($109.125)=market value ($13640.63—).

What is the market value

9. Of $18600 par of bonds, if the rate of premium is $3\frac{5}{16}$% ?
10. Of 30 shares of stock, if the rate of premium is 8⅝%?
11. Of $2800 par of stock, if the rate of discount is 4¼% ?
12. Of 159 shares of stock, if the rate of discount is $3\frac{1}{16}$% ?
13. Of $1700 par of bonds, if the quotation is 91⅜ ?
14. Of 29 shares of stock, if the quotation is 100⅞ ?

15. What is the cost of $7600 par of stocks bought at 25% premium, including ⅛% brokerage ?

Par value ($7600) × rate of cost (125⅛%) = cost ($9509.50).
No. of shares (76) × cost of 1 share ($125.125)=cost of all the shares ($9509.50).

Including ⅛% brokerage, what is the

16. Cost of $8500 par of stock bought at 15¾% premium?
17. Cost of 23 shares of stock bought at 8½% discount?
18. Cost of $3500 par of stock bought at 105⅜ ?
19. Net proceeds of 78 shares sold at 75¼ ?
20. Net proceeds of $5700 par of bonds sold at 18⅝% premium ?
21. Net proceeds of 85 shares of stock sold at 26⅞% discount ?

22. If the par value of stocks is $6100, and the dividend $427, what is the rate % of dividend ?

Dividend ($427) ÷ par value ($6100) = rate of dividend (7%).
Dividend ($427) ÷ No. shares (61) = rate of dividend per share ($7 = 7%).

If the par value of stock is

23. $6800, the assessment $204, what is the % of assessment ?

24. $5000, the dividend $318.75, what is the % of dividend ?

25. $8300, the premium $1182.75, what is the % of premium ?

26. $5400, the discount $681.75, what is the % of discount ?

27. $3600, and the market value $4243.50, what is the quotation ?

28. $1400, and the market value $1515.50, what is the rate % of premium ?

29. $8000, and the market value $5370, what is the % of discount ?

30. If the cost of 80 shares of stock, including ¼% brokerage, is $8760, at what quotation were they bought ?

Cost of all the shares ($8760) ÷ No. of shares (80) = cost of 1 share ($109½).
Gross cost per share ($109½) — brokerage per share ($¼) = prime cost per share ($109¼).

If, allowing ⅛% brokerage,

31. The cost of 52 shares of stock is $5869, at what quotation were they bought ?

32. The cost of 47 shares is $5804.50, at what % premium were they bought ?

33. The cost of 175 shares is $14743.75, at what % discount were they bought ?

34. The proceeds of 92 shares are $7693.50, at what quotation were they sold ?

35. The proceeds of 180 shares are $21465, at what % premium were they sold ?

36. The proceeds of 58 shares are $5031.50, at what % discount were they sold ?

37. What is the par value of stock bought for $7712, at 20½% premium ?

Market value ($7712) ÷ rate of market value ($1.20½) = par value ($6400).
Market val. ($7712) ÷ market val. per sh. ($120.50) = No. sh. (64 = $6400).

What is the par value of stock if the

38. Dividend is $170.50, and the rate of dividend 5½% ?

39. Assessment is $1031.25, and the rate of assessment 8¼% ?

40. Premium is $3293.75, and the rate of premium 19⅜% ?

41. Discount is $1550.25, and the rate of discount 13¼% ?

42. Market value is $7193.25, and the rate of premium 4¼% ?

43. Market value is $7182, and the rate of discount 14½% ?

44. Market value is $4812.50, and the quotation 68¾?

45. Cost is $1770.75, brokerage ¼%, and quotation 98⅛?

Cost ($1770.75) ÷ rate of cost (98⅛ + ¼% = 98⅜%) = par value ($1800).
Cost of all the sh. ($1770.75) ÷ cost per sh. ($98.375) = No. sh. (18).

46. Cost is $6631.25, brokerage ⅛%, and quotation 132½?

47. Cost is $13530, brokerage ⅜%, and premium 12⅜%?

48. Cost is $12487.50, brokerage ¼%, and discount 17%?

49. Net proceeds are $10436.25, brokerage ⅛%, and quotation 75¾?

50. Net proceeds are $4051.75, brokerage ¼%, and premium 6⅞%?

51. Net proceeds are $27320, brokerage ⅛%, and discount 14½%?

52. A man bought 150 shares of steamboat stock at 2⅜% discount, and sold the same at 1⅝% premium. What was his net gain, allowing ⅛% brokerage each way?

53. A speculator sold through a broker 74 shares of canal stock, and received $6336.25 as the net proceeds of the sale. At what quotation was the stock sold, allowing ¼% for brokerage?

54. I bought stock at 4⅝% discount and sold the same at 2⅛% premium, making a net profit of $975. The brokerage each way was ⅛%. How many shares were involved in the transaction?

55. A speculator sold 40 shares of stock at 15⅝% discount, paying ¼% brokerage. What were the net proceeds of the sale?

56. A retired merchant bought 124 shares of stock for $14337.50, including ¼% brokerage. At what % premium were they bought?

57. A broker, on his own account, sold stock at 6⅞% above par, and obtained $7908.75. How many shares did he sell?

58. A man bought canal stock at 98¾ and sold the same at 101⅝, receiving $6090 as the net proceeds of the sale. What was his net gain, allowing ⅛% brokerage each way?

59. A broker, on his own account, sold 160 shares of B. & O. R. R. stock for $31800, realizing a profit of $340. At what quotation did he buy the stock?

60. A gentleman sold 135 shares of railroad stock at 18¾% premium, paying ⅛% brokerage. What were the net proceeds of the sale?

61. A man bought 200 shares of manufacturing stock at 135⅜, sold 75 shares of the same at 136⅞, and the remainder at 138¼. What was his net gain, allowing ⅛% brokerage each way?

62. I bought stock at 28⅜% discount, received a dividend thereon of 6%, and then sold out at 31⅛% discount, netting a gain of $384 after paying my broker ⅛% for buying and the same for selling. What was the par value of the stock ?

63. A man bought 80 shares of N. C. R. R. stock for $5270, including brokerage at ¼%. At what % discount was the stock bought?

64. A capitalist drew a semi-annual dividend of 5¼% on his banking stock, and received $420. He afterwards sold 30 shares of that stock at 125⅜, and the remainder at 127⅞, paying a broker ⅛% for selling. What were the net proceeds of the sale?

296. The par value is always understood as the *normal* base in stock computations *when no other base is mentioned;* but in many problems, the investment, an *abnormal* base, is taken as the standard, in which case such abnormal base is always mentioned.

Rem. 1.—In the expression "8% stocks," "8% bonds," the base of 8% is understood to be the par value, since no other base is mentioned; but in the expression "stocks paying 8% *on the investment,*" the base of 8% is the investment mentioned.

Rem. 2.—When the rate % on par value and rate % on investment are both given in the same problem, the rate on par value should be considered as dollars of income per share (Rem. 3, **295**), and the rate on investment should be considered as the rate of this income.

EXAMPLES FOR PRACTICE.

1. At what price must 6% stock be purchased to realize an income of 8% on the investment?

Income per share ($6)÷rate of income on investment (8%) = investment per share ($75).

At what price must

2. 9% stock be bought to realize 12% on the investment?

3. 7% bonds be bought to secure 10% on the investment?

4. 6% stock be purchased to secure 4% on the investment?

5. 8% stock be purchased to secure 5% on the investment?

6. At what quotation must 5% stocks be bought to realize an income of 4% on the investment, brokerage for buying ¼%?

Income per share ($5)÷rate of income on investment (4%) = investment per share ($125).

Investment per share ($125)—brokerage per share ($¼) = quotation ($124¾).

If in each case the brokerage is ⅛%, at what quotation must

7. 7% stock be bought to realize 8% on the investment?
8. 3% stock be bought to realize 5% on the investment?
9. 8% stock be purchased to secure 10% on the investment?
10. 9% stock be purchased to secure 12% on the investment?
11. What % on investment is realized by buying 9% stock at 112½?

Income per share ($9)÷investment per share ($112.50) = rate of income on investment (8%).

What % on investment will be realized

12. By purchasing 4% stock at 80?
13. By purchasing 6% stock at 75?
14. By purchasing 8% stock at 120?
15. By purchasing 5% stock at 112⅜, brokerage ⅛% additional?

Income per share ($5)÷investment per share ($112⅜ + $⅛) = rate of income on investment (4$\frac{4}{9}$%).

16. By purchasing 7½% stock at 59½, brokerage ⅛% additional?
17. By purchasing 6% stock at 112, brokerage ⅛% additional?
18. By purchasing 5% stock at 139⅞, brokerage ⅛% additional?
19. What % stock can be bought at 80 to yield 5% on investment?

Investment per share ($80) × rate on investment (5%) = income per share ($4 = 4% on par, Rem. 3, **295**).

What per cent. stock can be bought

20. At 87½, and yield 8% income on the investment?
21. At 125, and yield 4% income on the investment?
22. At 60, and yield 5% income on the investment?
23. At 112¼, brokerage ¼% additional, to yield 8% on investment?

Investment per share ($112¼ + $¼) × rate on investment (8%) = income per share ($9 = 9%).

24. At 119⅞, brokerage ⅛% additional, to yield 5% on investment?
25. At 139¾, brokerage ¼% additional, to yield 8¼% on investment?

26. I bought 90 shares of stock at 125 and realized 6% per annum on my investment. What % dividend was annually declared thereon?

27. A gentleman paid $8960 for 112 shares of stock, and realized 5% on his investment when the annual dividends thereon were paid. What was the rate % of dividend received?

296

28. At what quotation must U. S. 4½% bonds be bought to enable the buyer to realize 3% on his investment, allowing ⅛% for brokerage?

29. If money is worth 3% per annum, what should be the quotation of stock which declares dividends of 5% regularly every year?

30. I realized 5% per annum on an investment in stock bought at 119⅞, brokerage ⅛%. What semi-annual rate of dividend was regularly declared thereon?

31. An investor bought sufficient 6% railroad bonds at 40% below par to secure a semi-annual income of $240. What was the amount of his investment, including ¼% brokerage?

32. I bought U. S. 4% bonds at 119⅞, brokerage ⅛% additional, and derive from the purchase a quarterly income of $300. How much did I invest?

33. A gentleman paid 86⅝ for canal stock paying 8% annual dividends, and received each year $480. What did the stock cost, including brokerage at ⅛%?

34. I sold $8600 par of 5% stock at 108⅛, and with the proceeds purchased 4% stock at 53⅞. What was my annual gain, ⅛% being charged for brokerage each way?

35. A steamboat company divided $14000 of its net earnings among its stockholders. What was the rate % of dividend, the capital stock being $350000? How much did a stockholder receive who held a certificate of 50 shares (par $25)?

36. I invested $7002.25 in 5% bonds bought at 5⅝% discount, brokerage ¼%. What is my annual income therefrom?

37. If stock actually worth $80 per share be watered by issuing a stock dividend of 10%, what is the actual value of the watered stock?

38. What is the value of a ground rent of $160 per annum, capitalized at 5%?

REM.—To capitalize a ground rent means to find a sum of money which, if placed at interest at the current rate % per annum, will produce the same income as the ground rent.

39. What is the value of a ground rent of $120 per annum, if capitalized at 6%?

40. Do I make a profit or suffer a loss by buying a $90 ground rent at a capitalization of 5% and afterwards selling it at a capitalization of 4½%, and how much?

41. I bought a ground rent of $80 per annum for $2000. At what rate % was the ground rent capitalized?

42. An investor purchased a ground rent for $6000. He afterwards sold it at a capitalization of 4%, and realized a gain of $1500. What annual income did the ground rent produce?

EXCHANGE.

297. Exchange is the system by which debts to persons in distant places are paid by means of written orders, called *Drafts* or *Bills of Exchange.*

REM.—Exchange is usually conducted through bankers or brokers, who buy commercial bills on distant cities and mail them for collection to their correspondents in those cities. Drafts or bills of exchange are then drawn on these correspondents for the whole or any required part of the sum thus placed to their credit, and sold to persons who have debts to pay in those cities.

298. A **Draft** or **Bill of Exchange** is an order written by one person (called the *drawer*) directing a second person (called the *drawee*) to pay a specified sum of money to a third person (called the *payee*), or to the payee's order.

REM. 1.—A *Foreign Bill of Exchange* is one in which the drawer and drawee reside in *different* countries; and a *Domestic* or *Inland Bill of Exchange* is one in which the drawer and drawee reside in the *same* country.

REM. 2.—Domestic or inland bills of exchange are usually called *drafts;* and are distinguishable as *sight drafts* when they are payable upon presentation, and as *time drafts* when they are payable at a certain time after the date of the draft, or after the date of its presentation for acceptance.

299. An **Acceptance** is an agreement by the drawee to pay the sum specified in the bill of exchange; and is denoted by writing the word "accepted" across the face of the instrument, with his signature; and if payable a given number of days *after sight,* adding the date of his acceptance.

REM. 1.—Bills of exchange are negotiable or non-negotiable upon the same conditions, and are subject to the same indorsements, and to the same protest if not paid at maturity, as promissory notes (see **267** to **269**, inclusive).

REM. 2.—The date of maturity of bills of exchange is ascertained in the same manner as that of notes (**271**); and, in the United States, excepting sight drafts, are allowed 3 days of grace, unless the phrase "without grace" is mentioned. In a few States and Territories, even sight drafts are allowed days of grace.

Rem. 3.—Time drafts if paid before maturity are discounted in the same manner as notes (**279**). In ascertaining the term of discount on time drafts, bankers sometimes estimate the full number of days during which they lose the use of the money advanced; that is, they include the days required for the return, by mail or express, of the money after the draft is paid; and if the draft be payable a certain time after sight, the time also that will intervene before the draft can reach the drawee for acceptance.

300. The **Face** or **Par** of a bill of exchange is the sum expressed therein in words (or both words and figures). It is the *original obligation* exclusive of interest, premium, discount, etc.

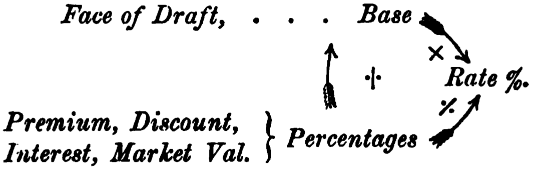
Rem. 1.—When exchange on a given place sells for *more* than its face or par value, exchange on that place is said to be *above par* or *at a premium;* and if it sells for *less* than its face, it is said to be *below par* or *at a discount.* The variation of exchange, whether at a premium or discount, between any two cities, will not exceed the cost of safely transmitting the money from one city to the other.

DOMESTIC EXCHANGE.

301. Domestic or **Inland Exchange** is exchange between two cities of the same country.

302. In domestic exchange, the face of the draft is regarded as the base; and the premium, discount, interest of a time draft, and market value (that is, prime cost of a draft if purchased, or gross proceeds if sold), as percentages. Hence,

General Formula.


Face of Draft, . . . Base
×
÷
Rate %.
%
Premium, Discount, Interest, Market Val.
Percentages

Rem. 1.—The rate % of market value of a sight draft is the rate of its face (100%) + the rate % of premium, or − the rate % of discount.

Rem. 2.—The rate % of market value of a time draft is the rate % of market value of a sight draft (Rem. 1) − the rate % of interest for the given time increased by 3 days for grace (Explanation, **253**).

Rem. 3.—Since drafts are bought and sold at their market value, the rate %

of market value will also be the rate % of prime cost if drafts are bought; or the rate % of gross proceeds if they are sold.

REM. 4.—Brokerage, if included, is usually estimated upon the *market value* of a draft; that is, upon its prime cost if purchased, or upon its gross proceeds if sold. Brokerage is added to the market value to find the gross cost of a purchase, or subtracted from the market value to find the net proceeds of a sale.

REM. 5.—Interest on the face of a time draft should be computed for the given time + 3 days for grace, at the legal rate of the State in which the draft is purchased if no special rate is designated, and deducted from the face of the draft. Hence, the cost of a time draft is less than the cost of a sight draft (by the interest on its face), if both are purchased at the same rate of exchange.

EXAMPLES FOR PRACTICE.

1. What is the cost of a sight draft of $960 at ⅝% premium?

Face of draft ($960) × rate of premium (⅝%) = premium ($6).
Face of draft ($960) + premium ($6) = cost of draft ($966).

What is the cost of a sight draft

2. Of $7200 at ½% premium?
3. Of $8500 at ¾% discount?
4. Of $3200 at ⅝% discount?
5. Of $1500 at ⅜% premium?

6. What is the cost of a 60-day draft of $1600 at ⅜% discount, interest 6%?

Face of draft ($1600) × rate of discount (⅜%) = discount ($6).
Interest of $1600 for 63 days at 6% = $16.80, discount for interest.
Face of draft ($1600) − total discount ($6 + $16.80) = cost ($1577.20).

7. What is the cost of a 30-day draft of $800 at ¾% premium, interest 6%?

Face of draft ($800) × rate of premium (¾%) = premium ($6).
Interest of $800 for 33 days at 6% = $4.40, discount for interest.
Face of draft ($800) + net premium ($6 − $4.40) = cost ($801.60).

What is the cost of

8. A 60-day draft of $3500 at ¾% premium, interest 6%?
9. A 30-day draft of $1800 at ½% premium, interest 8%?
10. An 18-day draft of $2500 at ¾% discount, interest 10%?
11. A 15-day draft of $750 at ⅝% premium, interest 5%?
12. A 30-day draft of $5000 at ⅛% discount, interest 6%?

13. What is the face of a sight draft which can be bought for $1492.50, if the course of exchange is ½% discount?

Cost of draft ($1492.50) ÷ rate % of cost (100% − ½%) = face ($1500).

What is the face of a sight draft which can be bought

14. For $829.15, if exchange is at ⅝% premium?

15. For $350.68, if exchange is at ⅜% discount?

16. For $7518.75, if exchange is at ¼% premium?

17. For $134.98, if exchange is at ¾% discount?

18. For $1210.50, if exchange is at ⅞% premium?

19. What is the face of a 30-day draft which can be bought for $4416.16, at ⅞% discount, interest 6%?

.33 ÷ 60 = .0055, rate of interest for 33 da. (Explanation, **253**).

Rate of face (100%) — rate of exchange disct. (⅞%) — rate of disct. for interest (.0055) = rate of cost (.98575).

Cost ($4416.16) ÷ rate of cost (.98575) = face of draft ($4480).

20. At ⅜% premium, what is the face of a 69-day draft which can be bought for $165.27, interest 10%?

.72 ÷ 36 = .02, rate of interest for 72 da. (Explanation, **253**).

Rate of face (100%) + rate of exchange premium (.00⅜) — rate of discount for interest (.02) = rate of cost (98⅜%).

Cost ($165.27) ÷ rate of cost (.98375) = face of draft ($168).

What is the face of a time draft which can be bought

21. For $120.84, if payable 30 days after sight; premium 1¼%, interest 6%?

22. For $1579.20, if payable 45 days after sight; discount ⅛%, interest 6%?

23. For $711.90, if payable 77 days after sight; premium ⅞%, interest 9%?

24. For $1179.90, if payable 60 days after sight; discount ⅝%, interest 6%?

25. A commission merchant of Charleston sold for a firm in Boston a consignment of English crockery. The sales amounted to $12240, and his commission 3%. With the proceeds, the commission merchant bought a 30-day draft on Boston at ⅝% discount and 6% interest. What was the face of the draft?

26. A commission merchant of San Francisco bought for a firm in Philadelphia a consignment of tea costing $15000. The rate of commission for buying was 2%. The firm in Philadelphia remitted a 15-day draft for the amount due, at ⅞% premium. What did the firm pay for the draft, rate of interest 6%?

FOREIGN EXCHANGE.

303. Foreign Exchange is that which is carried on between *different* countries.

Rem.—Foreign bills of exchange are usually drawn at sight (sometimes called "short exchange"), or at 60 days after sight (sometimes called "long exchange"). Three days of grace are usually allowed on foreign bills of exchange. The price of long exchange is less than that of short exchange by the interest on the face of the draft for the difference in time between the two dates of maturity.

304. The **Par of Exchange** is the established value of the monetary unit of one country, expressed in the currency of another.

Rem. 1.—The *intrinsic par* of exchange is the real or intrinsic value of coins estimated by the weight and purity of the metals of which they are composed. Thus, the intrinsic par of the sovereign or £ of Great Britain compared with the monetary unit of the United States (the gold dollar) is \$4.8665; the sovereign containing 113 grains of pure gold, and the dollar 23.22 grains, and $113 \div 23.22 = 4.8664\frac{1096}{1161}$, or practically 4.8665. The intrinsic value of the standard coins of foreign countries is estimated by the Director of the U. S. Mint, and on the 1st of January of each year is proclaimed by the Secretary of the Treasury.

Rem. 2.—The *commercial par of exchange* is the market value of the coin or currency of one country estimated by the requirements of trade between it and another country.

305. A **Set of Exchange** is a bill of exchange usually drawn in sets of three, of the same tenor and date, and so worded that when one of the set is paid, the others become void.

Rem.—The object of a set of exchange is to provide against loss in transmitting from one country to another. The bills are sent either by different routes, or by the same route at different dates. Sometimes only the first and second of the set are sent, and the third is preserved.

306. A **Letter of Credit** is a circular letter issued by a banking house to persons who wish to travel abroad, and addressed to bankers generally, and to its own correspondents particularly, in the different countries which the traveler is about to visit, requesting them to supply the traveler with money in sums to suit his convenience until the total amount specified in the letter has been obtained.

Rem.—The difference between a *bill of exchange* and a *letter of credit* is, that the former is paid at one designated place, at one fixed time, and in one stated amount, and the latter, at the option of the payee, is payable at several places, at different times, and in variable amounts.

307. The **Rate of Foreign Exchange** is the market value of the monetary unit of one country estimated in the currency of another.

Rem.—American exchange on Great Britain is quoted by giving the exchange value of £1 in dollars; on France, Belgium, and Switzerland by giving the exchange value of $1 in francs; on Holland by giving the exchange value of 1 guilder in cents; on Germany by giving the exchange value of *four* reichsmarks in cents.

308. To find the cost of a foreign bill of exchange.

ILLUSTRATIVE EXAMPLES.

1. How much must be paid for a bill of exchange on Liverpool of £320 17s. 6d., purchased at 4.86?

Solution.—£320 17s. 6d. = £320.875. If £1 of exchange is worth $4.86, £320.875 of exchange must be worth 320.875 times $4.86, or $1559.45+.

2. What is the cost of a bill of exchange on Paris of 1766.30 francs, bought at 5.19½?

Solution.—1766.30 ÷ 5.195 = 340. If 5.19½ francs of exchange are worth $1, 1766.30 francs of exchange are worth as many times $1 as 1766.30 fr. are times 5.19½ fr.; that is, 340 times $1, or $340.

3. What is the cost of a bill of exchange on Frankfort of 800 reichsmarks, purchased at 94⅞?

Solution.—($.94875 × 800) ÷ 4 = $189.75. If 4 marks of exchange are worth $.94⅞, 1 mark is worth ¼ of .94⅞; and 800 marks, 800 times ¼ of $.94⅞, or $189.75.

Rule.—*Divide the face of the bill by the quotation if both express similar monetary units; or multiply the quotation by the face of the bill if they express different monetary units. The result in either case will denote the market value in U. S. money.*

Note 1.—In German exchange, divide the result of the rule by 4.

Note 2.—Bills of exchange are sometimes bought and sold through brokers; in which case brokerage is computed on the market value (= prime cost if a purchase, or gross proceeds if a sale), and added to the market value to find the total cost to the purchaser, or deducted from the market value to find the net proceeds due the seller.

EXAMPLES FOR PRACTICE.

Find the cost. Find the cost.

Face of draft.	*Quotation.*	*Face of draft.*	*Quotation.*
1. £340	4.85¼	6. 750 guilders	40⅛
2. £350 2s. 6d.	4.85⅜	7. 1830 "	40¼
3. £730 12s. 6d.	4.85⅞	8. 1500 reichsmarks	94⅝
4. 3175.30 francs	5.18⅜	9. 1800 "	95¼
5. 418.75 "	5.19⅛	10. 3200.36 "	94⅞

11. A merchant of New York bought of Glenn & Co., Manchester, Eng., 840 yards cassimere at 16s. 8d. per yard; 500 yards tweed suitings at 12s. 6d. per yard; and paid by bill of exchange on Liverpool at 4.84¾. What was the cost of the bill of exchange?

12. I purchased, through a broker, a bill of exchange on London of £750 5s. at 4.85⅝. What was the total cost, brokerage ⅛%?

13. I sold, through a broker, a bill of exchange on Bremen of 815 reichsmarks at 95⅜. What were the net proceeds due me, brok. ⅛%?

309. To find the face of a foreign bill of exchange.

ILLUSTRATIVE EXAMPLES.

1. I paid $285 for a bill of exchange on Bremen bought at 95. What was the face of the bill?

SOLUTION.—Since the quotation denotes the market price of 4 marks, as many times 4 marks can be bought for $285 as $.95 is contained times in $285; that is, 300 times 4 marks, or 1200 marks.

2. I sold, through a broker, a bill of exchange on Havre and realized $3196 as net proceeds. What was the face of the bill, exchange being 5.19¼, and brokerage ⅛%?

SOLUTION.—$3196 ÷ (100% − ⅛%) = $3200, gross proceeds or market value. 5.19¼ fr. × 3200 = 16616 francs, face of bill.

RULE.—*Divide the market value by the quotation, if both express similar monetary units; or multiply the quotation by the market value, if they express different monetary units.*

NOTE 1.—If brokerage is included, divide the gross cost in purchasing a draft by 100% + rate of brokerage; or the net proceeds in selling a draft by 100% − rate of brokerage. The quotient in either case will be the market value of the draft.

NOTE 2.—**308** is applied when the remitter owes money which is payable where the payee resides, in which case the face of the bill should equal the debt, and all expenses of transmission should be defrayed by the remitter. **309** is applied when the remitter owes money which is payable where the remitter lives, in which case the total cost of the bill should equal the sum owed, and all expenses of transmission are deducted therefrom to find the face of the bill.

EXAMPLES FOR PRACTICE.

Find the face of a bill of exchange.

	Market value.	*Course of exchange.*		*Market value.*	*Course of exchange.*
1.	$6126.75	$4.86¼ to £1.	5.	$1489.50	41⅜¢ to 1 guilder.
2.	$2730	$4.87½ " "	6.	$181.44	40½¢ " "
3.	$800	5.20⅛ fr. to $1.	7.	$304.80	95¼¢ to 4 marks.
4.	$324	5.19¼ " " "	8.	$852.75	94¾¢ " "

What is the face of a bill of exchange which can be bought

9. For $5807.25, if quoted at 4.85 to the £, brokerage ⅛% ?

10. For $1834.80, if quoted at 5.19⅜ fr. to the dollar, brok. ¼%?

11. For $7584.32, if quoted at 40¾¢ to the guilder, brokerage $\frac{1}{16}$%?

12. For $8500, if quoted at 95⅛¢ to 4 marks, brokerage ⅛% ?

13. I bought, through a broker, a bill of exchange on Sheffield, and paid $3889.70. What was the face of the bill, the course of exchange being 4.85, and the brokerage ¼%?

14. I sold, through a broker, a bill of exchange on Bristol, Eng., at 4.85, and received $3580.02¾ as net proceeds. What was the face of the bill, brokerage ¼% ?

CUSTOM-HOUSE BUSINESS.

310. Duties or **Customs** are taxes levied by the general government on goods imported from foreign countries, to produce a revenue for the support of the government, to protect home industries, or for both purposes.

REM.—Duties are classified as *Ad Valorem* and *Specific.*

311. An **Ad Valorem Duty** is a duty levied upon imported goods *at a certain per cent.* of their market value at the time and place of purchase or of shipment; as, china ware, plain, 55%, ornamented, 60%; silks, 50%; jewelry, 25%; etc.

REM. 1.—By market value of goods is meant their invoice value after all discounts have been deducted and before any extra charges (such as commission, cartage, boxing, freight, etc.), have been added.

REM. 2.—Invoices are required to be made out in the weights, measures, and currency of the country from which the importation is made; the value in U. S. money of the foreign currency of the principal nations of the world being annually proclaimed for this purpose by the Secretary of the Treasury (**156**). If not thus proclaimed, invoices made out in a foreign currency must be accompanied by a consular certificate denoting its value in U. S. money.

REM. 3.—Duties are not computed on fractions of a dollar. If the cents are less than 50 they are rejected, if 50 or more the dollars are increased by 1.

312. A **Specific Duty** is a duty levied upon imported goods according to their weight or measure, and without reference to their value; as, camphor, 5¢ per pound; linseed oil, 25¢ per gallon; hemp carpeting, 6¢ per square yard; bituminous coal, 75¢ per ton, etc.

REM. 1.—In computing specific duties the ton is estimated at 2240 lb., the hundredweight at 112 lb., and the quarter at 28 lb.

REM. 2.—Some goods are subject to both a specific and an ad valorem duty; as, Axminster carpets, 45¢ per square yard and 30% ad valorem; cigars, $2.50 per pound and 25% ad valorem, etc.

313. **Allowances** are deductions made in estimating specific duties, and are distinguished as *Tare*, *Breakage*, and *Leakage*.

REM. 1.—*Tare* is a deduction from the gross weight of goods, allowed for the *weight* of the cask, box, bag, etc., in which they are contained.

REM. 2.—*Breakage* is a deduction allowed for the *waste* of liquids imported in bottles.

REM. 3.—*Leakage* is a deduction allowed for the waste of liquids imported in casks or barrels.

314. A **Tariff** is a list or schedule of goods, and the rates of duty imposed by law on the same.

315. Ad valorem duties are computed according to the principles of percentage, the market value at the place of purchase or export (reduced to U. S. money) being regarded as the base, and the ad valorem duty as the percentage.

ILLUSTRATIVE EXAMPLE.

What is the duty upon 800 yards of Brussels carpeting 27 inches wide, and invoiced at 11.25 francs per yard, the rate of duty being 30 cents per square yard and 30% ad valorem?

SOLUTION.

$.193 × (11.25 × 800) = $1737.

$1737 × .30 = $521, ad valorem duty.

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27 in. = $\frac{27}{36}$ = $\frac{3}{4}$ yd., width.

$\frac{3}{4}$ × 800 = 600 sq. yd.

$.30 × 600 = $180, specific duty.

$521 + $180 = $701, combined duty.

EXPLANATION.—Multiply 11.25 fr. by 800 to find the cost at the place of purchase (9000 fr.); this result by the custom-house value of 1 franc as shown in **156** ($.193) to find the dutiable value in U. S. money ($1737), and this result by the rate of ad valorem duty (30%) to find the ad valorem duty ($521).

Multiply the width of the carpet ($\frac{3}{4}$ yd.) by its length (800 yd.) to find the number of square yards (600), and this result by the specific duty per sq. yd. (30¢) to find the specific duty ($180). Add the ad valorem duty ($521) to the specific duty ($180) to find the total duty ($701).

RULE.—AD VALOREM DUTY.—*Reduce the net invoice price to U. S. money* (**156**)*; deduct all allowances* (**313**)*; and multiply the remainder by the rate of ad valorem duty.*

SPECIFIC DUTY.—*Deduct the total allowance from the total quantity specified in the invoice; reduce the remainder to the denomination on which the rate of specific duty is given, and multiply the result by the rate of specific duty.*

NOTE.—By net invoice price is meant the net cost or market value of the importation at the place of purchase (Rem. 1, **311**).

EXAMPLES FOR PRACTICE.

What is the ad valorem duty upon an importation invoiced

1. At £340 12s. 6d., allowing 10% for breakage, rate of duty 30%?
2. At 15378.40 marks, allowing 5% for leakage, rate of duty 15%?
3. At 9506.35 guilders, allowing 2% for tare, rate of duty 50%?
4. At £835 15s., rate of duty 10%?
5. At 8195 lira, allowing 8% for breakage, rate of duty 25%?
6. At 927.75 francs, if the rate of duty is 75%?
7. At 3500 yen, if the rate of duty is 12%?
8. At 475 pesos (Cuba), if the rate of duty is 40%?

315

What is the specific duty upon an importation of

9. 7835 sq. yd. of dress goods, rate of duty 25¢ per sq. yd.?

10. 300 tons of medicinal roots, rate of duty 1¢ per pound?

11. 1200 tons guano, rate of duty 75¢ per ton?

12. 30 doz. bottles wine, breakage 10%, duty $2.50 per doz.?

13. 650 gallons brandy, leakage 2%, rate of duty $1.50 per gal.?

14. 160 gross bottles, breakage 5%, rate of duty 10¢ per doz.?

15. A merchant in Paris consigned to a firm in Boston 3500 yards of 20-in. bunting, the market price of which in Paris, at the time of shipment, was 1.15 francs per yard. What was the duty at 10 cents per square yard and 35% ad valorem?

16. At 50% ad valorem, what is the duty upon an importation of 200 dozen kid gloves, invoiced at 73 francs per dozen?

17. What is the duty upon a consigment of watches, invoiced at £3560 10s., at 20% ad valorem?

18. If subject to a specific duty of $2 per gallon, what must be the selling price per gallon of an importation of 500 gallons French cognac to enable the importer to realize 15% profit, if the net invoice price is 18 francs per gallon, and the charges other than duty aggregate 95 francs in France, and $50 in the United States, the course of exchange on Paris being 5.20 francs to the dollar?

19. At 30% ad valorem, what is the duty upon an oil painting the net invoice of which is 3200 guilders?

20. At 1¼ cents per pound, what is the duty upon 3 T. 2 cwt. 1 qr. of cast-iron stove plates?

21. At 65 cents per cubic foot, what is the duty upon a block of marble 8 ft. × 2 ft. × 18 in.?

22. What is the duty upon an importation of 1500 yards flannel weighing 350 pounds net, and valued at 60 cents per yard, the rate of duty thereon being 24 cents per pound and 35% ad valorem?

23. A merchant of Baltimore imported from Sheffield a lot of pocket knives invoiced as follows: 50 doz. at 12s. 6d. per doz.; 25 doz. at 17s. 6d. per doz.; 15 doz. at £1 5s. 6d. per doz.; 20 doz. at £1 12s. 6d. per doz.; 8 doz. at £2 5s. per doz.; 15 doz. at £2 10s. per doz.; and 9 dozen at £3 12s. per doz. The charges in Sheffield amounted to £3 10s. What was the duty upon the importation at 50% ad valorem? What was the cost in Baltimore of remitting a bill of exchange in payment, the course of English exchange being 4.86½, brokerage ⅛% additional?

TAXES.

316. A **Tax** is a sum of money levied on the person, property, or income of an individual for the support of the government or for other public purposes.

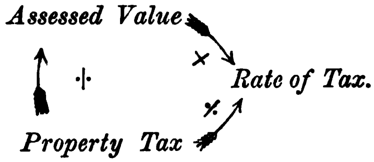
REM. 1.—A *Property Tax* is a sum levied on real estate or personal property.

REM. 2—A *Capitation* or *Poll Tax* is a sum levied, irrespective of property, on the person of every adult male citizen not specially exempt by law.

REM. 3.—A *License Fee* is a sum exacted by a city, state, or general government for the privilege of conducting a specified business.

317. Property taxes may be computed according to the principles of percentage, the assessed value of the property being regarded as the base, and the property tax as the percentage. Hence,

GENERAL FORMULA.—



EXAMPLES FOR PRACTICE.

1. The rate of taxation in a certain city is $1.15 on the $100. What tax should be paid by a person whose property is worth $18654, and is assessed at ⅔ of its value?

SOLUTION.—⅔ of $18654 = $12436, assessed value. $1.15 on $100 = $.0115 on $1. Hence, $.0115 × 12436 = $143.01+, required tax.

What is the tax, if the assessed value of the property is

2. $7200, and the rate of taxation ⅜%?

3. $9800, and the rate of taxation 12½ mills on the dollar?

4. $25300, and the rate of taxation 9 mills on the dollar?

5. $8758, and the rate of taxation $1.18 on $100?

6. $2940, and the rate of taxation 95 cents on $100?

7. I have property worth $19600, which was assessed at ¾ of its value. What was my entire tax at 7½ mills on the dollar, including 3 polls at 75 cents each?

8. My real estate was assessed at $7500, and my personal property $12500. What amount of tax did I pay thereon at $1.18 per $100, having been allowed a discount of 3% for prompt payment?

9. What is the rate of taxation in cents on $100, if the assessed valuation of property is $1875, and the total tax $27.50, including 1 poll at $1.25?

SOLUTION.—$27.50 − $1.25 = $26.25, property tax. $26.25 ÷ $1875 = $.014 on $1 = $1.40 on $100.

Find the rate of taxation in mills on $1 if the assessed valuation is

10. $514000, and the gross tax $6233, including 130 polls at 50¢.

11. $15000, and the gross tax $137.25, including 3 polls at 75¢.

12. $153387200, and the total property tax $169259.20.

Find the rate of taxation in cents on $100 if the assessed valuation is

13. $34000, and the total property tax is $469.20.

14. $1275000, and the total property tax is $12495.

The collector's commission is computed upon the gross amount of the tax collected. Hence, the net or available tax ÷ (100% − rate % of collector's commission) = gross amount of tax to be levied.

15. $78000, the net tax $837.33, and the collector's commission 5%.

16. $187500, the net tax $2793, and the collector's commission 2%.

17. The assessed value of real estate in a town is $986000, and of personal property $450000. The sum required by taxation is $12145.63 net, after allowing 2% for collection. What should be the rate of taxation on the entire property, if the number of polls be 375, and the tax for each poll 50 cents?

18. The cost of certain public improvements in a town is estimated at $4578.40; and to raise this sum, $1475000 worth of property is subject to taxation. The preceding estimate does not include the collector's commission of 3%. How much on $100 will each property holder be required to pay? What will be the tax of B, whose property is valued at $8300?

19. The assessed valuation of the property of a village is $832000. The estimate of corporate expenses includes $1500 for schools, $1600 for streets, $1200 for salaries and commissions, and $692 for sundry other expenses. What will be the rate of taxation expressed as mills on the dollar? What taxes will be required of A, whose real estate is assessed at $8000, and personal property at $450?

INSURANCE.

318. Insurance is a contract by which one party, for a stipulated sum, agrees to indemnify another for a specified loss or damage, if sustained within a given time.

Rem. 1.—Insurance is of two kinds: *Personal Insurance*, or indemnity to be paid in case of sickness, accident, or death of the person insured; and *Property Insurance*, or that which indemnifies the owner of real estate, merchandise, or other property for its loss or damage by fire, water, wind, or other specified casualty.

Rem. 2.—The *Insurer* or *Underwriter* is the person or company that agrees to indemnify for loss. The *Insured* is the person protected from loss. A *Policy* is a contract between the insurer and insured. The *Term of Insurance* is the time for which the property or person is insured.

319. The **Premium** is the sum paid by the insured to the insurer for the risks which the latter assumes.

Rem.—The premium is a certain rate per cent. of the amount of insurance, as ½%, ¾%, etc. Such rates are usually expressed by giving the cost of $100 of insurance; as 50 cents per $100, 75 cents per $100, etc.

FIRE AND MARINE INSURANCE.

320. Fire Insurance is a contract by which one party, for a stipulated payment, agrees to indemnify another for any loss or damage to a described piece of property, if caused by fire within a given time.

Rem. 1.—Fire insurance companies are also liable for loss or damage caused by water, when applied for the purpose of extinguishing a fire; and for the loss of buildings which have been torn down or otherwise damaged or destroyed to prevent a fire from spreading.

Rem. 2.—Fire insurance companies do not usually insure property for its *full* value, that the party insured may be interested in its protection. They also reserve the privilege of rebuilding or repairing damaged property, if it be to their interest.

Rem. 3.—In an ordinary fire policy, if the loss is only *partial*, the insurance company promises to pay the full value of the property destroyed, or the full depreciation of the property damaged, if it does not exceed the sum covered by insurance; but if insured in two or more companies, each company will pay its *pro rata* share of the loss.

321. Marine Insurance is a contract wherein the insurer, for a stipulated payment, agrees to indemnify the insured for any loss or damage to a certain vessel, cargo, or freight, by storms or other perils or risks of navigation to which it may be exposed during a particular voyage, or within a specified time.

Rem. 1.—*Transit Insurance*, or insurance of merchandise during shipment, is a form of marine insurance, whether the transportation be effected by rail or water route, or both; and so is *Inland Insurance*, or insurance on steamboats or other vessels navigating rivers, lakes, or other inland waters.

Rem. 2.—Marine policies contain an "average clause," under the terms of which the indemnity to be paid for a *partial loss* of property *partially insured* will be such a part of the loss, as the sum covered by insurance is of the full value of the property. Thus, if a vessel valued at $60000 is insured for one-half its value ($30000), and is *totally* destroyed, the owners will be entitled to the full amount covered by insurance ($30000); but if it is damaged to the extent of $8000, the owners will be entitled to only one-half of the partial loss, or $4000.

Rem. 3.—*Salvage* is a compensation allowed persons for their voluntary assistance in saving a ship, cargo, or other property from perils of fire or water.

322. Short Rates are certain rates of premium which are charged by companies when the period of insurance is less than one year.

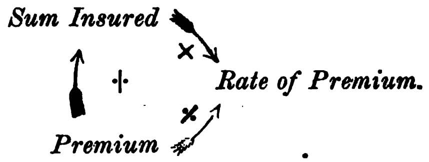
Rem.—It often happens that goods are insured for short periods of time, especially consignments. The rates for such periods are taken from the *Short Rate Tables*, which are higher, proportionately, than the rates for one year.

323. To **Cancel a Policy** is to annul the contract between the insurance company and the party insured.

Rem. 1.—When a policy is canceled before the expiration of the time for which it was issued, a *Return Premium* is paid by the insurance company to the party insured. When canceled at the instance of the insurance company, the return premium will be such a part of the entire premium, as the unexpired term is of the whole term; but when canceled at the request of the policy-holder, the return premium will be what is left of the total premium after deducting the "short rate premium" for the time the policy had been in force.

Rem. 2.—After a partial loss has been paid, the return premium will be such a part of the entire premium as the remainder of the policy (after deducting the partial loss paid) is of the entire or original policy.

324. Computations in fire and marine insurance are made in accordance with the principles of percentage; the sum insured being regarded as the base, and the premium as the percentage. Hence, FORMULA.—



REM.—The rate of premium is usually called rate of insurance.

EXAMPLES FOR PRACTICE.

1. A merchant insured his warehouse worth $10800 at $\frac{3}{5}$ of its value. What was paid for premium if the rate of insurance was $\frac{5}{8}$%?

Sum insured ($\frac{3}{5}$ of $10800) × rate of premium ($\frac{5}{8}$%) = premium (?).

2. The stock of a manufacturing company was insured for $75000 at the following rates: $\frac{1}{2}$ of the stock at $\frac{5}{8}$%, $\frac{2}{3}$ of the remainder at $\frac{3}{4}$%, and the remainder at $\frac{7}{8}$%. What was the total premium paid?

3. A stock of goods valued at $45000 was insured in the Eagle Fire Insurance Company for $10000, in the Manhattan F. I. Co. for $15000, and in the Globe F. I. Co. for $9000. If the stock was damaged by fire to the extent of $18000, what proportion of the loss should be paid by each company (Rem. 3, **320**).

4. A store and contents were insured as follows: $10000 in the Home Insurance Co., $6000 in the Essex Insurance Co., and $8000 in the Howe Insurance Co. A partial loss of $6000 having been caused by fire, how much should each company contribute?

5. A ship worth $75000 was insured for $20000 in one insurance company at $\frac{3}{4}$% premium, and for $35000 in another at $\frac{7}{8}$% premium. She was afterwards damaged by a storm to the extent of $3000. What insurance did the owners of the ship receive from each company?

6. A vessel worth $30000 was insured for $5000 in each of four companies. What insurance would its owners receive from each company if the vessel sustain a partial loss of $8000? If the vessel be totally destroyed?

7. A stock of goods was insured for one year for $12000 at $\frac{5}{8}$%. Six months thereafter the policy was canceled at the request of the insured. What was the return premium, the short rate for six months being $\frac{3}{8}$% (Rem. 1, **323**)? What would have been the return premium if the policy had been canceled at the request of the underwriters?

324

8. I paid $175 premium on a schooner worth $25000, which was insured at ⅘ of its value. What was the rate % of insurance? What was the rate of insurance in cents on the $100?

Premium ($175) ÷ sum insured (⅘ of $25000) = rate of premium (?).

9. An insurance company received $120 premium on property insured for $16000. If a building be insured for $40000 in another company at the same rate, what will be the premium on the latter?

10. A marine insurance company received $484.50 for insuring a vessel worth $80000 at ⅘ of its value. What was the rate % of insurance if $4.50 was charged for the policy and survey?

11. The stock of a wholesale notion house is insured in several companies for $12000, and is damaged by water, while extinguishing a fire, to the extent of $960. What % of its risk should be paid by each company? What amount should be paid by a company which had a risk of $5000 upon the stock?

12. For what sum is a house insured if the premium paid is $75, and the rate of insurance ⅝%?

Premium ($75) ÷ rate of premium (⅝%) = sum insured (?).

13. I paid $19.20 for having my house insured, the rate of insurance being 60 cents per $100. For what sum was my house insured?

14. A man owns ⅘ of a steamboat, and insures ¾ of his interest at 1¼%, paying $112.50 premium. What was the value of his interest in the boat? If, before the policy lapses, the boat should be damaged by fire to the extent of $4000, what sum will he receive from the insurance company?

15. The invoice price of a shipment of goods is $7930. The shipper wishes to have them insured at such a sum as will cover both the invoice price and the premium in case of loss. What should be the sum insured, if the rate of insurance is ⅞%?

Value ($7930) ÷ 100% − rate% of insurance (99⅛%) = sum to insure which will include both value and premium (?).

This kind of insurance is not applicable to real estate or other risks in which insurance for a greater sum than the value of the property will offer an incentive to negligence in its protection. It is allowed in transit and some other kinds of marine insurance.

16. The value of a certain consignment is $711.72. For what sum must the shipper have it insured to obtain both the value of the consignment and the premium thereon if it should be totally lost, the rate of insurance being $1.15 per $100?

17. The value of a schooner's cargo is $8372.50, and the owner wishes to insure it for a sum which will cover the value of the cargo and the cost of insurance. If the rate of insurance is 1½%, and the cost of the policy $1.75 additional, what should be the sum insured?

18. A ship is insured for enough more than its value to include the cost of insurance at 1¼%, and $6 for the policy and survey. If the ship is worth $51350, what should be the sum insured?

RATIO AND PROPORTION.

325. Ratio is the relative value of one quantity as compared with another quantity of the same kind.

REM. 1.—The *ratio* of 15 ft. to 5 ft. means the *relative value* of 15 ft. as compared with 5 ft., and this ratio or relative value is found by dividing the first of these quantities by the second. Thus, 15÷5 = 3.

REM. 2.—The two quantities compared, taken separately, are called *terms* of the ratio; and taken together, are called a *couplet.*

REM. 3.—The left-hand term of a couplet is called its *antecedent,* and the right-hand term its *consequent.*

REM. 4.—The *symbol of ratio* is the sign of the operation (division) with the horizontal line omitted (:). It is placed between the antecedent and consequent, means *divided by*, and is read *is to.*

326. A Simple Ratio is a ratio of two quantities; as $6 : $2.

REM. 1.—The learner should discriminate between a ratio and its mere expression. Thus, $6 : $2 is simply the expression of the ratio, the ratio itself being the resultant quotient (3).

REM. 2.—The terms of a ratio are usually *concrete.* If concrete, they should be reduced to the same denomination, if necessary. The ratio is *abstract.*

327. A Compound Ratio is the product of two or more simple ratios; as $\begin{array}{c}\$8 : \$4\\ \$3 : \$2\end{array}$. It is read, abstractly, 8 times 3 is to 4 times 2.

328. A Proportion is an expression of equality between two ratios.

Rem. 1.—The *symbol* of proportion is the double colon (::); and it is placed between two ratios to show that they are equal. The sign of equality is often used as a substitute for the symbol of proportion.

Rem. 2.—*Simple Proportion* is the expression of equality between two simple ratios. Thus, 8 lb. : 24 lb. :: 5 da. : 15 da. is a simple proportion, consisting of the two simple and equal ratios (8 lb. : 24 lb.) and (5 da. : 15 da.) united by the symbol (::). It may be read, 8 lb. are to 24 lb. as 5 da. are to 15 da.; or, the ratio of 8 lb. to 24 lb. equals the ratio of 5 da. to 15 da. This proportion is sometimes written $\frac{8}{24} = \frac{5}{15}$.

Rem. 3.—*Compound Proportion* is the expression of equality between two ratios, one or both of which are compound. Thus $\left\{\begin{matrix} 12 \text{ da.} : 8 \text{ da.} \\ 5 \text{ hr.} : 6 \text{ hr.} \end{matrix}\right\}$ :: \$150 : \$120 is a compound proportion. It is usually read, 12 times 5 is to 8 times 6 as \$150 is to \$120.

Rem. 4.—The *extremes* of a proportion are its first and fourth terms, that is, its two outside terms; and the *means* of a proportion are its second and third terms, that is, its two inside terms. Thus, in the proportion, 8 lb. : 3 lb. :: 64¢ : 24¢, 8 lb. and 24¢ are the extremes, 3 lb. and 64¢ the means.

329. The product of the extremes of a proportion equals the product of its means. Thus, in the proportion, 8 lb. : 3 lb. :: 64¢ : 24¢, the product of the extremes ($24 \times 8 = 192$) equals the product of its means ($3 \times 64 = 192$). Hence,

Principles.—1. *If the product of the extremes of a proportion be divided by one of its means, the quotient will be its other mean.*

2. *If the product of the means of a proportion be divided by one of its extremes, the quotient will be its other extreme.*

330. To find an omitted term of a proportion.

ILLUSTRATIVE EXAMPLE.

Find the omitted term in the proportion, 9 yd. : 12 yd. :: \$6 : (\$?).

Explanation.—Applying **329**, take the product of the two given means ($12 \times 6 = 72$) as the product of the two extremes. Hence, divide the known product of both extremes (72) by the given extreme (9), and the quotient must equal the other extreme (8).

Solution.

$12 \times 6 = 72$,

$72 \div 9 = 8$. Hence,

9 yd. : 12 yd. :: \$6 : \$8.

EXAMPLES FOR PRACTICE.

Find the omitted term of the following proportions:

1. 4 oz. : 9 oz. :: ? : 25¢.
2. $3 : ? :: 9 yd. : 12 yd.
3. ? : 21 mi. :: $4 : $12.
4. 10 gal. : 9 gal. :: $30 : ?.
5. $\frac{3}{4}$ bu. : $\frac{3}{5}$ bu. :: ? : $\$\frac{4}{5}$.
6. 9.4 bbl. : 10.25 bbl. :: $4.70 : ?.
7. £4 : £36 :: ? : 18 doz.
8. ? : $65 :: 9 men : 45 men.
9. $\left\{\begin{matrix}30 \text{ men} : 5 \text{ men} \\ 4 \text{ da.} : 8 \text{ da.}\end{matrix}\right\} :: \$18 : ?.$
10. $\left\{\begin{matrix}75 \text{ rd.} : 25 \text{ rd.} \\ 54 \text{ bu.} : 9 \text{ bu.}\end{matrix}\right\} :: ? : 11 \text{ men.}$
11. $\left\{\begin{matrix}3 \text{ mi.} : ? \\ 28 \text{ hr.} : 6 \text{ hr.}\end{matrix}\right\} :: \$160 : \$80.$
12. $\left\{\begin{matrix}9 \text{ da.} : 15 \text{ da.} \\ ? : 3 \text{ hr.}\end{matrix}\right\} :: \$28.80 : \$18.$

SINGLE RULE OF THREE.

331. The **Single Rule of Three** is a process of obtaining a result which depends upon the comparison of one pair of quantities of the same kind.

Rem.—The *Single Rule of Three* is applicable to problems having *three* given quantities, two of which are of the same kind and form a certain ratio, and it is required to find a fourth quantity which will form an equal ratio with the remaining quantity. For this reason it is also called Simple Proportion (Rem. 2, **328**).

ILLUSTRATIVE EXAMPLES.

1. If 50 bushels of wheat cost $48, what will 73 bushels of the same wheat cost?

SOLUTION.

$$\begin{array}{r|l} & \$48 \\ 50 & 73 \end{array}$$

$$(48 \times 73) \div 50 = \$70.08.$$

Explanation.—Since a *cost* is required, place the given cost ($48) at the right of a perpendicular line. There are now only two terms (50 bu. and 73 bu.) remaining to be arranged.

Since the cost of 73 bu. is required, and this required cost is *greater* than the given cost of 50 bu., *place the greater* of the two remaining terms (73 bu.) *at the right* of the line, and the less (50 bu.) on the opposite side.

Multiply the two right-hand terms ($48 × 73 = $3504), and divide the product by the left-hand term (50), obtaining $70.08 as the required cost.

Rem.—As a proportion, the required cost is to the given cost as the quantity of which the cost is required is to the quantity of which the cost is given; that is, $? : $48 :: 73 bu. : 50 bu. Since two means and one extreme are given, apply Prin. 2, **329**, to find the other extreme, obtaining $70.08.

2. If 300 acres of a field will produce 16500 bushels of corn, how many bushels of corn will 93 acres of the same field produce?

SOLUTION.

$$\begin{array}{r|l} & 165 \\ 1 & \cancel{16500} \text{ bu.} \\ \cancel{300} & \cancel{93} \\ & 31 \end{array}$$

$165 \times 31 = 5115$ bu.

Explanation.—Since bushels are required, place the given bushels (16500) at the right of the line.

Since bushels from 93 acres are required, and the required bushels must be *less* than the given bushels from 300 acres, *place the less* of the two remaining terms (93) at the right of the line, and the other term (300) on the opposite side.

Cancel and proceed with the uncanceled terms as in Ill. Ex. 1, obtaining 5115 as the required number of bushels. If 1 is obtained as the result of cancellation, omit it from the left-hand terms, and retain it in those on the right.

3. If 24 men can do a given piece of work in 9 days, in how many days can 18 men do it?

SOLUTION.

$$\begin{array}{r|l} 1 & \cancel{9} \text{ da.} \\ \cancel{18} & \cancel{24} \\ & 12 \end{array}$$

Ans. 12 da.

Explanation.—Since days are required, place the given days (9) at the right of the line.

Since days for 18 men to do the work are required, and the required days are *greater* than the given days for 24 men (for when *fewer* men are employed on a piece of work, the *greater* must be the time required to complete it), *place the greater* of the two remaining terms (24) at the right of the line, and the other term (18) on the left.

Canceling out all the terms, it is found that 12 is the only uncanceled factor. Therefore the required answer is 12 times 1 da. or 12 da.

Rule.—I. *At the right of a perpendicular line, place that one of the three given terms which is of the same kind as the required answer.*

II. *If, from the nature of the question, it is seen that the required answer will be greater than the term already written, place the greater of the two remaining terms at the right of the line.*

III. *If, however, it is seen that the required answer will be less than the term already written, place the less of the two remaining terms at the right of the line.*

IV. *After placing one of the two remaining terms at the right of the line, place the other term on the left.*

V. *Multiply the two right-hand terms, and divide the product by the left-hand term, first canceling factors which are common to the left-hand term and either of the right-hand terms.*

EXAMPLES FOR PRACTICE.

1. What is the cost of 13 pounds of sugar if 8 pounds cost 72 cents?

2. If 12 pounds of beef cost $1.80, how much will 9 pounds cost?

3. A farmer raised 1750 bushels of wheat on 70 acres of land. At the same rate how many bushels could he have raised on 87 acres?

4. If 50 bushels of wheat cost $75, what will 80 bushels cost at the same rate?

5. If I am charged $12.25 for 35 yards of goods, how much should I be charged for 47 yards of the same goods?

6. If a garrison consume 102 barrels of flour in 17 days, how many barrels of flour will it consume in one year?

7. What will 30 pounds of butter cost, if 18 lb. 3 oz. cost $5.82?

8. If 82 sheep cost $287, how many sheep at the same price will cost $52.50?

9. If $320 will accrue $9.60 interest, how much interest will $240 accrue in the same time and at the same rate %?

Sometimes, as in the preceding example, all of the three given terms express the same kind of units; but in such cases one of the three given terms will be found to agree more specifically with the required answer. Thus, *dollars of interest* are required, and the given dollars of interest ($9.60) will be the upper right-hand term. The two remaining terms specifically agree as *dollars of principal.*

10. What principal will accrue $1.59 interest, if $820 will accrue $5.30 interest in the same time and at the same rate %?

11. A received a dividend of $420 on $8400 of stock. What would he have received if he had owned $9300 of the same stock?

12. How long a shadow will be cast by a vertical staff 11 feet high, if at the same moment a staff 6 feet high cast a shadow 4 feet long?

13. A pole 18 feet high casts a shadow 6 feet long. How high is a tree which, at the same time, casts a shadow 13 feet long?

14. How many men can do the same work in 20 days that 45 men can do in 16 days?

15. A contractor agreed to build a house in 45 days, and estimated that 6 men would be sufficient to do the work. Should he desire to complete the job in 30 days, how many men must he employ?

16. A invested $5400 in a partnership, and B $3600; and the contract between them specified that each partner's gain or loss should be proportional to his investment. If the total gain is $1500, what should be each partner's share?

17. A merchant sells $3600 worth of goods and gains $648. At the same % of profit, what amount of goods must he sell to gain $1620?

18. What should be the tax on a farm assessed at $32500, if a farm assessed at $7500 is taxed $18.21?

19. A merchant sold an invoice of goods at 15% profit and gained $450. What % would he have gained by selling at a profit of $750?

20. If A lend B $800 for 18 days, how long should B lend A $300 to offset his claim for interest?

21. If the rate of freight is 18¢ per bushel for 1326 miles, what will be the proportionate rate for 884 miles?

22. If a vessel is provisioned for a crew of 30 men and a voyage of 168 days, how long a voyage can she make with the same provisions if the crew be increased to 35 men?

DOUBLE RULE OF THREE.

332. **The Double Rule of Three** is a process of obtaining a result which depends upon the comparison of two or more pairs of quantities of the same kind.

REM.—If the terms of a problem in Double Rule of Three are arranged in the form of a proportion, the first couplet will contain as many simple ratios as there are pairs of given quantities of the same kind. For this reason it is also called *Compound Proportion* (Rem. 3, **328**).

ILLUSTRATIVE EXAMPLE.

If 4 men can earn $84 in 7 days by working 10 hours a day, how many dollars can 6 men earn in 10 days by working 9 hours a day?

SOLUTION.

$$\begin{array}{r|l} & \$\cancel{84}\ \ \overset{3}{\cancel{12}} \\ \cancel{4} & 6 \\ \cancel{7} & \cancel{10}\ 1 \\ \cancel{10} & 9 \end{array}$$

$\$3 \times 6 \times 9 = \$162.$

EXPLANATION.—Since dollars are required, place the given dollars ($84) at the right of the perpendicular line.

Since dollars are required that six men can earn, and 6 men can earn more dollars than 4 men, place the greater of these two similar terms (6 men) at the right of the line, and the other (4 men) at the left.

Since dollars are required that 6 men can earn in 10 days, and 6 men in 10 days can earn more dollars than in 7 days, place the greater of these two similar terms (10 days) at the right of the line, and the other (7 days) at the left.

Since dollars are required that 6 men can earn in 10 days by working 9 hours, and 6 men in 10 days by working 9 hours will earn less dollars than by working 10 hours, place the less of these two similar terms (9 hours) at the right of the line and the other (10 hours) at the left.

Cancel all factors common to terms which are on opposite sides of the line, and divide the product of the uncanceled numbers on the right ($3 × 6 × 9) by the product of the uncanceled numbers on the left (1), obtaining $162 as the required answer.

RULE.—I. *At the right of a perpendicular line, place that given term which is of the same kind as the required answer.*

II. *With the remaining terms form pairs of the same kind, and place the terms of each pair at the right and left of the line by II, III and IV of the Single Rule of Three, as if the required answer depended on that pair alone.*

III. *Cancel all factors which are common to terms on opposite sides of the line, and divide the product of the uncanceled numbers at the right of the line by the product of the uncanceled numbers at the left.*

NOTE.—In deciding upon the position of the terms of each pair, uniformly compare the term in the question with the similar term in the statement or supposition.

EXAMPLES FOR PRACTICE.

1. A farmer's expenses for feeding 27 horses for 17 weeks were $91.80. What would be his expenses for feeding 23 horses for 13 weeks?

2. The freight on 12250 pounds of goods was $73.50 for 240 miles. A *pro rata* sum was charged on 28250 pounds of the same class of goods for 90 miles. What was paid for freight for the shorter distance?

3. If from 650 reams of paper 4000 copies of a book containing 420 pages can be printed, how many reams will be required to print 7000 copies of a book containing 528 pages of the same size?

4. If $600 will accrue $9.60 interest at 6% per annum in 3 months and 6 days, at what rate per annum will $240 accrue $7.20 interest in 4 months?

5. An excavator agreed to dig two cellars at the same price per cubic yard, and obtained $120 for one which was 40 feet long, 30 feet wide, and 9 feet deep. The other cellar was 36 feet long, 27 feet wide, and 12 feet deep. How much was received for digging the latter?

6. What should be the cost of pasturing 73 head of cattle for 7 weeks, if the cost of pasturing 95 head of cattle for 9 weeks is $513?

7. If $840 will accrue $19.60 interest in 105 days at 8% per annum, in what time will $576 accrue $1.84 interest at 5%?

8. A contractor purchased a pile of stone 24 feet long, 12 feet high, and 9 feet wide for $120, and paid $300 for another pile of the same quality of stone which was 12 feet wide and 36 feet long. How high was the latter pile?

9. If 300 men can perform a certain work in 45 days by laboring 10 hours a day, how many hours a day will 600 men have to labor to perform the same work in 25 days?

10. If 20 men can earn $1260 in 15 days by working 12 hours a day, how many dollars can 17 men earn in 23 days by working 10 hours a day?

11. How many half-barrels of flour will supply a gang of 9 laborers for 20 weeks, if 3 barrels will supply a gang of 5 laborers for 15 weeks and 3 days?

12. If 7 men can plow a field 144 rods long and 105 rods wide in 9 days by using horses whose working gait is 5 miles per hour, in how many days can 5 men plow a field 216 rods long and 60 rods wide with horses whose gait is 4 miles per hour?

BANKRUPTCY.

333. A **Bankrupt** or **Insolvent** is a person who fails in business or is unable to meet his pecuniary obligations.

REM. 1.—The *assets* of a bankrupt are his entire property, including debts due to him by others.

REM. 2.—The *liabilities* of a bankrupt are the pecuniary obligations due by him to others.

334. A **Schedule** or **Statement** is a list setting forth the assets and liabilities of the bankrupt, containing the name of each of his creditors and debtors, their places of business or residence, and the sum due to or from each.

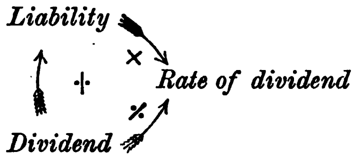
REM.—A *debtor* of a bankrupt is a person or firm who is indebted to the bankrupt; and a *creditor* is one to whom the bankrupt is indebted.

335. An **Assignee** is a person appointed to take charge of the property of a bankrupt for the purpose of converting it into cash, and, after deducting the expenses of the assignment, to pay such a proportion of the bankrupt's liabilities as the available assets will allow.

Rem.—A *dividend* is the proportion of available assets which is paid by the assignee to the creditors of a bankrupt.

336. Computations in bankruptcy may be made in accordance with the principles of percentage; the liability being regarded as the base, and the dividend as the percentage. Hence,

Formula.—



Rem.—Total liability × rate of dividend = total dividend to *all* creditors; and liability to one creditor × by rate of dividend = dividend to that creditor.

ILLUSTRATIVE EXAMPLE.

A merchant failed in business and rendered the following schedule: Assets—cash, $1346.10; real estate, $3578.64; personal accounts, $2125; merchandise on hand, $1517.36. Liabilities—promissory notes outstanding, $3021; due J. H. Baldwin & Co., $1673; D. S. Rogers, $1177; S. R. White, $6052. The expenses of the assignment were $221. How much should each creditor receive?

Solution.—$1346.10 + $3578.64 + $2125 + $1517.36 = $8567.10, total assets. $8567.10 − $221 = $8346.10, net assets. $3021 + $1673 + $1177 + $6052 = $11923, total liability.

$8346.10 ÷ $11923 = .70, or 70%, rate of dividend.

$1673 × .70 = $1171.10, dividend to Baldwin & Co.; $1177 × .70 = $823.90, dividend to D. S. Rogers; $6052 × .70 = $4236.40, dividend to S. R. White.

Rule.—I. *From the total assets deduct all expenses, and divide the remainder by the total liabilities. The quotient carried to hundredths will be the rate per cent. of dividend.*

II. *Multiply the amount of each creditor's claim by the rate per cent. of dividend.*

Note.—The same result may be obtained by the "Rule of Three" by taking the net assets and liability due each creditor as right-hand terms, and the total liability as the left-hand term (**331**).

EXAMPLES FOR PRACTICE.

1. J. H. Bell & Co. failed with liabilities amounting to $150000 and assets amounting to $93147. How much should each creditor receive on the dollar, and what sum was allowed J. E. Arnold & Bro., whose claim was $17314, if the expenses of settlement were $3147?

2. A coal firm failed with liabilities amounting to $49000. The assets were, cash, $2160; real estate, $16500; notes on hand, $2340. The expenses of the assignment were 2% of the assets. How much should Wright & Co. receive, whose claim against the firm was $12617?

3. G. W. Smith's claim against a bankrupt firm was $2400, and J. M. Owens's $1680. After the expenses of the assignment were deducted from the assets, there remained $6120. If the total liabilities were $8160, how much did Smith and Owens receive?

4. A lumber firm failed with assets amounting to $23072, and liabilities amounting to $56000. The firm owed Joel Manning & Co. $7327.50; M. J. Whiting & Co., $3725; and Robert French & Sons, $2375. How much did each of these creditors receive, if the expenses of bankruptcy were $672?

5. The Empire Knitting Mills Company failed with liabilities amounting to $274320, and assets amounting to $125912.88. If the expenses for settling were $2468.88 how much should W. A. Miles, J. R. Thomas, and W. G. Lewis receive, whose claims were respectively $34620, $15234, and $8750?

6. A firm failed with liabilities amounting to $52250, and with assets, exclusive of real estate, amounting to $2042.50. The assignee obtained for the real estate $31350, and deducted for settling the bankruptcy $475. W. H. Devon's claim against the firm was $7284; W. A. Heitmueller's, $6382; W. H. Patrick's, $5794; S. F. Lackland's, $4767; and J. K. Hopper's, $3964. How much did each of these creditors receive?

7. A bankrupt firm owed J. P. Jordan & Co. $1874; W. H. Henson, $675; T. A. Smith & Son, $385; and C. E. Sinclair, $284. The assets of the firm were $48700, and their liabilities $59048.75. How much did each of the above creditors receive, if the expenses of settling the bankruptcy were 3% of the assets?

8. The real estate of a bankrupt firm was sold by their assignee for $12000, and their goods for $6122. There were collected on notes due the firm $2107, and on personal accounts $2673. The total lia-

bilities of the firm are $27033, and the expenses of settling $734.94. How much on the dollar can be paid, and what should M. E. Whitney receive, whose claim is $6215?

EQUATION OF PAYMENTS.

337. Equation of Payments is the process of finding the date at which one payment can be made in settlement of two or more debts falling due at different dates, without loss to the debtor or creditor.

REM.—The date at which two or more debts can be equitably discharged by one payment is called the *average* or *equated date;* and the number of months or days before they can be so discharged, the *average* or *equated time.*

338. The **Focal Date** is any assumed date taken as a standard to which the dates of two or more payments may be compared to find their average date.

REM.—Though any date may be assumed as the standard of comparison, the most convenient focal date is the last day of the month which immediately precedes the date of the earliest item. Other good focal dates are the earliest or the latest of the several dates in the given account.

339. The **Term of Credit** is the time allowed for the payment of a debt, or of goods purchased on account.

REM.—The average time allowed for the payment of the sum of two or more debts due at different dates is called the *average term of credit.*

340. To find the average date for the payment of two or more debts due at different dates is an application of the following

PRINCIPLES.—1. *Money is worth its interest for the time it is used.*

2. *The loss of the use of money paid before it is due is offset by the gain in the use of an equal sum kept an equal time after it is due.*

3. *Assuming any common date of comparison, the prepayment multiplied by the time of prepayment should equal the after-payment multiplied by the time of after-payment.*

341. To find the average date for the payment of two or more debts contracted at the same time but due at different dates.

ILLUSTRATIVE EXAMPLE.

A merchant sold goods amounting to $1800, as follows: $400 on 3 months' credit, $600 on 5 months' credit, and $800 on 6 months' credit. What is the equated time for the payment of the whole?

SOLUTION.

Debts.	*Terms of Credit.*	*Use of $1 for*
400 ×	3 months	= 1200 months.
600 ×	5 "	= 3000 "
800 ×	6 "	= 4800 "
1800 ×	? months	= 9000 "

9000 months ÷ 1800 = 5 months.

EXPLANATION.—Assume the entire bill ($1800) to have been paid on the day of purchase. The buyer would then have been deprived of the use, as per terms of sale, of $400 for 3 mo., or of $1 for 1200 mo.; of $600 for 5 mo., or of $1 for 3000 mo.; of $800 for 6 mo., or of $1 for 4800 mo.; or, altogether, of $1 for 1200 + 3000 + 4800, or 9000 mo. To be deprived of the use of $1 for 9000 months is equivalent to being deprived of the use of 1800 times $1, or $1800, for $\frac{1}{1800}$ of 9000 months, or 5 months. Hence, the correct or equated time for paying the entire amount should be 5 months after the date of purchase.

RULE.—I. *Multiply each item by its term of credit, and divide the sum of the products by the sum of the items to find the average term of credit.*

II. *Add the average term of credit to the date of purchase to find the average date.*

NOTE.—Fractions of a month should be reduced to days; and fractions of a day should be discarded if less than ½, or considered an entire day if ½ or more.

EXAMPLES FOR PRACTICE.

Find the average term of credit of the following bills:

(1.)

Items.	*Terms of Credit.*
$700	5 months.
350	4 "
550	2 "
400	5 "

(2.)

Items.	*Terms of Credit.*
$1000	3 months.
2500	5 "
1500	4 "
4000	6 "

(3.)

Items.	*Terms of Credit.*
$800	4 months.
750	2 "
325	5 "
585	3 "

Find the equated date of payment of the following:

(4.)	(5.)
May 12, $150 on 30 days' credit.	June 20, $1200 on 90 days' credit.
" 400 " 90 " "	" 2400 " 30 " "
" 300 " 60 " "	" 1800 " 60 " "
" 500 " 30 " "	" 1500 " 90 " "

6. A wholesale merchant sold goods amounting to $3600, of which $1800 was to be paid in 6 months, $900 in 8 months, and the remainder in 9 months. What was the average term of credit?

7. An invoice of goods amounting to $1200 was purchased May 30, 1889, on the following terms: $600 payable in 30 days, $400 in 60 days, and $200 in 90 days. What is the equated date for the payment of the entire bill?

8. If I buy goods amounting to $1600, and agree to pay half in 3 months, and the remainder in 5 months, and afterward wish to pay the entire debt in one amount, when should the payment be made?

9. A plantation was sold for $18000 upon the following terms: one-third in cash, one-half of the remainder in 12 months, and the balance in 18 months, without interest. What is the equated time for the payment of the entire sum?

10. On July 15, 1889, an importer sold the following bills to a wholesale firm: No. 1, $500 on 30 days' credit; No. 2, $792 on 60 days; No. 3, $820 on 90 days; No. 4, $624 on 120 days. What is the equated time for the payment of the entire purchase?

11. On Sept. 18, 1889, a wholesale dealer sold the following merchandise: $718.25 on 60 days' credit, $239.70 on 30 days' credit, and $538.42 on 90 days' credit. What is the equated date for paying the sum of the three items?

342. To find the average date for paying two or more debts contracted at different dates, if no terms of credit are included.

ILLUSTRATIVE EXAMPLE.

J. M. Gaulden bought of Geo. W. Keller & Co., the following merchandise: May 16, 1889, $586; June 21, 1889, $375; July 10, 1889, $418. No credit having been allowed, what is the average date for paying the sum of the several bills?

FIRST SOLUTION.—*Product Method.*

Focal Date, April 30, 1889 (= May 0, 1889).

Dates.	*Items.*	*Days.* (*from focal date*)	*Products.* (*use for 1 day of*)	
1889, May 16,	$586 ×	16 =	$3516	= ($586 × 6 units).
			586	= (586 × 1 ten).
June 21,	375 ×	52 =	750	= (375 × 2 units).
			1875	= (375 × 5 tens).
July 10,	418 ×	71 =	418	= (418 × 1 unit).
			2926	= (418 × 7 tens).
	$1379 ×	(?) =	$58554	for 1 day.

58554 ÷ 1379 = 42 + (days after focal date). Hence,

Average date: 42 days after Apr. 30, 1889, or June 11, 1889.

EXPLANATION.—For its convenience as a basis from which to compute time, assume April 30, 1889, or its equivalent, May 0, 1889, as the required average date.

As the first item ($586) was not due until May 16 (16 days later than the assumed date), to have paid it on Apr. 30 would have deprived Gaulden of the use of $586 for 16 days, or of 16 times $586 for 1 day; as the second item ($375) was not due until June 21 (52 days after the assumed date), to have paid it on Apr. 30 would have deprived Gaulden of the use of $375 for 52 days, or of 52 times $375 for 1 day; and, similarly, to have paid the third item ($418) on Apr. 30 would have deprived Gaulden of the use of 71 times $418 for 1 day. Hence, to have paid all three of the items on Apr. 30 would have deprived Gaulden of the use of $58554 for 1 day, or of $1379 (which is $\frac{1}{42}$ of $58554) for 42 times 1 day, or 42 days.

The correct date at which Gaulden can pay his total indebtedness without loss to himself or Keller, is thus found to be 42 days after the assumed date (Apr. 30, 1889), or June 11, 1889.

PRODUCT RULE.—I. *Take as the focal date the last day of the calendar month preceding the date of the earliest item.*

II. *Multiply each item by the difference in days between the focal date and the date at which such item falls due; and divide the sum of the obtained products by the sum of the items.*

III. *The quotient will denote the number of days to count forward from the focal date to find the equated date.*

NOTE 1.—The date at which the *latest* item falls due is another convenient focal date much used by accountants. The quotient of dividing the sum of the products by the sum of the items will then denote the number of days to count *backward* from the focal date to find the equated date; because, in this case, the sum of the products will denote the number of days during which the buyer has withheld $1 *beyond* the equated date.

SECOND SOLUTION.—*Interest Method.*

Focal date, Apr. 30, 1889 (= May 0, 1889). Rate of Interest 6%.

Mo.	*Dates.*	*Items.*	*Interest.*				
0	May 16,	$586	$.977—,	int.	for	10 da.	($\frac{1}{6}$ of $5.86).
			.586	"	"	6 "	($\frac{1}{10}$ of $5.86).
1	June 21+1,	375	2.625	"	"	42 "	($.375 × 7).
			.625	"	"	10 "	($\frac{1}{6}$ of $3.75).
2	July 10+1,	418	4.18	"	"	2 mo.	($\frac{1}{100}$ of $418).
			.697—,	"	"	10 da.	($\frac{1}{6}$ of $4.18).
			.070—,	"	"	1 "	($\frac{1}{10}$ of $.697).
		$1379	$9.760				
			× 6				

Int. of $1379 for 6 da. = $1.379) $58.560 (42+ (days after focal date).

Average date: 42 days after Apr. 30, 1889, or June 11, 1889.

EXPLANATION.—Assume Apr. 30, 1889 (= May 0, 1889), as the date at which the entire amount should be paid; and to the left of the date of each item, write the number of complete calendar months between it and the assumed date. For convenience in calculating interest, suppose each of these intervening complete months to be 30-day months; and in correction, plus or minus each date with the excess over, or deficiency under, 30 days of each intervening complete month.

By paying the first item ($586) on Apr. 30, the buyer loses the use of $586 for 16 days (= 10 da. + 6 da.) which, at 6% per annum, is equivalent to a loss in interest of $.977— for 10 days and of $.586 for the remaining 6 days.

By paying the second item on Apr. 30, the buyer loses the use of $375 for the 31-day month of May and 21 days in June (= 1 30-day month and 22 days), which, at 6% per annum, is a loss in interest of $2.625 for 1 30-day month 12 days and of $.625 for the remaining 10 days.

By paying the third item on Apr. 30, the buyer loses the use of $418 for the 31-day month of May, the 30-day month of June, and 10 days in July (= 2 30-day months and 11 days), which, at 6% per annum, is a loss in interest of $4.18 for the 2 30-day months, of $.697 for 10 days, and of $.070 for the remaining 1 da.

If by paying the sum of the three items ($1379) on Apr. 30, 1889, the buyer loses the sum of the obtained interest ($9.760), the proper date for paying this sum ($1379) without loss of interest must be as many days after Apr. 30, 1889, as will enable $1379 to produce $9.760 interest.

At 6% per annum, the interest of $1379 for 1 day is $\frac{1}{6}$ of $1.379, or $.229⅚ (**243**). Hence, applying **252**, divide the given interest ($9.760) by this interest for 1 day ($\frac{1}{6}$ of $1.379), or to avoid fractions, divide 6 times the given interest ($58.560) by the entire $1.379, obtaining practically 42 days as the time after Apr. 30 to pay the sum of the items without loss of interest to either buyer or seller.

INTEREST RULE.—I. *Select the focal date as directed in the Product Rule.*

II. *Find the interest of each item for the time between the*

342

focal date and the date at which such item falls due; and divide six times the sum of the obtained interests by .001 of the sum of the items.

III. *The quotient will denote the number of days to count forward from the focal date to find the equated date.*

NOTE 2.—The focal date may be selected as directed in Note 1. The final quotient will then denote the number of days to count backward from the focal date to find the equated date.

NOTE 3.—At 6% per annum, the interest for 2 30-day months is 1% of the principal (obtained by mentally moving the decimal point of the principal *two* places to the left); for 4 30-day months take twice the preceding result; for 6 30-day months take three times the preceding result; etc.; etc. For 6 days, the interest is .001 of the principal (obtained by mentally moving the decimal point of the principal *three* places to the left); for twice 6 days take twice the preceding result; for 3 times 6 days take 3 times the preceding result; etc.; etc. For less than 6 days, take aliquot parts of any of the preceding results.

NOTE 4.—Since, at 6% per annum, the interest is 1% of the principal for 60 days or for two 30-day months, if the time be expressed in months and days, change the calendar months to 30-day months by increasing the time 1 day for each included 31-day month, or by decreasing the time 2 days (1 day for leap year) if February is included.

EXAMPLES FOR PRACTICE.

Find the equated date of purchase of the following bills:

(1.)

1889.	June 5,	$500,	no credit.
	July 20,	300,	"
	Aug. 15,	200,	"

(2.)

1889.	Jan. 3,	$250,	no credit.
	Feb. 18,	375,	"
	Apr. 4,	530,	"

(3.)

1888.	Mar. 9,	$735,	no credit.
	" 31,	840,	"
	Apr. 8,	750,	"
	May 15,	475,	"

(4.)

1889.	Aug. 12,	$690,	no credit.
	Sept. 4,	325,	"
	Oct. 21,	512,	"
	Dec. 8,	450,	"

5. I bought of Simmons & Bro., Pensacola, the following invoices of merchandise:

Jan. 3, 1887,	invoice amounting to	$750.
Feb. 9,	" " " "	195.
Mar. 11,	" " " "	275.
Apr. 18,	" " " "	525.

What was the amount of my bill, and the average date of purchase?

342

6. J. P. Jordan & Co.'s sales to T. A. Smith are as follows:

Mar.	19,	1889, a bill of	$275.
“	31,	“ “ “	340.
Apr.	25,	“ “ “	520.
May	15,	“ “ “	150.
June	12,	“ “ “	260.

What is the average date of purchase?

7. A bought of B the following merchandise:

Feb.	5,	1889, a bill amounting to	$420.
Mar.	13,	“ “ “ “	762.
Apr.	14,	“ “ “ “	578.
May	21,	“ “ “ “	284.
June	6,	“ “ “ “	512.

What is the average date of purchase?

8. A sold goods to B as follows: May 30, $525.75; June 15, $325.40; June 28, $275; July 7, $214.30; July 20, $390.15; Aug. 12, $125.45; and Sept. 10, $263.20. What is the equated date of total sales?

9. W. J. Robb & Co. sold to L. W. Bristol the following invoices of tobacco: July 7, 3840 lb. @ 27¢; July 19, 2640 lb. @ 26½¢; Aug. 4, 3752 lb. @ 28¾¢; Aug. 15, 1464 lb. @ 29¢; and Sept. 3, 3890 lb. @ 28¼¢. What was the average date of sale?

If, as in the following examples, *all* the items have the same term of credit expressed in days, proceed as before to find the equated *time* in days, add the uniform term of credit to the result, and then find the equated *date;* but if the uniform term of credit is expressed in months, first find the equated *date* as if there were no term of credit, and advance this equated date as many calendar months as are contained in the uniform term of credit.

10. S. H. Hanson sold to a retail merchant merchandise as follows:

1888.	Feb.	6,	an invoice amounting to	$673.40.
“	Mar.	24,	“ “ “ “	462.60.
“	Apr.	18,	“ “ “ “	327.90.
“	June	24,	“ “ “ “	128.10.

If 60 days' credit was allowed on each item, what was the average date for the payment of the entire amount?

11. A wholesale merchant sold to a customer the following:

1888. Sept. 4, an invoice of merchandise amounting to $624.20.
" Oct. 12, " " " " " " 920.80.
" Nov. 26, " " " " " " 440.72.
" Dec. 18, " " " " " " 364.28.

A credit of 90 days being allowed on each item, what was the equated date for the payment of the amount of the several bills?

12. M. A. Davenport bought of Johnson & Co.:

Mar. 12, 1889, Merchandise amounting to $742.83.
April 4, " " " " 926.17.
May 24, " " " " 432.25.
June 17, " " " " 327,75.

What is the equated date for payment of the amount of the several bills, a credit of 30 days being allowed on each item?

13. J. W. Miller bought of Freeman & Co.:

May 30, 1888, Merchandise amounting to $345.30.
June 19, " " " " 576.75.
Aug. 3, " " " " 453.40.

A credit of 2 months having been allowed on each item, what was the equated date for paying the entire amount?

14. I bought the following bills of goods on 3 months' credit: Aug. 18, 1889, $430.80; Sept. 8, 1889, $215.65; Sept. 25, 1889, $300; and on Oct. 15, 1889, $591.20. What was the equated date of total purchase?

343. To find the average date for paying two or more debts contracted at different dates, and having different terms of credit.

ILLUSTRATIVE EXAMPLE.

PHILADELPHIA, *Dec. 1, 1889.*

E. G. CHURCH,

To R. S. SCOTT & Co., *Dr.*

1889.					
June 25	To Mdse. per inventory @ 60 da.	$625	85		
July 8	" " " " " 30 "	731	40		
Aug. 19	" " " " " 90 "	574	65		
Sept. 13	" " " " net,	382	35	$2314	25

FIRST SOLUTION.—*Product Method.*

Focal date, May 31, 1889 (=June 0, 1889).

Dates of Sale.	*Credits.*	*Time.*	*Items.*	*Products.*
June 25 +	60 da. =	85 da. ×	$625.85 =	3129 = $625.85 × 5 units. 50068 = 625.85 × 8 tens.
30 + July 8 +	30 da. =	68 da. ×	731.40 =	5851 = 731.40 × 8 units. 43884 = 731.40 × 6 tens.
61 + Aug. 19 +	90 da. =	170 da. ×	574.65 =	40226 = 574.65 × 7 tens. 57465 = 574.65 × 1 hundred.
92 + Sept. 13	=	105 da. ×	382.35 =	1912 = 382.35 × 5 units. 38235 = 382.35 × 1 hundred.
			$2314.25)	$240770 (104 + days.

Average date: 104 days after May 31, 1889, or Sept. 12, 1889.

EXPLANATION.—The above solution is similar to that by the product method, **342**, except that each item is multiplied by the number of days between the focal day (May 31 or June 0) and the date at which such item falls due, obtained by adding the term of credit (if any) to the number of days between the focal date and the date of sale.

SECOND SOLUTION.—*Interest Method.*

Focal date, May 31, 1889 (= June 0, 1889). Rate per annum, 6%.

Mo.	*Dates of Sale.*	*Items.*	*Credits.*	*Interest.*
0	June 25	$625.85	@ 60 da.	$6.259, int. for 60 da. ($625.85 × .01). 2.503, " " 24 da. ($.62585 × 4). .104, " " 1 da. ($.62585 + 6).
1	July 8	731.40	@ 30 da.	7.314, " " 2 mo. ($731.40 × .01). .731, " " 6 da. ($731.40 × .001). .244, " " 2 da. ($.73140 + 3).
2	Aug. 19 + 1	574.65	@ 90 da.	11.493, " " 4 mo. ($5.7465 × 2). 4.597, " " 48 da. ($.57465 × 8). .192, " " 2 da. ($.57465 + 3).
3	Sept. 13 + 2	382.35		3.824, " " 2 mo. ($382.35 × .01). 2.676, " " 42 da. ($.38235 × 7). .191, " " 3 da. ($.38235 + 2).
		$2314.25		$40.128 × 6

Int. of $2314.25 for 6 da. = $2.314) $240.768 (104 + days.

Average date: 104 days after May 31, 1889, or Sept. 12, 1889.

EXPLANATION.—The above solution is like that of the interest method, **342**, except that the interest is found on each item from the focal date to its due date, obtained by adding the term of credit to the date of purchase, considering each 30 days of the term of credit as an additional 30-da. month.

343

RULE.—*Add the term of credit to the time from the focal date to the date of the item; then proceed by either the product or interest Rule,* **342.**

NOTE 1.—When the product rule is used and an item contains cents, it is customary to discard decimal results from the products. To secure integral products, first multiply the *units' order* of the item by the *units' order* of the multiplier; then the *tenths' order* of the item by the *tens' order* of the multiplier; lastly the *hundredths' order* of the item by the *hundreds' order* of the multiplier; and to each result add the carrying figure obtained by multiplying the adjoining discarded figure of the item.

NOTE 2.—If the interest rule is used and a term of credit is given in months, they should be reduced to 30-day months by Note 4, **342.**

NOTE 3.—In dividing by the sum of the items, it is usual to discard the cents from the divisor if less than 50, and to increase the integral divisor by 1 if the cents are 50 or more. Thus, in the Ill. Ex., 2314 was taken as the divisor.

NOTE 4.—In obtaining the due date of notes, drafts, etc., add 3 days for grace to their nominal time of payment.

EXAMPLES FOR PRACTICE.

Find the equated date of payment.

(1.)

1889.	Mar.	3,	$300	@ 60 da.
	Apr.	15,	475	@ 30 "
	May	5,	520	@ 30 "

(2.)

1888.	Aug.	7,	$625	@ 90 da.
	Sept.	3,	550	@ 30 "
	Oct.	12,	225	@ 60 "

(3.)

1888.	Sept.	20,	$750	@ 60 da.
	"	28,	825	@ 90 "
	Oct.	15,	575	@ 30 "
	Nov.	1,	150	@ 30 "

(4.)

1889.	Jan	18,	$380	@ 45 da.
	Feb.	3,	720	@ 60 "
	Mar.	9,	260	@ 30 "
	Apr.	20,	440	@ 60 "

(5.)

1889.	Mar.	21,	$825.30	@ 3 mo.
	Apr.	3,	512.75	@ 1 "
	"	29,	250.90	@ 2 "
	May	8,	175.25	@ 3 "
	"	19,	520.00	@ 2 "

(6.)

1888.	May	2,	$150.35	@ 1 mo.
	June	4,	275.58	@ 3 "
	"	30,	182.15	@ 4 "
	Aug.	1,	325.75	@ 2 "
	Sept.	15,	415.10	@ 3 "

343

7. W. H. Ransom & Co. bought merchandise, as follows:

Mar. 15, 1888, a bill of $324 on 60 days' credit.
May 14, " " 676 " 30 " "
June 12, " " 480 " 3 months' "
July 15, " " 225 " 90 days' "
Aug. 4, " " 422 " 3 months' "

What is the equated date of payment?

8. Average the following statement of account:

NEW YORK, *Oct. 1, 1889.*

W. W. TREVOR & Co.,

In account with H. A. FAIRMAN. *Dr.*

1889.	Mar. 25,	To Mdse. per bill rendered,	$340.72	@	30	da.
"	Apr. 14,	" " "	671.34	@	60	"
"	May 14,	" " "	374.62	@	90	"
"	June 2,	" " "	517.27	@	30	"
"	" 24,	" " "	243.25	@	3	mo.
"	July 18,	" " "	721.80	@	30	da.

9. On Jan. 25, 1889, a grocer sold goods amounting to $1340, one-half to be paid in 90 days, one-fourth in 4 months, and the remainder in 5 months. What was the equated date of payment?

10. On March 24, 1888, M. A. Davenport bought of J. R. Compton merchandise amounting to $1644, one-fourth payable in 60 days, one-fourth in 3 months, and the remainder in 4 months. At what date could the entire debt be equitably paid?

11. E. I. Collier sold E. A. Ellicott & Co., Feb. 4, 1889, 200 bbl. flour at $6 per bbl., on 30 days' credit; Mar. 12, 1889, 23 bbl. sugar, averaging 240 lb. per bbl., at 7½¢ per pound, on 60 days' credit; Apr. 16, 1889, 320 lb. N. Y. butter, at 22½¢ per pound, on 3 months' credit; Mar. 22, 1889, 640 doz. eggs at 12½¢ per dozen. When should E. A. Ellicott & Co.'s entire indebtedness to E. I. Collier be paid by average?

12. F. H. Smith & Son sold H. W. Shaw the following bills of merchandise: Mar. 3, 1888, $467.20 @ 30 days; Mar. 25, 1888, $562.75 @ 60 days; Apr. 12, 1888, $210.58 @ 4 months; May 3, 1888, $535.10 @ 30 days; and May 15, 1888, $285 @ 90 days. What is the equated date for paying the sum of the several bills?

AVERAGING ACCOUNTS.

344. The **Average of an Account** is the average or equated time for the payment of the balance of an account which contains both debit and credit items.

Rem.—In Equation of Payments, only *one* side of an account is considered, the items being either all debits or all credits; but when an account involves both debit and credit items, it is necessary to consider *both* sides of the account in ascertaining the equated date to pay the balance due. The former is sometimes called *Simple Average*, and the latter *Compound Average.*

345. Accounts are averaged upon the following principles:

1. *Goods delivered and money paid a certain time* BEFORE *they are due should equitably be* CREDITED *with interest for that time.*

2. *Goods delivered and money paid a certain time* AFTER *they are due should equitably be* DEBITED *with interest for that time.*

3. *The equated time for paying the balance of an account is that date at which the credit interest of Prin. 1 will offset the debit interest of Prin. 2.*

Rem.—If the balance of an account is paid before the equated date, it is customary to deduct bank discount on the balance from the date of its payment to the equated date; and if paid after the equated date, to add interest to the balance from the equated date to the date of its payment. If an interest-bearing note is given in settlement of an account, the equated date is taken as the date of the note.

346. To find the equated date for paying the balance of any account.

ILLUSTRATIVE EXAMPLE.

Average the following ledger account:

Dr. R. H. Carter. *Cr.*

1889.				*1889.*			
Oct. 10	To Mdse.,	$418	25	Nov. 24	By Cash,	$275	18
Nov. 8	" "	586	72	Dec. 13	" "	482	95
Dec. 23	" "	392	40	" 28	" "	368	35

FIRST SOLUTION.—*Product Method.*

Focal Date, Sept. 30, 1889 (= Oct. 0, 1889).

Dates.	*Time.*	*Items.*	*Products.*	*Dates.*	*Time.*	*Items.*	*Products.*
Oct. 10 =	10 da.	× $418.25 =	$4183	31 + Nov. 24 =	55 da.	× $275.18 =	$1376 13759
31 + Nov. 8 =	39 "	× 586.72 =	5280 17602	61 + Dec. 13 =	74 "	× 482.95 =	1932 33807
61 + Dec. 23 =	84 "	× 392.40 =	1570 31392	61 + " 28 =	89 "	× 368.35 =	3315 29468
		$1397.37	$60027			$1126.48	$83657
		1126.48					60027
		$270.89					$23630

($23630 + $271 = 87 + da.)

Average date: 87 days before Sept. 30, 1889, or July 5, 1889.

EXPLANATION.—Equating the debit side of the account by Product Rule **342**, it is seen that if the entire bill ($1397.37) had been paid on Sept. 30, 1889, Carter would have been deprived of the use of $60027 for 1 day; and similarly equating the credit side, it is found that Carter had the use of $83657 for 1 day. Hence, on the basis of a settlement on Sept. 30, 1889, Carter *gains* the use of $83657 − $60027, or $23630 for 1 day, and he should therefore pay the balance of his account ($270.89) as many days before Sept. 30, 1889, as will make the use of this balance to his creditor equal to the use of $23630 for 1 day; that is, 87 days before Sept. 30, or July 5, 1889.

PRODUCT RULE.—I. *Take as the focal date the last day of the calendar month preceding the date of the earliest item of the account, and multiply each item of the account by the difference in days between this focal date and the date at which such item falls due.*

II. *Separately add the obtained products of each side of the account, and divide the difference between the sum of the debit products and credit products by the balance of the account.*

III. *The quotient will denote the number of days between the focal date and equated date, which should be counted forward from the focal date (added to it) if the product balance and balance of items are on the same side of the account; or be counted backward from the focal date (subtracted from it) if the product balance and balance of account are on opposite sides.*

NOTE 1.—The date of the latest item, or any convenient subsequent date, may be taken as the focal date. The quotient, as obtained by the above rule, will then

346

denote the number of days to be counted backward if both balances are on the same side of the account, or to be counted forward if on opposite sides—the reverse of what is required by the rule.

SECOND SOLUTION.—*Interest Method.*

Focal Date, Sept. 30, 1889 (= Oct. 0, 1889).—Rate per annum, 6%.

Mo.	*Dates.*	*Items.*	*Interest.*	*Mo.*	*Dates.*	*Items.*	*Interest.*
0	Oct. 10,	$418.25 =	$.697 (10 da.)	1	Nov. 24+1,	$275.18 =	$2.477 (54 da.) .046 (1 da.)
1	Nov. 8+1,	586.72 =	3.520 (36 da.) .293 (3 da.)	2	Dec. 13+1,	482.95 =	5.795 (72 da.) .161 (2 da.)
2	Dec. 23+1,	392.40 =	3.924 (2mo.) 1.570 (24 da.)	2	Dec. 28+1,	368.35 =	3.684 (2mo.) 1.473 (24 da.) .307 (5 da.)
		$1397.37	$10.004			$1126.48	$13.943
		1126.48					10.004
		$270.89					3.939
							×6
							271)23.634(87+da.

87 da. before Sept. 30, 1889 = July 5, 1889, *Ans.*

EXPLANATION.=Equate the debit side of the account by Interest Rule **342**, finding that if the entire bill ($1397.37) had been paid on Sept. 30, Carter would have lost interest amounting to $10.004 at 6% per annum. Then similarly equate the credit side, finding that Carter had the use of $1126.48 of his debits after Sept. 30 until it produced $13.943 interest, or $3.939 more than enough to offset his loss. Hence, Carter should pay the balance of his account ($270.89) as many days before Sept. 30, 1889, as will enable it by Sept. 30 to accrue $3.939, and thus offset the interest balance against him. At 6%, it will require $270.89 practically 87 days to accrue $3.939 interest; it should therefore be paid 87 days before Sept. 30, 1889, or on July 5, 1889.

INTEREST RULE.—I. *Select the focal date as directed in the preceding Rule; and find the interest of each item of the account for the time between the focal date and its due date.*

II. *Find the difference between the total debit interest and total credit interest, and divide six times this difference by one thousandth of the balance of the account. The quotient will denote the time in days between the focal date and equated date, which should be counted forward from the focal date if the interest balance and balance of account are on the same side, or counted backward from the focal date if the interest balance and balance of account are on opposite sides.* **346**

NOTE 2.—The focal date may be selected by Note 1, in which case the process of counting backward or forward must be in agreement with said note.

EXAMPLES FOR PRACTICE.

Find the equated date for paying the balances of the following:

Dr. (No. 1.) J. A. HARRISON. *Cr.*

1890.				*1890.*			
May 12	To Mdse.,	$575	00	June 8	By Cash,	$150	00

Dr. (No. 2.) T. C. BRUFF & CO. *Cr.*

1889.				*1889.*			
Oct. 18	To Mdse.,	$348	00	Nov. 5	By Cash,	$130	00

Dr. (No. 3.) E. BENSEL & CO. *Cr.*

1890.				*1890.*			
Mar. 20	To Mdse.,	$695	00	May 3	By Cash,	$340	00

Dr. (No. 4.) O. FOREMAN & BRO. *Cr.*

1890.				*1890.*			
Apr. 5	To Mdse.,	$415	00	May 4	By Cash,	$275	00
May 21	" "	327	00	" 15	" "	112	00

Dr. (No. 5.) J. J. JANNEY. *Cr.*

1889.				*1890.*			
Nov. 8	To Mdse.,	$380	00	Jan. 3	By Cash,	$219	00
Dec. 2	" "	516	00	Feb. 25	" "	410	00

Dr. (No. 6.) JOHN B. HUNTON. *Cr.*

1889.				*1889.*			
June 15	To Mdse.,	$375	00	July 3	By Cash,	$150	00
July 8	" "	250	00	Aug. 12	" "	230	00
Sept. 23	" "	190	00	Oct. 9	" "	210	00

Dr. (No. 7.) R. W. Merchant. *Cr.*

1890.				*1890.*			
Aug. 4	To Mdse.,	$815	00	Aug. 15	By Cash,	$500	00
Sept. 30	" "	350	00	Sept. 6	" Mdse.,	250	00
Oct. 8	" "	480	00	Oct. 18	" Cash,	390	00

Dr. (No. 8.) J. H. Butler. *Cr.*

1888.				*1888.*			
Apr. 19	To Mdse.,	$175	18	May 8	By Mdse.,	$450	00
May 5	" "	325	25	June 2	" "	351	40
" 30	" "	536	80	" 17	" "	258	90
June 10	" "	275	40				

Dr. (No. 9.) L. J. Ives. *Cr.*

1889.				*1889.*			
Sept. 18	To Mdse.,	$925	12	Sept. 30	By Cash,	$325	00
" 25	" "	381	75	Oct. 15	" Sundries,	600	00
Oct. 8	" "	426	60	Nov. 20	" Cash,	500	00
" 19	" "	241	15				

Dr. (No. 10.) Jno. W. Graves. *Cr.*

1890.				*1890.*			
Jan. 21	To Mdse.,	$125	00	Feb. 6	By Cash,	$100	00
Feb. 15	" "	284	75	Mar. 18	" "	220	00
Apr. 7	" "	190	45	Apr. 3	" "	130	00
May 27	" "	325	65	" 23	" "	200	00

Dr. (No. 11.) E. C. Berkeley. *Cr.*

1888.				*1888.*			
July 27	To Mdse.,	$358	30	Aug. 1	By Cash,	$350	00
Aug. 17	" "	215	75	Sept. 20	" "	275	00
Sept. 3	" "	426	40	Oct. 15	" "	250	00
" 19	" "	372	95				

Dr. (No. 12.) L. L. Osmond. *Cr.*

1887.				*1887.*			
Feb. 15	To Mdse.,	$825	15	Apr. 3	By Cash,	$500	00
Mar. 9	" "	324	58	May 19	" "	375	00
" 30	" "	175	25	" 31	" "	250	00
Apr. 8	" "	215	45	June 8	" "	340	00
May 21	" "	348	30	July 15	" "	210	00
June 15	" "	278	25	Aug. 20	" "	125	00

Dr. (No. 13.) C. E. Nicol. *Cr.*

1888.				*1888.*			
May 12	To Mdse., 30 da.	$700	00	June 18	By Cash,	$200	00
June 5	" " 60 "	450	00	Aug. 20	" "	400	00
July 8	" " 90 "	825	00	Sept. 3	" Note, 30 da.,	300	00
Aug. 6	" " Net	340	00				

Dr. (No. 14.) F. A. Sadler. *Cr.*

1888.				*1888.*			
June 15	To Mdse. @ 30 da.	$285	00	July 1	By Cash,	$200	00
July 18	" " " 10 "	340	00	Aug. 10	" "	300	00
Aug. 4	" " " 60 "	782	00	Sept. 1	" Note, 60 da.,	1000	00
" 17	" " " 90 "	945	00	" 10	" Cash,	400	00

Dr. (No. 15.) W. H. Patrick. *Cr.*

1888.				*1888.*			
Sept. 5	To Mdse. @ 60 da.	$376	00	Oct. 9	By Cash,	$350	00
Oct. 9	" " " 30 "	428	00	Nov. 10	" "	275	00
Nov. 14	" " " 60 "	763	00	Dec. 10	" "	360	00
Dec. 7	" " " 10 "	470	00	*1889.*			
				Jan. 1	" "	200	00

Dr. (No. 16.) J. O. Moul. *Cr.*

1889.				*1889.*			
Jan. 18	To Mdse. @ 3 mo.	$619	35	Feb. 18	By Note, 3 mo.,	$375	40
Feb. 6	" " " 2 "	475	48	Mar. 10	" Cash,	418	25
Mar. 25	" " " 4 "	293	70	Apr. 5	" Note, 2 mo.,	249	75
Apr. 15	" " " 1 "	527	65	May 20	" " 1 "	318	20

Dr. (No. 17.) M. E. WHITNEY. *Cr.*

1888.				*1888.*			
July 6	To Mdse. @ 2 mo.	$465	45	Aug. 25	By Cash,	$275	40
Aug. 20	“ “ “ 1 “	538	90	Sept. 30	“ Note, 2 mo.,	398	25
Sept. 15	“ “ “ 3 “	392	75	Oct. 12	“ “ 1 “	427	60
Oct. 5	“ “ “ 4 “	682	50	Dec. 31	“ “ 3 “	329	15

Dr. (No. 18.) W. P. RICHARDSON. *Cr.*

1889.				*1889.*			
Feb. 4	To Mdse., 90 da.	$1284	64	Mar. 12	By Note, 60 da.,	$1298	12
Mar. 12	“ “ “ “	1262	28	June 17	“ Cash,	2480	00
April 8	“ “ “ “	1480	37	July 24	“ Note, 3 mo.,	2648	88
May 12	“ “ “ “	2474	88	“ 31	“ Cash,	1000	00
June 8	“ “ “ “	1320	83				

Dr. (No. 19.) R. M. BROWNING. *Cr.*

1889.				*1889.*			
Apr. 12	To Mdse., 30 da.	$648	78	May 24	By Cash,	$542	36
May 23	“ “ 60 “	212	22	Aug. 24	“ “	367	27
June 10	“ “ 30 “	284	17	Sept. 4	“ “	763	74
July 28	“ “ 90 “	374	83	“ 20	“ “	650	63

Dr. (No. 20.) E. G. TAYLOR & CO. *Cr.*

1888.				*1888.*			
Sept. 3	To Mdse., 60 da.	$963	50	Oct. 1	By Cash,	$1000	00
“ 9	“ “ 30 “	432	82	Nov. 4	“ “	500	00
Oct. 13	“ “ 4 mo.	496	37	“ 15	“ Dft., 30 da.,	260	53
“ 17	“ “ 10 da.	786	39	Dec. 17	“ Cash,	1000	00
Nov. 3	“ “ 3 mo.	512	40				
“ 24	“ Cash,	1000	00	*1889.*			
Dec. 4	“ Mdse., 90 da.	826	52	Jan. 5	“ Acceptance, 60 da.	1250	00
“ 21	“ “ Net.	973	86	Feb. 15	“ Cash,	500	00
				Mar. 3	“ Note, 3 mo.,	1000	00
1889.				Apr. 15	“ Cash,	200	00
Jan. 15	“ Acceptance, 3 mo.	1000	00	May 2	“ “	1000	00
Feb. 3	“ Mdse., 4 mo.	821	82				
Mar. 7	“ “ 30 da.	723	67				

21. If, on March 3, 1889, I purchase $800 worth of merchandise on 90 days' credit, and on April 15, 1889, make a partial payment of $400, what is the equated date for paying the balance?

In the above problem, make a ledger statement by placing the purchase with its date and term of credit on the debit side of the account; and the payment with its date on the credit side. Then find the equated date for paying the balance, as in the accounts which precede it.

NOTE.—While it is customary as well as equitable to proportionately extend the term of credit on the balance of a debt if a part thereof has been prepaid, the law does not enforce such extension if the creditor objects.

22. On Aug. 15, 1888, C. F. Dickinson sold an invoice of merchandise for $1200 on 90 days' credit, and on Sept. 20 he received from the purchaser $400. When in equity should the balance be paid?

23. I sold on Sept. 5, 1888, a bill of goods for $875 on 3 months' credit. When in equity should I receive the balance if on Oct. 25, 1888, the purchaser made a partial payment of $375?

24. On May 5, 1887, James Atwater sold to H. P. Underhill an invoice of goods amounting to $1500, on 1 month's credit. On May 10, H. P. Underhill paid $300, and on May 20, $570. On the date of the second payment, H. P. Underhill gave an interest-bearing note, properly dated, for the unpaid balance. What was the date of the note?

25. On Dec. 8, 1887, a manufacturer sold an invoice of goods to a merchant for $1800, on 3 months' credit; and on Jan. 15, 1888, the manufacturer bought from the merchant a bill of goods for $2500, on which he was allowed no credit. What is the equated date for the merchant to receive the balance due from the manufacturer?

26. On Jan. 12, 1888, J. Fagan & Son sold Ferguson Bros. & Co. a bill of merchandise amounting to $1825, on 3 months' credit; and on Feb. 11, 1888, J. Fagan & Son borrowed of Ferguson Bros. & Co. $2500. What should be the date of an interest-bearing note to be given by J. Fagan & Son in settlement of the balance due Ferguson Bros. & Co.?

27. A has three of B's notes: one of $375, dated Aug. 21, 1888, and payable in 3 months; one of $530, dated Sept. 3, 1888, and payable in 60 days; and one of $425, dated Oct. 15, 1888, and payable in 4 months. On Nov. 1, 1888, B paid A $600, canceled his three notes, and in exchange gave a single note for the remainder. What was the date of the latter note?

AVERAGING ACCOUNTS SALES.

347. When a consignment arrives, the consignee pays the freight charges and other necessary items of expense, and frequently makes cash advances on the consignment, charging interest on all advances or retaining the net proceeds sufficiently long to offset the accrued interest.

ILLUSTRATIVE EXAMPLE.

Find the equated date for paying the net proceeds of the following account sales of 255 bbl. of pork, received from E. Meredith & Co., Chicago, to be sold on their account and risk :

1889.	April 12,	Sold 75 bbl. @ $12.00,	Cash,	$900.00	
	May 9,	" 80 " @ 13.00,	"	1040.00	
	June 17,	" 60 " @ 13.50,	"	810.00	
	Aug. 28,	" 40 " @ 12.50,	"	500.00	$3250.00
		Charges:			
	April 3,	To cash for freight and cartage,		$252.00	
	" 3,	" cooperage..................		12.00	
	" 3,	" advertising and insurance.		15.00	
	June 20,	" cash advanced consignor..		300.00	
	Aug. 28,	" storage......		14.00	
		Commission at 4% (on $3250)		130.00	$723.00
		Net proceeds,....			$2527.00

SOLUTION.—*Focal date, Mar. 31, 1889.*

Dates of Sale.	*Sales.*	*Interest.*
0 Apr. 12	$ 900	$ 1.80 (12 da.)
1 May 9	1040	6.24 (36 ") .52 (3 ")
2 June 17+1	810	8.10 (2 mo.) 2.43 (18 da.)
4 Aug. 28+2	500	10.00 (4 mo.) 2.50 (1 mo.)
	$3250	$31.59
		6

$3.25) $189.54 (58+da.

EXPLANATION.— First find the equated date of sales by the interest rule, **342**, and take it (May 28) as the date of the total sales ($3250) and of the total commission ($130).

Then average the account by the interest rule, **346**, obtaining May 31, 1889, as the equated date for paying the net proceeds ($2527).

Mar. 31+58 da. = May 28, equated date of commission and sales.

Mo.	*Dates.*	*Charges.*	*Interest.*
0	Apr. 3,	$252	
	" 3,	12	
	" 3,	15	
		$279	$0.140 (3 da.)
2	June 20+1	300	3.00 (2 mo.) .90 (18 da.) .15 (3 da.)
4	Aug. 28+2	14	.28 (4 mo.) .07 (1 mo.)
1	May 28	130	1.04 (48 da.) .217 (10 da.)
		$723	$5.797

Mo.	*Date.*	*Sales.*	*Interest.*
1	May 28	$3250	$26.00 (48 da.)
		723	5.417 (10 da.)
		$2527	$31.417
			5.797
			$25.620
			×6
		$2.527)	$163.720 (61 da.

Mar. 31+61 da. = May 31, 1889, equated date for paying the net proceeds ($2527).

RULE.—I. *If the sales have been effected at different dates, find the equated date of sales by* **342,** *and take the result as the date of commission and of total sales.*

II. *Write the total sales as a credit item, and the several charges as debit items; and find the equated date for paying the net proceeds or balance of the account by* **346.**

EXAMPLES FOR PRACTICE.

1. Find the equated date for paying the net proceeds of the following account sales:

BALTIMORE, MD., *May 1, 1888.*

Account Sales of Flour,

Sold for account of JNO. G. HIXON & CO.,

By J. J. DAVIES & SON.

1888.					
Jan. 18	150 bbl. to C. M. Bennett @ $6, cash........	***	**		
Feb. 5	160 " " D. E. Pine @ $7, "	****	**		
Mar. 19	90 " " R. C. Wheeler @ $6, "	***	**		
Apr. 27	200 " " R. M. Weir @ $\$6^{25}$, "	****	**	****	**
	Charges.				
Jan. 6	Freight and drayage........................	$175	00		
" 6	Advertising and insurance..................	25	00		
" 30	Advance on Consignment.....................	600	00		
Feb. 13	Cooperage..................................	16	70		
	Commission 2%..............................	**	**	***	**
	Net Proceeds per Average...................			****	**

347

2. W. H. Rogers & Co., of Philadelphia, sold for account of R. O. Phelps, of Boston, the following consignment of Young Hyson tea: 1889—Mar. 3, 20 half-chests, 1210 lb., at 65¢; Apr. 12, 44 half-chests, 2640 lb., at 72¢; May 5, 130 half-chests, 7635 lb., at 68¢. The charges are: Feb. 24, freight, $25; Mar. 1, cartage, $2.25; Mar. 20, cash advanced on consignment, $250; commission, 3%.

3. W. Tippett, of New York, sold for account of R. Moss, of Chicago, the following: 1889—May 12, 9280 lb. bacon at 6½¢; May 25, 15370 lb. bacon at 6⅜¢; and June 18, 25396 lb. bacon at 6⅞¢. The charges were: May 15, cash advanced Moss, $500; May 2, drayage, $18; commission, 1½%.

CASH BALANCE.

348. The **Cash Balance** is the sum due on an account current at a specified date.

REM. 1.—An *Account Current* is a statement of the gross debits and credits of each business transaction, within a certain period, between two houses; and may be rendered annually, semi-annually, quarterly, or monthly.

REM. 2.—Interest is usually, though not invariably, charged on the balance of an account; that being regulated by a previous understanding between the parties thereto, or by the usage of each particular business.

REM. 3.—To charge interest on periodical balances and carry the amount to a new account, is virtually compounding interest. Hence, the oftener accounts are closed, the more unfavorable it will prove to the debtor. Some merchants, however, who render accounts current oftener than once a year, do not carry the interest balance to the main column of the account until the end of the year.

ILLUSTRATIVE EXAMPLE.

Find the cash balance, Sept. 30, 1889, on the following account, including interest at 6%:

Dr. WM. CALLEN in acct. current with H. A. RICE. *Cr.*

1889.		*Days.*	*Interest.*	*Am'nts.*	*1889.*		*Days.*	*Interest.*	*Am'nts.*
Feb. 5	To Mdse., 30 da.			$615 75	Apr. 27	By Note, 3 mo.			$215 40
Mar. 7	" " 3 mo.			398 40	May 18	" " 60 da.			485 98
Apr. 12	" " 60 da.			416 65	Aug. 28	" Cash,			362 85
May 4	" " Net.			582 30	Sept. 15	" "			580 20

INTEREST SOLUTION.—*Rate 6% per annum.*

Dr. 1889.			*Days.*	*Interest.*	*Am'nts.*	1889.		*Cr.*	*Days.*	*Interest.*	*Am'nts.*
Feb.	5	To Mdse., 30 da.,	207	$21 24	$615 75	Apr.	27	By Note, 3 mo.,	62	$2 23	$215 40
Mar.	7	" " 3 mo.,	115	7 64	398 40	May	18	" " 60 da.,	72	5 83	485 98
Apr.	12	" " 60 da.,	111	7 71	416 65	Aug.	28	" Cash,	33	2 00	362 85
May	4	" " Net,	149	14 46	582 30	Sept.	15	" "	15	1 45	580 20
Sept.	30	" *Bal. of Int.*,			39 54	"	30	" *Bal. of Int.*,		39 54	
						"	"	" " " *Acct.*,			408 21
				$51 05	$2052 64					$51 05	$2052 64
Sept.	30	To Balance,			$408 21						

EXPLANATION.—In the "time column" place the number of days between the due date of each item and the date of finding the cash balance (Sept. 30, 1889); and in the "interest column" write the interest of each item for the number of days found in the "time column."

Next balance the interest columns by writing on the less side the interest necessary to balance the greater side ($39.54); and carry this interest balance as an additional debit item to the "amount column" on the debit side.

Finally balance the amount columns by writing on the less side the sum necessary to balance the greater side; and this sum ($408.21) will be the cash balance.

Dr. PRODUCT SOLUTION. *Cr.*

1889.			*Days.*	*Products.*	*Amounts.*	1889.			*Days.*	*Products.*	*Amounts.*
Feb.	5	To Mdse., 30 da.	207	127460	$615 75	Apr.	27	By Note, 3 mo.,	62	13355	$215 40
Mar.	7	" " 3 mo.	115	45816	398 40	May	18	" " 60 da.,	72	34991	485 98
Apr.	12	" " 60 da.	111	46248	416 65	Aug.	28	" Cash,	33	11974	362 85
May	4	" " Net,	149	86763	582 30	Sept.	15	" "	15	8703	580 20
Sept.	30	" *Bal. of Int.*,			39 54	"	30	" *Bal. of Products*,		237264	
						"	30	" " *Account*,			408 21
				306287	$2052 64					306287	$2052 64
Sept.	30	To Balance,			$408 21						

EXPLANATION.—In the product column write the product from multiplying each item, by the number of days between its due date and the date of finding the cash balance (Sept. 30, 1889).

Next balance the product columns by writing on the less side the sum necessary to balance the greater side ($237264 for 1 day); change this balance of products to balance of interest by finding the interest of $237264 for 1 day,

$237.264 being the interest for 6 days, and ⅙ of $237.264, or $39.54, the interest for 1 day (**243**); and carry this interest balance as an additional debit item to the "amount column" on the debit side.

Finally, balance the amount columns as explained in the Interest Solution, obtaining $406.21 as the cash balance.

REM.—The above product solution is an application of the product rule for finding interest (**242**); hence, to secure the correct cash balance, the *decimal part* of each item in the amount column must be multiplied *as it stands.* It should be remembered that the required interest is *one mill* for every *six dollars* of the obtained products.

INTEREST RULE.—I. *Compute the interest of each item from its due date to the date of settlement. The difference between the total debit interest and total credit interest will be the balance of interest.*

II. *To find the cash balance, add the balance of interest to the balance of items if both are on the same side; or subtract if on opposite sides.*

PRODUCT RULE.—I. *Multiply each item by the number of days between its due date and the date of settlement. The difference between the total debit product and total credit product will be the balance of products.*

II. *Divide one-thousandth of the balance of products by 6 to find the balance of interest at 6% per annum* (**243**); *then find the cash balance as by II., Interest Rule.*

NOTE 1.—The interest of items which fall due after the date of taking the cash balance should be placed in the opposite interest column ; or such interest may be written with *red ink* on the same side as the items, and when the totals are taken, this red interest can be omitted from its own side and added with the interest on the opposite side, thus avoiding awkward transfers.

NOTE 2.—When monthly statements are rendered, accountants frequently include the average due date. The cash balance at any earlier or later date is then usually found by Rem., **345.** The cash balance obtained by average is, however, not strictly correct, owing to the fraction of a day which is discarded in finding the average time.

TO THE TEACHER.—The following examples for practice are arranged for the interest method of finding the cash balance. If preferable, however, the product method can be used by substituting "products" for the heading of the interest columns, and proceeding as in the product solution of the Illustrative Example.

348

EXAMPLES FOR PRACTICE.

1. Find the cash balance of the following on July 1, 1889, at 6% :

Dr. W. WALLACE *in account with* R. C. COLE. *Cr.*

		Days.	Interest.	Amounts.			Days.	Interest.	Amounts.
1889.					*1889.*				
Jan. 1	To Balance,	***	* **	$325 00	Feb. 1	By Cash,	***	* **	$325 00
Feb. 8	" Mdse.,	***	** **	450 00	Mar. 1	" "	***	* **	400 00
Mar. 20	" "	***	* **	386 00	May 13	" "	**	* **	250 00
May 2	" "	**	* **	520 00	June 5	" "	**	* **	300 00
July 1	" *Bal. of Int.*,			** **	July 1	" *Bal. of Interest*,		** **	
					" 1	" " *Acct.*,			*** **
			** **	**** **				** **	**** **
July 1	To Balance,			*** **					

2. Find the cash balance of the following on Oct. 5, 1889, at 6% :

Dr. S. R. RICE *in account with* J. SMOOT. *Cr.*

		Days.	Interest.	Amounts.			Days.	Interest.	Amounts.
1889.					*1889.*				
May 10	Mdse. 30 da.,	***	* **	$298 75	June 18	Cash,	***	* **	$275 45
June 8	" 90 "	**	* **	315 28	July 8	"	**	* **	341 60
July 20	" 60 "	**	* **	586 30	Aug. 2	"	**	* **	485 25
Aug. 25	" net,	**	* **	472 50	Oct. 5	*Bal. of Interest*,			* **
Oct. 5	*Bal. of Interest*,		* **		" 5	" " *Account*,			*** **
			** **	**** **				** **	**** **
Oct. 5	Balance,			*** **					

3. Find the cash balance of the following on Jan. 1, 1890, at 6% :

Dr. J. WHITE *in account with* E. HUNTON. *Cr.*

		Days.	Interest.	Amounts.			Days.	Interest.	Amounts.
1839.					*1889.*				
July 1	Balance,	***	** **	$416 80	Aug. 6	Cash,	***	* **	$380 10
" 18	Mdse. 3 mo.,	**	* **	391 15	Sept. 2	Note, 3 mo.,	**	* **	351 75
Aug. 12	" 1 "	***	* **	513 45	Oct. 19	" 1 "	**	* **	250 00
Sept. 25	" 2 "	**	* **	486 90	Nov. 1	Cash,	**	* **	425 50
1890.					*1890.*				
Jan. 1	*Bal. of Interest*,			** **	Jan. 1	*Bal. of Interest*,		** **	
					" 1	" " *Account*,			*** **
			** **	**** **				** **	**** **
Jan. 1	Balance,			*** **					

4. The following account was settled on Aug. 15, 1888. What was the amount paid, interest 6% ?

Dr. C. F. HAINES *in account with* ROBERT TYLER. *Cr.*

1888.		*Days.*	*Interest.*		*Amounts.*		*1888.*		*Days.*	*Interest.*		*Amounts.*	
Jan. 18	Mdse., 30 da.,	***	**	**	$450	00	Feb. 2	Cash,	***	*	**	$300	00
Feb. 10	" 10 "	***	**	**	872	00	Mar. 13	"	***	**	**	1000	00
" 16	" 60 "	***	**	**	920	00	Apr. 5	"	***	*	**	200	00
Mar. 8	Cash,	***	**	**	500	00	May 7	"	***	*	**	500	00
Apr. 5	Mdse., 30 da ,	***	**	**	638	00	June 19	"	**	*	**	400	00
May 29	" net,	**	*	**	424	00	Aug. 15	*Bal. of Interest,*		**	**		
Aug. 15	*Bal. of Interest,*				**	**	" 15	" " *Account,*				****	**
					****	**				**	**	****	**
Aug. 15	Balance,				****	**							

5. Find the amount due on the following account current on Sept. 1, 1889, including interest at 7%:

Dr. E. A. ELLICOTT *in a/c current with* R. S. BRISTOL. *Cr.*

1889.		*Days.*	*Interest.*		*Amounts.*		*1889.*		*Days.*	*Interest.*		*Amounts.*	
Jan. 1	Balance,	***	**	**	$1421	52	Jan 10	Cash,	***	**	**	$1000	00
" 5	Mdse., 30 da.,	***	*	**	127	60	Feb. 15	"	***	**	**	675	00
" 17	" 60 "	***	*	**	220	40	Mar. 7	Note, 90 da.,	**	*	**	500	00
Feb. 9	Cash,	***	**	**	500	00	Apr. 15	Cash,	***	*	**	300	00
" 15	Mdse., net,	***	**	**	783	50	May 20	"	***	*	**	250	00
Mar. 7	" 30 da.	***	*	**	176	25	June 14	"	**	*	**	200	00
Apr. 8	" 30 "	***	*	**	217	60	" 25	"	**	*	**	360	00
May 19	Bill, J. C. S.,	***	*	**	75	85	Sept. 1	*Bal. of Interest,*		**	**		
June 14	Mdse., 10 da.,	**	*	**	126	84	" 1	" " *Account,*				***	**
" 23	" 30 "	**	*	**	185	90							
Sept. 1	*Bal. of Interest,*				**	**							
			***	**	****	**				***	**	****	**
Sept. 1	Balance,				***	**							

6. W. P. Hill bought of W. G. Cowman the following merchandise: 1889.—May 23, $718.25 @ 90 days; June 13, $425.36 @ 60 days; Aug. 8, $381.75 @ 2 months; and Sept. 2, $482.60 @ 3 months. Hill paid Cowman $325 cash on June 2, 1889, and $471 cash on Aug. 3, 1889. Arrange the above in the form of an account current, and find the cash balance on Dec. 15, 1889, at 6% interest.

PARTNERSHIP.

349. A **Partnership** is an association of two or more persons who combine their capital, skill, labor, or all of them, for the purpose of conducting business; each person sharing the gains or losses in a certain proportion as stipulated in their agreement.

Rem. 1.—Such an association is called a *firm*, a *house*, or a *company*, and derives its name from the parties of whom it is composed; as J. E. Scott & Co., Wood & Fisher, etc. Each individual of the association is called a *partner*.

Rem. 2.—An *Active Partner* is one who is publicly known as such, sharing in all the profits or losses of the business. A *Silent Partner* is one who, though actually a partner, is not publicly known as such, his name being concealed to avoid liability to the creditors of the firm. If, however, it should become known that he has an actual interest in the business, he may be held equally liable with the other partners for all debts of the firm. A *Special Partner* is one who furnishes a certain portion of the capital of the firm, and holds himself liable for that amount only. If, however, the extent of his liability is not published and legally recorded he can be held equally liable with the other partners. A *Nominal Partner* is one who has no real interest in the business, but who assumes the responsibility of a partner by lending his name and credit to the partnership.

350. The **Capital** of a firm is the money or property invested in the business.

Rem. 1.—The *Assets* or *Resources* of a firm are its entire property, including all debts or obligations due the firm.

Rem. 2.—The *Liabilities* of a firm embrace all the debts or obligations due by the firm to its creditors.

Rem. 3.—The *Net Capital* of a firm is the excess of its assets or resources over its *outside* liabilities; that is, other liabilities than such as stand to any partner's credit.

351. The **Insolvency** of a firm is the total sum due the creditors in excess of the firm's assets or resources.

Rem. 1.—A firm is said to be insolvent when it owes more than its assets will enable it to pay.

Rem. 2.—When a firm commences business insolvent and realizes a gain which is less than the original insolvency, the remaining insolvency is called their *net insolvency*.

352. The **Investment** of a partner is the aggregate of the money or property contributed by him to the capital of the firm.

REM. 1.—The *Net Investment* of a partner is the difference between the sum of his investments and the sum of his withdrawals.

REM. 2.—The *Average Investment* of a partner is the average sum in continuous use of the firm, when several investments, withdrawals, or both, have been made by him at different times.

353. The **Net Gain** is the excess of the total gains of a firm over its total losses, within a certain period.

354. The **Net Loss** of a firm is the excess of its total losses over its total gains, within a given period.

355. A **Partnership Contract** is a written instrument, setting forth the agreement between the partners, specifying the amount of each partner's investment, the limitation of the partnership, the proportion allowed to be withdrawn by each partner, the proportion of profit or loss to each partner, and such other particulars as may be deemed necessary.

REM.—A partner may receive a regular salary in addition to his proportionate share of the profits, for taking charge of the books, or for some other special duty. A salary to a partner is an indebtedness of the *firm*, which is placed to the credit of the salaried partner at the close of each month, at the close of the year, at its average due date, etc., according to the partnership contract. In the following examples, salaries are supposed to be charged at the close of the year.

356. Computations in Partnership are made in accordance with the following formulas:

(*1.*) *Present worth = resources — liabilities.*

(*2.*) *Present insolvency = liabilities — resources.*

$$(3.)\ \textit{Gain} = \left\{\begin{array}{l} \textit{greater capital at closing} - \textit{capital at commencing.} \\ \textit{insolvency at commencing} + \textit{capital at closing.} \\ \textit{greater insolvency at commencing} - \textit{insolvency at closing.} \end{array}\right.$$

$$(4.)\ \textit{Loss} = \left\{\begin{array}{l} \textit{greater capital at commencing} - \textit{capital at closing.} \\ \textit{capital at commencing} + \textit{insolvency at closing.} \\ \textit{greater insolvency at closing} - \textit{insolvency at commencing.} \end{array}\right.$$

$$(5.)\ \textit{Capital at commencing} = \left\{\begin{array}{l}\textit{greater capital at closing} - \textit{gain.}\\ \textit{less capital at closing} + \textit{loss.}\\ \textit{loss} - \textit{insolvency at closing.}\end{array}\right.$$

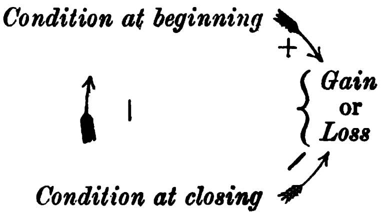
$$(6.)\ \textit{Capital at closing} = \left\{\begin{array}{l}\textit{less capital at commencing} + \textit{gain.}\\ \textit{great capital at commencing} - \textit{loss.}\\ \textit{gain} - \textit{insolvency at commencing.}\end{array}\right.$$

$$(7.)\ \textit{Insolvency at commencing} = \left\{\begin{array}{l}\textit{gain} - \textit{capital at closing.}\\ \textit{less insolvency at closing} + \textit{gain.}\\ \textit{greater insolvency at closing} - \textit{loss.}\end{array}\right.$$

$$(8.)\ \textit{Insolvency at closing} = \left\{\begin{array}{l}\textit{loss greater than capital} - \textit{capital.}\\ \textit{less insolvency at commencing} + \textit{loss.}\\ \textit{greater insolvency at commencing} - \textit{gain.}\end{array}\right.$$

To THE TEACHER.—The many formulas given above are likely to confuse the student. Much better results can be obtained by considering all profits or solvent conditions as ***plus quantities*** and all losses or insolvent conditions as ***minus quantities***. In a few minutes a class can learn enough of algebraic addition and subtraction, that is, addition and subtraction when one or both of the given terms are minus quantities, to apply the following

GENERAL FORMULA.



If any two of the three terms of this formula are given, perform the operations indicated by the sign between them, following the direction of the arrows, and the result will be the third term.

Thus, if a firm is solvent $5000 at commencing and loses $3000, its condition at closing will be +$5000 added to −$3000, or +$2000, that is, solvent $2000. If insolvent $5000 at commencing and the firm loses $3000, its condition at closing will be −$5000 added to −$3000, or −$8000, that is, insolvent $8000; but if the firm gains $3000, its condition will be −$5000 added to +$3000, or −$2000, that is, insolvent $2000. *Vice versa*, if a firm is insolvent $5000 at commencing and insolvent $3000 at closing, subtract −$5000 from −$3000, obtaining +$2000, or $2000 gain; but if the condition at commencement had been solvent $5000, subtracting +$5000 from −$3000 will produce −$8000, or $8000 loss, etc., etc.

To take the algebraic sum of two quantities if both are plus or both minus, add them and prefix the common sign; if one is minus and the other plus, subtract the less quantity from the greater and prefix the sign of the greater.

To take the algebraic difference of two quantities, change the sign of the subtrahend and proceed as in algebraic addition.

EXAMPLES FOR PRACTICE.

What is the gain or loss in 1888?

1. Capital, Jan. 1, '88, $4150.75; capital Jan. 1, '89, $8175.09.
2. Insolv., Jan. 1, '88, $2375.80; capital, Jan. 1, '89, $2846.30.
3. Insolv. Jan. 1, '88, $12420.38; insolv. Jan. 1, '89, $5407.25.
4. Capital Jan. 1, '88, $7250.19; capital Jan. 1, '89, $3251.28.
5. Capital Jan. 1, '88, $5260.80; insolvency Jan. 1, '89, $1380.30.
6. Insolv. Jan. 1, '88, $3243.25; insolv. Jan. 1, '89, $5124.50.

What is the capital or insolvency on Jan. 1, 1888?

7. Gain in 1888, $1825.50; capital Jan. 1, '89, $4760.50.
8. Loss in 1888, $3257.25; capital Jan. 1, '89, $5172.90.
9. Loss in 1888, $8254.20; insolvency Jan. 1, '89, $2125.
10. Gain in 1888, $5139.25; capital Jan. 1, '89, $4150.25.
11. Gain in 1888, $2730.45; insolvency Jan. 1, '89, $9140.25.
12. Loss in 1888, $2172.80; insolvency, Jan. 1, '89, $5282.30.

What is the capital or insolvency on Jan. 1, 1889?

13. Capital Jan. 1, '88, $9284.16; gain in '88, $4264.15.
14. Capital Jan. 1, '88, $18275.32; loss in '88, $2769.40.
15. Insolvency Jan. 1, '88, $391.26; gain in '88, $2825.50.
16. Capital Jan. 1, '88, $1126.40; loss in '88, $3285.16.
17. Insolvency Jan. 1, '88, $1926.42; loss in '88, $1815.20.
18. Insolvency Jan. 1, '88, $1538.25; gain in '88, $1282.95.
19. Capital Jan. 1, '88, $9260.25; loss in '88, $850.30.
20. Insolvency Jan. 1, '88, $875.20; loss in '88, $2937.50.

21. The firm of Sonn & Jones commenced business with the following resources: Cash $13500, real estate $12000, merchandise $17000. They owed on personal accounts $7500, and on bills payable $10000. What was their net capital?

22. In the above example what was the net gain or loss at the end of the first year, if they then had in stock merchandise $21000, due them on personal accounts $13522, bills receivable $9527, real estate $17000; and they owed on personal accounts $14712, and on notes $12356.50?

23. A firm commenced business as follows: Assets—cash $7500, merchandise on hand $4500, personal accounts due the firm $3200, bills receivable $875. Liabilities—due by the firm on personal accounts $2500, bills payable $3600. At the end of the first year they had $8000 in cash, $5600 in merchandise, $4100 due the firm on personal accounts; and they owed $4000 on personal accounts, and $900 on outstanding notes. What was the net gain or loss of the firm?

24. J. B. Will and A. Meyers formed a copartnership, the former investing $\frac{5}{8}$ of the capital, and the latter $\frac{3}{8}$, and agreed to share the gains and losses in proportion to their investments. At the date of settlement the resources of the firm consisted of cash $7240, bills receivable $1420, real estate $11600, merchandise $9320; and their liabilities were bills payable $6200, due Benton & Co., $2720, due on bond and mortgage $2250. If the net gain was $2410, how much did each partner invest in the business?

25. Wells & Green lost in the dry goods business $4342. Their resources are cash $1875, real estate $12500, merchandise $4700, personal accounts $2840, bills receivable $2400; and their liabilities are bills payable $3560, mortgage on real estate $8000, interest on mortgage $480, personal accounts $4625. If Wells invested $\frac{3}{5}$ of the capital and Green $\frac{2}{5}$, what was the investment of each partner?

26. S. R. Waite, H. Foley, and M. R. Wiley are partners in a firm. At commencing the firm was insolvent, Waite's insolvency being $\frac{2}{5}$ of the whole, Foley's $\frac{2}{5}$, and Wiley's $\frac{1}{5}$. They agreed to share the gains and losses equally. At closing, their books showed a net gain of $10779, their resources being, bills receivable $2417, cash $1560, merchandise $3320, real estate $4490, J. Nolan's account $6850, Miller & Co.'s account $2231; and their liabilities, notes and acceptances outstanding $1260, due to T. E. Taylor $3240, to H. Carter $3816, to W. E. Norman $2192, to J. E. Lowe $4581. What was each partner's insolvency at commencing? What was each partner's capital at closing?

357. To divide the gain or loss among partners in proportion to their investments, if contributed for equal periods.

ILLUSTRATIVE EXAMPLE.

A and B engaged in business, A investing $8500 and B $6000. The net profit for the first year was $2500, which was divided between them in proportion to their respective investments. What was each partner's share of the gain?

SOLUTION.

$\$8500 + \$6000 = \$14500$, total capital.

$$\begin{array}{r|l} & \overset{5}{\cancel{\$2500}} \\ \underset{29}{\cancel{\$14500}} & \$8500 \end{array} \qquad\qquad \begin{array}{r|l} & \overset{5}{\cancel{\$2500}} \\ \underset{29}{\cancel{\$14500}} & \$6000 \end{array}$$

$(\$5 \times 8500) + 29 = \$1465.52-$, A's profit. $(\$5 \times 6000) + 29 = \$1034.48+$, B's profit.

RULE.—*Find each partner's gain or loss by the Rule of Three,* **331**, *taking the total profit or loss and each partner's investment as right-hand terms, and the total investment as the left-hand term.*

EXAMPLES FOR PRACTICE.

1. J. W. Long, J. C. Shimer, and C. F. Dorr formed a partnership, and invested respectively $8400, $9600, $10800. The net profit for the first year was $7200, which was divided in proportion to the partners' investments. What was each partner's share of the gain ?

2. Three partners invested respectively $2900, $3400, and $4200 in business, and agreed to share the gains or losses in proportion to their investments. If the net profit is $3150, how much should be each partner's share ?

3. W. Russell and M. Florence are partners in the grain business. Russell invested 8000 bushels of wheat, invoiced at 90 cents per bushel, and Florence's investment embraced personal accounts previously due him amounting to $500, bills receivable $800, cash $700, and a warehouse valued at $4500. Their profits for the year were $4425. In proportion to their investments, how much of the gain should each receive ?

4. S. Arnold and R. A. Brown engaged in business with a joint capital of $13100. What was the investment of each, if Arnold's gain was $1800 and Brown's $3440 ?

5. J. K. Reid and L. J. Haines engaged in business, the former investing $18000 and the latter $16000, and agreed to share the gains and losses in proportion to investment. At the end of the year their resources were $58000 and their liabilities $9000. If Reid was to receive a salary of $1500 per annum for taking charge of the books, and nothing was withdrawn by either during the year, what was the interest of each partner at the end of the year ?

358. To divide the gain or loss among partners in proportion to their investments, if contributed for unequal periods.

ILLUSTRATIVE EXAMPLE.

J. W. Bond and N. A. Wright engaged in business on Jan. 1, 1889, each investing $6000. On March 1, Wright made an additional investment of $3000, and Bond withdrew from the firm $1500; on July 1, Bond invested $2900, and Wright withdrew $3000. If the profits during the year, $4620, were to be divided between the partners in proportion to their investments, what was the share of each?

SOLUTION.

From Jan. 1 to Mar. 1 = 2 mo.	Hence, $6000 × 2 =	$12000 for 1 mo.
	Withdrawal, 1500	
" Mar. 1 to July 1 = 4 mo	Hence, $4500 × 4 =	18000 " " "
	Investment, 2900	
" July 1 to Jan. 1 = 6 mo.	Hence, $7400 × 6 =	44400 " " "
	Bond's investment =	$74400 " " "

$74400 ÷ 12 = $6200, Bond's average investment through the year.

From Jan. 1 to Mar. 1 = 2 mo.	Hence, $6000 × 2 =	$12000 for 1 mo.
	Investment, 3000	
" Mar. 1 to July 1 = 4 mo.	Hence, $9000 × 4 =	36000 " " "
	Withdrawal, 3000	
" July 1 to Jan. 1 = 6 mo.	Hence, $6000 × 6 =	36000 " " "
	Wright's investment =	$84000 " " "

$84000 ÷ 12 = $7000, Wright's average investment through the year.

$6200 + $7000 = $13200, average investment of the firm for the year.

$$\begin{array}{r|l} & \overset{35}{\$\cancel{4620}} \\ \overset{\cancel{132}}{\$\cancel{13200}} & \underset{62}{\$\cancel{6200}} \end{array} \qquad \begin{array}{r|l} & \overset{35}{\$\cancel{4620}} \\ \overset{\cancel{132}}{\$\cancel{13200}} & \underset{70}{\$\cancel{7000}} \end{array}$$

$35 × 62 = $2170, Bond's gain. $35 × 70 = $2450, Wright's gain.

EXPLANATION.—Bond's capital on Jan. 1 ($6000) remained unchanged to March 1, or for 2 months; and $6000 invested for 2 months equals 2 times $6000, or $12000, invested for 1 month. On March 1, Bond withdrew $1500, leaving $4500 in the business, which remained unchanged to July 1, or for 4 months, and $4500 invested for 4 months equals 4 times $4500, or $18000 invested for 1 month. On July 1, Bond invested $2900, making his investment $7400, which remained unchanged to the end of the year, or for 6 months, and $7400 invested for 6 months equals 6 times $7400, or $44400 invested for 1 month. Hence, Bond's total investment equals $12000 + $18000 + $44400, or $74400 invested for 1 month, or an average investment of $\frac{1}{12}$ of $74400, or $6200 for the year.

Similarly, Wright's average investment for the year is found to be $7000.

Divide the gains for the year ($4620) between the partners in proportion to their average investments, by Rule, **357**.

RULE.—I. *Multiply each partner's original investment by the months or days between the date of such investment and the date of his next withdrawal or investment.*

II.—*Multiply the increased or decreased investment then in the business by the number of months or days between the date of such increase or decrease and the date of the next increase or decrease; and so continue to the date of adjusting the gains or losses.*

III. *Divide the sum of the products thus obtained by the number of months or days in the period of adjustment, to find each partner's average investment.*

IV. *Divide the gains or losses among the partners in proportion to their average investments, by Rule,* **357**.

NOTE.—To avoid fractions, the gains or losses may be adjusted by the Rule of Three by taking the total gain or loss and each partner's average investment per month or day as the right-hand terms, and the average capital per month or day as the left-hand term.

EXAMPLES FOR PRACTICE.

1. A, B, and C formed a copartnership, and invested respectively $9600, $8400, and $7200. At the end of 4 months, A invested $2000 additional, B $1400, and C $800. If the net gains during the year were $12800, and all gains and losses were to be divided in proportion to their average investments, what was each partner's share of the profits?

2. Wilson, Holmes, and Farr engaged in business on Jan. 1, 1889, Wilson investing $8000, Holmes $5000, and Farr $6000. On May 1, Wilson put in $2000 additional; on July 1, he withdrew $3000; and on Oct. 1, he put in $1500. On Feb. 1, Holmes withdrew $1000; on Apr. 1, he put in $4000; on Aug. 1, he withdrew $500; and on Sept. 1, he put in $3000. On July 1, Farr put in $2500 additional. At the end of the year the net gain was found to be $5000. What was each

partner's share of the gain if it was divided in proportion to their average investments ?

3. Harvey and Roberts engage in business Jan. 1, 1889, Harvey investing $5000, and Roberts $8000. On May 1, Harvey invested $2000 additional; and on June 1, Roberts withdrew $1000. On Aug. 1, they took in Williams as a third partner, who invested $7000. On Jan. 1, 1890, their net gain was $5000. What was the share of each partner ?

4. On Jan. 1, S. H. Merritt and J. L. Reeves formed a copartnership for one year, each investing $8000. On Apr. 1, Merritt withdrew $3200, and Reeves $1600 ; on Sept. 1, each withdrew $2400 ; and on Nov. 1, Merritt withdrew $900, and Reeves $700. At the end of the year, the partnership was dissolved with a capital of $6000. What proportion of the capital should each partner then receive ?

359. To adjust the partners' accounts when the proportion of profit or loss is fixed, and interest is allowed on the excess and charged on the deficiency of each partner's required investment.

ILLUSTRATIVE EXAMPLE.

On Jan. 1, 1887, E. J. Ayres, R. W. Malley, and J. A. Manning entered into copartnership. It was agreed that Ayers should contribute $\frac{4}{9}$ of the capital and receive $\frac{4}{9}$ of the gains ; that Malley should contribute $\frac{3}{9}$ of the capital and receive $\frac{3}{9}$ of the gains ; and that Manning should contribute $\frac{2}{9}$ of the capital and receive $\frac{2}{9}$ of the gains. Interest at the rate of 10% per annum was to be allowed on the excess, and charged on the deficiency of each partner's required investment. A settlement was effected at the end of the year, at which time the net gain was found to be $3600. Find Ayres's and Malley's net investment, and Manning's net insolvency on Jan. 1, 1888, if the following is a statement of each partner's account :

Dr. E. J. AYRES. *Cr.*

1887—Apr. 23,	Withdrawal,	$1500	1887—Jan. 1,	Investment,	$16000
" June 16,	"	800	" Mar. 18,	"	2400
" Aug. 17,	"	900	" Oct. 20,	"	3000
	Total withdrawal,	$3200		Total investment,	$21400

R. W. MALLEY.

Dr.			Cr.		
1887—July 28,	Withdrawal,	$600	1887—Jan. 1,	Investment,	$12000
" Dec. 4,	"	800	" " 21,	"	1800
			" May 17,	"	600
	Total withdrawal,	$1400		Total investment,	$14400

J. A. MANNING.

Dr.			Cr.		
1887—Mar. 30,	Withdrawal,	$6000	1887—Jan. 1,	Investment,	$6000
" Sept. 5,	"	4000	" Aug. 3,	"	600
	Total withdrawal,	$10000		Total investment,	$6600

SOLUTION.

(While partnership settlements are usually made on Jan. 1 of each year, and are so dated, they include no entries later than those of the preceding day (Dec. 31). Hence, in finding the time between the date of an investment or withdrawal and the date of settlement, it is customary among business men to consider Dec. 31, inclusive, as the last day of the interval.)

Int. of $16000 from Jan. 1, '87, to Jan. 1, '88, at 10%,	$1600.00	
" 2400 " Mar. 18, " " " " (288 da.)	192.00	
" 3000 " Oct. 20, " " " " (72 da.)	60.00	
Int. of Ayres's total investment, ...		$1852.00
Int. of $1500 from Apr. 23, '87, to Jan. 1, '88, at 10% (252 da.)	$105.00	
" 800 " June 16, " " " " (198 da.)	44.00	
" 900 " Aug. 17, " " " " (136 da.)	34.00	
Int. of Ayres's total withdrawal, ...		$183.00
" " average investment, . . .		$1669.00

Int. of $12000 from Jan. 1, '87, to Jan. 1, '88, at 10%,	$1200.00	
" 1800 " Jan. 21, " " " " (344 da.)	172.00	
" 600 " May 17, " " " " (228 da.)	38.00	
Int. of Malley's total investment, ...		$1410.00
Int. of $600 from July 28, '87, to Jan. 1, '88, at 10% (156 da.)	$26.00	
" 800 " Dec. 4, " " " " (27 da.)	6.00	
Int. of Malley's total withdrawal, ...		$32.00
" " average investment, . . .		$1378.00

Int. of $6000 from Jan. 1, '87, to Jan. 1, '88, at 10%,	$600.00	
" 600 " Aug. 3, " " " " (150 da.)	25.00	
Int. of Manning's total investment, . . .		$625.00
Int. of $6000 from Mar. 30, '87, to Jan. 1, '88, at 10% (276 da.)	$460.00	
" 4000 " Sept. 5, " " " " (117 da.)	130.00	
Int. of Manning's total withdrawal, . . .		$590.00
" " average investment, . .		$35.00

$1669 + $1378 + $35 = $3082, interest of firm's average capital.
$\frac{4}{9}$ of $3082 = $1369.78, interest of Ayres's required investment.
$\frac{1}{3}$ of $3082 = $1027.33, " Malley's " "
$\frac{2}{9}$ of $3082 = $684.89, " Manning's " "

$21400—$3200 = $18200.00, Ayres's net investment, as per his account.
$\frac{4}{9}$ of $3600 = 1600.00, " proportion of the total profits.
$1669—$1369.78 = 299.22, int. of Ayres's excess over required investment.
$20099.22, amount due Ayres, Jan. 1, 1888.

$14400—$1400 = $13000.00, Malley's net investment, as per his account.
$\frac{1}{3}$ of $3600 = 1200.00, " proportion of the total profits.
$1378—$1027.33 = 350.67, int. of Malley's excess over required investment.
$14550.67, amount due Malley, Jan. 1, 1888.

$10000—$6600 = $3400.00, Manning's indebtedness, as per his account.
$684.89—$35 = 649.89, int. of Manning's deficiency of required investment.
$4049.89, Manning's total indebtedness.
$\frac{2}{9}$ of $3600 = 800.00, " proportion of the total profits.
$3249.89, " net indebtedness, Jan. 1, 1888.

EXPLANATION.—Separately compute the interest on the investments and withdrawals of each partner, and obtain the interest balance of each account, which will also be the interest of each partner's average investment. Then add the interest of Ayres's average investment ($1669), of Malley's ($1378), and of Manning's ($35), to find the interest of their total average investment, or of the firm's average capital ($3082). Since Ayres agreed to contribute $\frac{4}{9}$ of the capital of the firm, $\frac{4}{9}$ of the interest of the average capital of the firm, or $1369.78, must be the interest of Ayres's required investment; but the interest of Ayres's actual investment ($1669) is $299.22 greater than the interest of his required investment ($1369.78). Therefore the interest of this excess of investment ($299.22) should be placed to his credit as per agreement, which ($299.22) with his share of the profits ($1600) should be added to the balance of his account ($18200), to find the sum due him by the firm ($20099.22).

The sum due Malley is similarly obtained. Manning's indebtedness, as per account, is $1400; besides which he owed the remaining partners $649.89 as interest on the deficiency of his required investment. His total indebtedness would therefore have been $4049.89, were it not for his share of the gain ($800), which reduced his indebtedness to $3249.89.

RULE.—I. *Compute the interest on the investments and withdrawals of each partner from the time they were made to the date of settlement, and subtract the debit interest from the credit interest of each partner's account to find his net interest, or the interest of his net average investment* (see *a*, Note).

II. *Add the balances of interest thus found to obtain the interest on the firm's net average capital* (see *b*, Note); *and take such proportions of this result as each partner agreed to contribute to the capital of the firm.*

III. *If the interest of any partner's net average investment exceeds the interest of his required investment, the difference will be the interest due to such partner; if less, the difference will be the interest due from such partner* (*c*, Note).

IV. *Find the capital or indebtedness of each partner at closing by Formula 6 or 8,* **356.**

NOTE.—(*a*) Should any partner's total debit interest be greater than his total credit interest, the difference will be the interest of such partner's net average indebtedness to the firm; (*b*) which should be subtracted from the sum of the interest balances to the credit of the remaining partners, to find the interest of the firm's net average capital. (*c*) The interest of any partner's net average indebtedness to the firm should be added to the interest of his required investment to find the total interest due from such partner.

EXAMPLES FOR PRACTICE.

1. J. R. Cross and J. J. Davies engaged in business on Jan. 1, 1888, and agreed to invest equally and share equally in gains and losses. Interest at 8% was to be allowed on the excess, and charged on the deficiency of each partner's required investment. The following is a statement of each partner's account:

Dr. J. R. CROSS. *Cr.*

1888—Apr. 3,	Withdrawal,	$3000	1888—Jan. 1,	Investment,	$6000
" June 7,	"	1500	" Mar. 14,	"	4000
" Oct. 15,	"	800	" July 3,	"	3000
			" Sept. 18,	"	5000

Dr.			J. J. DAVIES.		Cr.
1888—Mar. 8,	Withdrawal,	$2000	1888—Jan. 1,	Investment,	$4000
“ Aug. 15,	“	1000	“ May 12,	“	5000
“ Sept. 30,	“	1200	“ July 16,	“	2000
			“ Oct. 5,	“	1500

If the net gains during the year were $5800, what was the condition of each partner's account on Jan. 1, 1889?

2. L. A. Horn and J. R. McLane engaged in business on Jan. 1, 1888, each agreeing to furnish $\frac{1}{2}$ the capital of the firm and to share $\frac{1}{2}$ the gains and losses, McLane to receive a salary of $800 per annum for taking charge of the books; and interest at 8% per annum to be allowed on the excess, and charged on the deficiency of either partner's required investment. On Jan. 1, 1888, Horn invested $9000; on Mar. 1, $5000; on Aug. 3, $3000. On Jan. 1, 1888, McLane invested $8000; on July 20, $3000; on May 15, he withdrew $500, and on Oct. 8, $300. The net gains during the year were $6200. What was each partner's interest in the firm on Jan. 1, 1889? How much interest did McLane owe Horn?

3. On Jan. 1, 1889, T. Herndon, W. Newman, and B. P. Dulin engaged in business, Herndon agreeing to furnish $\frac{3}{7}$ of the capital and receive $\frac{3}{7}$ of the gains; Newman and Dulin each to furnish $\frac{2}{7}$ of the capital and receive $\frac{2}{7}$ of the gains; and interest at 10% per annum was to be allowed on the excess, and charged on the deficiency of each partner's required investment. On Jan. 1, 1889, Herndon invested $8000; on Aug. 12, $6000; and on July 3, withdrew $3000. On Jan. 1, 1889, Newman invested $5000; and on July 3, withdrew $800. On Jan. 1, 1889, Dulin invested $5000; on Mar. 15, $2000; on June 18, $1500; on Sept. 1, $800; on May 1, he withdrew $1200; and on Sept. 9, $500. The net gain during the year was $5300. What was each partner's interest in the firm on Jan. 1, 1890? How much interest did Newman owe Herndon and Dulin?

4. H. Gould, J. Hunter, and S. W. Nichols formed a copartnership on Jan. 1, 1889, and agreed to invest equal sums, and share equally in gains and losses; interest at 9% per annum being allowed on the excess, and charged on the deficiency of each partner's required investment. H. Gould invested $8000 on Jan. 1, $6400 on Apr. 15, $4000 on May 20, $1600 on June 24; and withdrew $1200 on May 5,

$1800 on Aug. 13, and $2400 on Nov. 16. J. Hunter invested $7000 on Jan. 1, $9000 on July 24, $1600 on Oct. 12; and withdrew $6000 on Mar. 26, $8000 on May 5, $5500 on June 14, and $1200 on Sept. 2. S. W. Nichols invested $7500 on Jan. 1, $6000 on Jan. 25, $4000 on Feb. 14, $2800 on May 25; and withdrew $2000 on Apr. 27, $3600 on June 4, and $800 on Aug. 2. If the net gain during the year was $6842.19, what was each partner's interest in the firm on Jan. 1, 1890? How much interest did Hunter owe the remaining partners?

5. G. W. Hixon, R. Merritt, and W. Pickett entered into copartnership to invest equally, and share equally in gains or losses, and agreed to allow interest on each partner's excess, or charge interest on each partner's deficiency of required investment. At the date of adjustment, Hixon's account showed an excess of withdrawals of $900; of debit interest of $314.62. Merritt's excess of debit interest was $214.42; but his investments exceeded his withdrawals by $1200. Pickett's excess of debit interest was $312.80, and he overdrew his investment by $1400. If there was a net loss of $1800, what was the condition of each partner's account at the date of adjustment?

6. J. P. Smith, J. A. Harrison, and W. Yeatman formed a copartnership, Smith agreeing to furnish $\frac{6}{15}$ of the capital, Harrison $\frac{5}{15}$, and Yeatman $\frac{4}{15}$. At the end of the first year a settlement was effected, the books showing that Smith's account was overdrawn by $1500, Harrison's by $1200, and Yeatman's by $960. There was a net gain of $6000, which was shared in proportion to each partner's required investment. Interest was allowed on the excess, and charged on the deficiency of each partner's required investment. Smith's excess of debit interest was $124.62, Harrison's $120.81, and Yeatman's $98.07. How much was due each partner at the date of settlement?

7. S. R. White and J. Moore jointly purchased the steamer Excelsior for $90000. White paid $\frac{2}{3}$ of this amount, and Moore paid $\frac{1}{3}$. During the season White paid from his private funds, for wages, fuel, etc., $960, and drew out $300. Moore paid steamer's expenses, amounting to $680, and drew out $150. The partners realized a net gain of $8400, and then sold the steamer for $92000, receiving $46000 in cash, and a note for $46000, which White accepted at 5% discount, to apply on his account. The gains and losses were divided equally. How much was due each partner at closing?

APPENDIX.

360. To add by grouping figures into convenient sets.

ILL. EXAMPLE.

$$\begin{array}{lll}
\begin{array}{l}
\left.\begin{array}{r}3\\2\\1\\4\end{array}\right\}10\\
\left.\begin{array}{r}3\\5\end{array}\right\}8\\
9\\
\left.\begin{array}{r}7\\3\end{array}\right\}10\\
9\\
\left.\begin{array}{r}6\\2\end{array}\right\}8\\
8\\
\left.\begin{array}{r}3\\2\\4\end{array}\right\}9\\
8\\
\left.\begin{array}{r}6\\3\end{array}\right\}9
\end{array}
&
\begin{array}{l}
\left.\begin{array}{r}6\\3\end{array}\right\}9\\
8\\
\left.\begin{array}{r}5\\2\end{array}\right\}7\\
\left.\begin{array}{r}1\\4\\5\end{array}\right\}10\\
\left.\begin{array}{r}3\\4\\2\end{array}\right\}9\\
8\\
\left.\begin{array}{r}3\\5\end{array}\right\}8\\
\left.\begin{array}{r}1\\2\\5\end{array}\right\}8\\
\left.\begin{array}{r}2\\7\end{array}\right\}9
\end{array}
&
\begin{array}{l}
\left.\begin{array}{r}2\\5\\3\end{array}\right\}10\\
\left.\begin{array}{r}6\\2\end{array}\right\}8\\
7\\
\left.\begin{array}{r}4\\1\\3\end{array}\right\}8\\
\left.\begin{array}{r}5\\4\end{array}\right\}9\\
\left.\begin{array}{r}6\\2\end{array}\right\}8\\
\left.\begin{array}{r}3\\5\\1\end{array}\right\}9\\
8\\
\left.\begin{array}{r}3\\4\end{array}\right\}7
\end{array}
\\ \hline
96 & 3 & 4
\end{array}$$

Explanation.—To add rapidly, it is necessary to acquire facility in grouping the figures of a column into convenient sets, whose sum can be determined at a glance. The greater number of skilled accountants add but one column at a time; yet, by combining two or more figures whenever possible, and adding these combinations instead of the figures of which they are composed, results are obtained so quickly as to excite the surprise of those unacquainted with the process. Thus: to add the columns herewith appended, group into convenient sets (which are here enclosed in braces to be better understood by the pupil, but which in practice should be discarded) and add the numbers represented by these sets; as follows, 7+8+9+8+9+8+7+8+10 = 74 units, or 7 tens and 4 units. Carry the 7 tens to the tens' column, obtaining 7+9+8+8+8+9+10+7+8+9 = 83 tens, or 8 hundreds and 3 tens. Carry the 8 hundreds to the hundreds' column, obtaining 8+9+8+9+8+8+9+10+9+8+10 = 96 hundreds, making the sum of the three columns equal to 9634.

In grouping, it is best for the learner at first not to form sets in excess of 10.

To add two columns at once.—When the addends are few and consist of two or three figures each, students should acquire facility in obtaining the totals in the following manner: \$58+\$46+\$35 = 50+40+30+8+6+5 = \$139. \$2.75 +\$3.28+\$4.67 = \$2+\$3+\$4+70¢+20¢+60¢+5¢+8¢+7¢ = \$10.70. Similarly add the following:

\$65	\$41	\$92	\$85	\$51	\$2.48	\$5.31	\$7.15
32	68	57	73	87	3.27	2.95	9.27
78	34	46	26	34	8.52	4.86	6.78

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
3	7	5	41	31	85	253	516
8	3	2	58	27	54	138	273
6	4	1	34	62	48	847	832
5	8	8	72	16	32	413	127
2	5	2	67	75	76	272	983
7	3	7	25	83	27	765	275
1	2	1	13	28	63	994	764
5	7	6	92	54	84	283	353
3	2	2	84	12	12	812	626
8	1	8	38	47	41	647	189
6	4	1	12	83	38	733	841
2	8	4	79	35	29	528	237
1	3	5	23	21	75	264	723

(9.)	(10.)	(11.)	(12.)
224639865	3878945	72813456	538193716
442314143	2127624	29738268	445362437
343823234	5622346	46382759	758275354
827231421	8246393	58349271	394583725
214482178	2933528	43627183	876345678
145358723	7285342	29148364	691827364
528442837	3424637	75314627	253458926
393214253	4379464	28392673	514875343
724283645	5232521	84736829	482692736
272828432	3743255	41538294	854829176
489273229	6122732	32839267	382453826
162791318	1898326	75482739	623432714
323185986	2313652	34265873	255783468
873923882	3635218	82895852	987375384
418146179	5252867	24376437	142839267
152439652	9718343	65843745	483948945
237397254	2383734	46278426	827658372
689281543	6522425	58745954	586573486
341813437	3274612	93426374	712843943
734252154	7432536	35786453	347391658
125658415	8256541	87438265	439247386

361. To find the remainder in one operation, when a minuend is to be diminished by the sum of two or more subtrahends.

ILLUSTRATIVE EXAMPLE.

A merchant had bought 7135 bushels of corn, of which he sold 1204 bushels to A, 934 bushels to B, 796 bushels to C, and 1439 bushels to D. How many bushels had he remaining ?

SOLUTION.
7135
1204
934
796
1439
2762

EXPLANATION.—Write the minuend (7135) and draw a line underneath, then write the several subtrahends, placing units under units, tens under tens, etc. Then add the units' orders of the subtrahends, producing 9+6+4+4 = 23. The additional units required to make the right-hand figure of this result (23) correspond with the units' order of the minuend being 2 (23+2 = 25), 2 will be the units' order of the final remainder. Carry 2 tens to the sum of the tens' order of the subtrahends, and add, producing 2+3+9+3+0 = 17 tens. The additional tens required to make the right-hand figure of this sum (17) correspond with the tens' order of the minuend (3), being 6 (17+6 = 23), 6 will be the tens' order of the final remainder. Proceeding in this manner, it will be found that the sum of the hundreds' column (24) lacks 7 (24+7 = 31) of corresponding with the hundreds' order of the minuend (1); and the sum of the thousands' column (5) lacks 2 (5+2 = 7) of corresponding with the thousands' order of the minuend (7). Therefore 7 is the hundreds' order, and 2 the thousands' order of the final remainder, and 2762 bushels of corn is the total remainder.

Find the balances of the following ledger accounts :

(1.)		(2.)		(3.)	
Dr.	*Cr.*	*Dr.*	*Cr.*	*Dr.*	*Cr.*
$7143.28	$1456.30	$3127.48	$7124.58	$2783.40	$4121.38
273.82	283.95	286.32	8296.25	251.31	5938.24
9549.37	2736.72	750.45	432.30	1586.90	726.35
4378.62	45.34	1430.15	50.80	732.35	4829.60
529.15	383.27	8274.20	1437.15	4829.17	2748.42
75.32	987.36	326.75	2137.80	1238.25	3214.50
918.24	2478.31	2183.92	450.16	2874.32	291.32
46.38	*****.**	7189.35	****.**	****.**	450.38
$22914.18	$22914.18	$23568.62	$23568.62	$22320.19	$22320.19

4. A merchant's total deposits at bank were $15372.65, and he withdrew at different times $875.92, $439.25, $6281.50, and $2738.65. How much had he remaining in bank ?

362. To multiply when the multiplier is 11, and the multiplicand consists of two figures whose sum is less than 10.

ILLUSTRATIVE EXAMPLE.

Multiply $35 by 11.

Explanation.—$3+5=8$; and 8 placed between 3 and 5 produces $385, the product. Hence,

Rule.—*Between the two figures of the multiplicand write their sum.*

Note.—When the sum of the two orders of the multiplicand is more than 10, add the excess to the left-hand order.

EXAMPLES FOR PRACTICE.

Multiply

1. $52 by 11.
2. $72 by 11.
3. $36 by 11.
4. $53 by 11.
5. 25 rods by 11.
6. 18 miles by 11.

Multiply

7. $16 by 11.
8. $81 by 11.
9. $27 by 11.
10. $45 by 11.
11. $34 by 11.
12. $62 by 11.

363. To multiply any number by 11.

ILLUSTRATIVE EXAMPLE.

Multiply 4762 pounds by 11.

Explanation.—Take the units' figure of the multiplicand (2) as the units' figure of the product; and for the remaining figures of the product take the units' figure (2) plus the tens' figure (6), obtaining 8; then the tens' figure (6) plus the hundreds' figure (7), obtaining 13; write 3, and carry 1 to the hundreds' figure (7) plus the thousands' figure (4), obtaining 12; write 2, and carry 1 to the thousands' figure (4), obtaining 5; making the total product 52382 pounds. Hence,

Rule.—*To the left of the units' order write successively the units plus the tens, the tens plus the hundreds, the hundreds plus the thousands, etc., carrying as usual, and terminating at the left with the highest order, or the highest order plus the carrying figure.*

EXAMPLES FOR PRACTICE.

Multiply

1. $5136 by 11.
2. $72638 by 11.
3. $514832 by 11.

Multiply

4. $37281.05 by 11.
5. $41268.45 by 11.
6. $3275.75 by 11.

Multiply

7. $48263.25 by 11.
8. $94873.30 by 11.
9. $15238.65 by 11.

364. To multiply when the multiplier consists of two figures and its units' order is 1.

ILLUSTRATIVE EXAMPLE.

Multiply $387 by 41.

Explanation.—Take the units' figure of the multiplicand (7) as the units' figure of the product, and for the remaining figures of the product take 4 times the units' figure (7), plus the tens' figure (8), obtaining: 36. Write 6, and carry 3 to 4 times the tens' figure (8), plus the hundreds' figure (3), obtaining 38. Write 8, and carry 3 to 4 times the hundreds' figure (3), obtaining 15; making the complete product $15867. Hence,

Rule.—*Take the units' order of the multiplicand as the units' order of the product; and multiply the tens' order of the multiplier into each order of the multiplicand, adding the next higher order of the multiplicand to the successive products thus obtained, and carrying as in ordinary multiplication.*

EXAMPLES FOR PRACTICE.

Multiply
1. $2735.50 by 51.
2. $1824.75 by 81.
3. $5104.30 by 31.

Multiply
4. $6238.65 by 21.
5. $7415.40 by 61.
6. $8291.25 by 91.

Multiply
7. $2134.15 by 71.
8. $5827.80 by 41.
9. $2584.93 by 11.

365. To multiply when the multiplier consists of two figures and its tens' order is 1.

ILLUSTRATIVE EXAMPLE.

Multiply $873 by 19.

Explanation.—Multiply the units' figure of the multiplicand (3) by the units' figure of the multiplier (9), obtaining 27. Write 7, and carry 2 to 9 times the tens' figure of the multiplicand (7), increased by its units' figure (3), obtaining 68. Write 8, and carry 6 to 9 times the hundreds' figure of the multiplicand (8), increased by its tens' figure (7), obtaining 85. Write 5, and carry 8 to the hundreds' figure of the multiplicand (8), obtaining 16; making the complete product $16587. Hence,

Rule.—*Multiply the units' order of the multiplier into each order of the multiplicand, adding the next lower order of the multiplicand to the successive products thus obtained, carrying* *as in ordinary multiplication, and finally adding the last carrying figure to the left-hand order of the multiplicand.*

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply
1. $7125.45 by 16.	5. $2580.95 by 18.	9. $9125.25 by 17.
2. $815.30 by 13.	6. $1497.60 by 15.	10. $8174.85 by 19.
3. $758.42 by 14.	7. $917.36 by 16.	11. $415.18 by 18.
4. $925.35 by 18.	8. $532.16 by 19.	12. $214.46 by 15.

366. To multiply when an aliquot part or a multiple of the multiplier will form a more convenient multiplier.

ILLUSTRATIVE EXAMPLE.

Multiply $75 by 56, and $628 by 25.

FIRST SOLUTION.

$$\begin{array}{lrl} 8 \text{ times } \$75 & = & \$600 \\ \frac{1}{8} \text{ of } \quad 56 & = & \underline{\quad 7} \\ & & \$4200 \end{array}$$

SECOND SOLUTION.

$$\begin{array}{lrl} \frac{1}{4} \text{ of } \quad \$628 & = & \$157 \\ 4 \text{ times } 25 & = & \underline{\quad 100} \\ & & \$15700 \end{array}$$

EXPLANATION.—Both of the above solutions are practical applications of the principle that to multiply one factor and divide the other by the *same* number does not affect the value of their product.

RULE.—*Multiply one factor and divide the other by such a number as will simplify the multiplication.*

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply	Multiply
1. $35 by 15.	5. $72 by 75.	9. $150 by 48.	13. $36 by 33⅓.
2. $42 by 18.	6. $36 by 25.	10. $64 by 25.	14. $72 by 16⅔.
3. $24 by 16.	7. $52 by 250.	11. $35 by 18.	15. $24 by 13⅓.
4. $18 by 45.	8. $42 by 350.	12. $28 by 450.	16. $96 by 12½.

367. To multiply when one part of the multiplier is an aliquot of another part.

ILLUSTRATIVE EXAMPLE.

Multiply $2574 by 186, and $5462 by 856.

FIRST SOLUTION.

$2574
186
15444
46332
$478764

SECOND SOLUTION.

$5462
856
43696
305872
$4675472

EXPLANATION.—In the first solution, since one part of the multiplier (18) is *three times* the remaining part (6), first multiply $2574 by 6, obtaining $15444; then take three times 15444 as the product by 3 times 6, or 18 tens, obtaining 46332 tens; and add the two results.

In the second solution, since one part of the multiplier (56) is *seven times* the remaining part (8), first multiply $5462 by 8 hundreds, obtaining 43696 hundreds; then take seven times 43696 as the product by 7 times 8, or 56 units, obtaining 305872 units; and add the two results. Hence,

RULE.—*Multiply by the selected order of the multiplier; and take as many times the obtained result as the remaining orders of the multiplier are times the order used. Add the partial products.*

EXAMPLES FOR PRACTICE.

Multiply
1. $385.85 by 642.
2. $742.50 by 328.
3. $82615 by 729.

Multiply
4. $4265.25 by 255.
5. $9541.12 by 546.
6. $172.48 by 763.

Multiply
7. $2153.28 by 18624.
8. $530.75 by 315.
9. $618.40 by 16412.

368. To multiply when the multiplier is a little more or a little less than 100, 1000, 10000, etc.

ILLUSTRATIVE EXAMPLE.

Multiply $385.20 by 998.

SOLUTION.

$385200
770.40.
$384429.60. *Ans.*

EXPLANATION.—Multiply $385.20 by 1000; the resulting product (385200) will be 1000 − 998, or 2 times $385.20 too great. Hence deduct 2 times $385.20, or $770.40, from $385200, obtaining the true product, $384429.60.

RULE.—*Multiply by the nearest number of 100's, 1000's, 10000's, etc.; and add to the result as many times the multiplicand as the true multiplier is units more than the assumed multiplier; or subtract from the result as many times the multiplicand as the true multiplier is units less than the assumed multiplier.*

EXAMPLES FOR PRACTICE.

Multiply
1. $425 by 999.
2. $865 by 98.
3. $735.25 by 998.

Multiply
4. $258.30 by 104.
5. $827.58 by 1003.
6. $516 by 1.02.

Multiply
7. $989 by 992.
8. $99 by 97.
9. $1005 by 1002.

369. To multiply any number of two or three figures by itself.

ILLUSTRATIVE EXAMPLE.

Multiply 48 by itself.

SOLUTION.

$$\begin{array}{r} 48 \\ \underline{48} \\ 2304 \end{array}$$

EXPLANATION.—$8 \times 8 = 64$ units. Write 4 and carry 6.
$(4+4) \times 8 = 64+6$ to carry $= 70$. Write 0 and carry 7.
(Or, $\overline{8+8} \times 4 = 64+6$ to carry $= 70$.)
$4 \times 4 = 16+7$ to carry $= 23$. Write 23.

RULE.—*First, multiply the units' order by itself; then twice the tens' order by the units' order (or twice the units' order by the tens' order, if more convenient); then the tens' order by itself, carrying as usual.*

EXAMPLES FOR PRACTICE.

Multiply
1. $36 by 36.
2. $42 by 42.
3. $56 by 56.

Multiply
4. $85 by 85.
5. $74 by 74.
6. $93 by 93.

Multiply
7. $82 by 82.
8. $64 by 64.
9. $38 by 38.

Multiply
10. $19 by 19.
11. $113 by 113.
12. $122 by 122.

370. To find the product of two numbers when each consists of two or three orders, and their units' orders are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $74 by 94.

SOLUTION.

$$\begin{array}{r} \$74 \\ \underline{94} \\ \$6956 \end{array}$$

EXPLANATION.—The solution of this example depends upon the same principle as that of Ill. Ex., **369.** Hence, $4 \times 4 = 16$ units. Write 6, and carry 1. $(7+9) \times 4 = 64+1$ to carry $= 65$. Write 5, and carry 6. $7 \times 9 = 63+6$ to carry $= 69$.

Rule.—*First find the product of the units' orders; then the product of the sum of the tens' orders into the common units' order; then the product of the tens' orders, carrying as usual.*

EXAMPLES FOR PRACTICE.

Multiply
1. $42 by 72.
2. $38 by 58.
3. $61 by 21.
4. $34 by 64.

Multiply
5. $93 by 43.
6. $74 by 24.
7. $85 by 35.
8. $52 by 72.

Multiply
9. $51 by 31.
10. $92 by 52.
11. $132 by 92.
12. $135 by 75.

Multiply
13. $153 by 73.
14. $145 by 85.
15. $128 by 88.
16. $156 by 76.

371. To find the product of two numbers when each consists of two or three orders, and their tens' orders are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $83 by 89.

Solution.
$83
89
$7387

Explanation.—The solution of this example is a modification of that of Ill. Ex., **369, 370.** The orders which are *unlike* being added together, and then multiplied by the common order. Thus, $3 \times 9 = 27$; $(9+3) \times 8 + 2$ to carry $= 98$; $(8 \times 8) + 9$ to carry $= 73$.

EXAMPLES FOR PRACTICE.

Multiply
1. $58 by 53.
2. $75 by 79.
3. $32 by 34.

Multiply
4. $81 by 85.
5. $64 by 67.
6. $25 by 23.

Multiply
7. $18 by 19.
8. $48 by 41.
9. $92 by 93.

Multiply
10. $62 by 65.
11. $84 by 83.
12. $52 by 55.

372. To multiply any number of two or three figures and ending in 5, by itself.

ILLUSTRATIVE EXAMPLE.

Multiply $75 by 75.

Solution.—$7 \times (7+1) = 56$; annex 25, obtaining $5625, the answer. Hence,

Rule.—*Multiply the tens' order by itself increased by 1, and annex 25.*

EXAMPLES FOR PRACTICE.

Multiply
1. $25 by 25.
2. $65 by 65.

Multiply
3. $45 by 45.
4. $85 by 85.

Multiply
5. $35 by 35.
6. $75 by 75.

Multiply
7. $55 by 55.
8. $115 by 115.

9. What is the square of 95; that is, what is the product of 95 multiplied by itself?

373. To find the product of two numbers whose tens' orders are alike, and the sum of their units' orders is 10.

ILLUSTRATIVE EXAMPLE.

Multiply $83 by 87, and $114 by 116.

SOLUTIONS.

$$\begin{array}{rr} \$83 & \$114 \\ 87 & 116 \\ \hline \$7221 & \$13224 \end{array}$$

EXPLANATION.—Both solutions are modifications of **369**, and the answers are obtained as follows:

$(8+1)\times 8 = 72$, annex $3\times 7 = 21$, obtaining $7221.

$(11+1)\times 11 = 132$, annex $4\times 6 = 24$, obtaining $13224.

RULE.—*Multiply one of the tens' orders plus 1 by the other tens' order, and annex the product of the units' orders.*

EXAMPLES FOR PRACTICE.

Multiply
1. $82 by 88.
2. $73 by 77.
3. $54 by 56.

Multiply
4. $96 by 94.
5. $23 by 27.
6. $41 by 49.

Multiply
7. $25 by 25.
8. $36 by 34.
9. $103 by 107.

Multiply
10. $17 by 13.
11. $98 by 92.
12. $112 by 118.

374. To find the product of two numbers whose tens' orders differ by 1, and the sum of their units' orders is 10.

ILLUSTRATIVE EXAMPLE.

Multiply $73 by 67.

SOLUTION.

$$\begin{array}{rlr} \$73 & 7^0 \times 7^0 = & 49^{00} \\ 67 & 3\ \times 3\ = & 9 \\ \hline \$4891 & & 4891 \end{array}$$

EXPLANATION.—Disregard the less number, and multiply the tens' order (7) of the greater number by itself, obtaining 49 (hundreds). Next multiply the units' order (3) of the greater number by itself, obtaining 9. Subtract 9 from 4900, obtaining $4891, the required product.

RULE.—*Multiply the tens' order of the greater number by itself; and from the product (considered as hundreds), subtract the product of the units' order of the greater number multiplied by itself.*

EXAMPLES FOR PRACTICE.

Multiply
1. $42 by 58.
2. $75 by 65.
3. $39 by 41.

Multiply
4. $72 by 68.
5. $63 by 77.
6. $75 by 65.

Multiply
7. $84 by 96.
8. $53 by 47.
9. $81 by 99.

Multiply
10. $124 by 116.
11. $102 by 118.
12. $106 by 94.

375. To find the product of two mixed numbers when their terminal common fractions are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $8½ by 4½, and $9¼ by 7¼.

FIRST SOLUTION.

$$\begin{array}{rl}\$8\frac{1}{2} & \\ 4\frac{1}{2} & \\ \hline 32 & = 8\times 4 \\ 6 & = (8+4)\times\frac{1}{2} \\ \frac{1}{4} & = \frac{1}{2}\times\frac{1}{2} \\ \hline \$38\frac{1}{4} & \end{array}$$

SECOND SOLUTION.

$$\begin{array}{rl}\$9\frac{1}{4} & \\ 7\frac{1}{4} & \\ \hline 63 & = 9\times 7 \\ 4 & = (9+7)\times\frac{1}{4} \\ \frac{1}{16} & = \frac{1}{4}\times\frac{1}{4} \\ \hline \$67\frac{1}{16} & \end{array}$$

RULE.—*Find the product of the integers; then the product of the sum of the two integers into the common terminal fraction; then the product of the two fractions. Add the three results.*

NOTE.—When both integers are alike, and their terminal common fractions are ½, annex the product of the terminal common fractions to the product of one integer plus 1 multiplied by the other integer. Thus, $8½ × 8½ = (9 × 8) + (½ × ½), or $72¼.

The above note will not hold good with any other fraction than ½.

EXAMPLES FOR PRACTICE.

Multiply
1. $4¼ by 8¼.
2. $6½ by 8½.
3. $5⅛ by 3⅛.

Multiply
4. $5¼ by 7¼.
5. $3½ by 9½.
6. $7⅓ by 5⅓.

Multiply
7. $5½ by 5½.
8. $7½ by 7½.
9. $8½ by 8½.

376. To find the product of two mixed numbers whose integers are alike and whose terminal common fractions have, or may be conveniently reduced to, a common denominator.

ILLUSTRATIVE EXAMPLE.

Multiply $\$9\frac{1}{8}$ by $9\frac{7}{8}$, and $\$8\frac{3}{4}$ by $8\frac{5}{8}$.

FIRST SOLUTION.

$$\begin{array}{ll} \$9\frac{1}{8} & \\ \underline{9\frac{7}{8}} & \\ 90 & = (9+\frac{1}{8}+\frac{7}{8}) \times 9 \\ \underline{\frac{7}{64}} & = \frac{1}{8} \times \frac{7}{8} \\ \$90\frac{7}{64} & \end{array}$$

SECOND SOLUTION.

$$\begin{array}{ll} \$8\frac{6}{8} & \\ \underline{8\frac{5}{8}} & \\ 75 & = (8+\frac{6}{8}+\frac{5}{8}) \times 8 \\ \underline{\frac{15}{32}} & = \frac{6}{8} \times \frac{5}{8} \\ \$75\frac{15}{32} & \end{array}$$

Rem.—The above solutions are self-explanatory.

Rule.—*To the integer of the multiplicand, add the sum of the terminal common fractions, and multiply the result by the integer of the multiplier. To the product thus obtained, annex the product of the fractions.*

EXAMPLES FOR PRACTICE.

Multiply
1. $\$9\frac{1}{3}$ by $9\frac{2}{3}$.
2. $\$10\frac{3}{5}$ by $10\frac{2}{5}$.
3. $\$12\frac{5}{6}$ by $12\frac{1}{2}$.
4. $\$18\frac{1}{2}$ by $18\frac{1}{3}$.

Multiply
5. $\$.03\frac{1}{3}$ by $3\frac{2}{3}$.
6. $\$60\frac{3}{5}$ by $60\frac{3}{4}$.
7. $\$.40\frac{3}{8}$ by $40\frac{1}{4}$.
8. $\$25\frac{3}{5}$ by $25\frac{2}{5}$.

Multiply
9. $\$.05\frac{1}{4}$ by $5\frac{1}{2}$.
10. $\$.08\frac{3}{4}$ by $8\frac{1}{3}$.
11. $\$6\frac{5}{8}$ by $.06\frac{3}{8}$.
12. $\$12\frac{1}{9}$ by $12\frac{5}{9}$.

377. To multiply when the multiplier lacks 1 fractional unit of being an integer.

ILLUSTRATIVE EXAMPLE.

What is the cost of 64 yards of silk at $\$3\frac{7}{8}$ per yard ?

SOLUTION.

$$\begin{array}{lll} 64 \times \$4 = & \$256, & \text{cost at } \$4. \\ 64 \times \$\frac{1}{8} = & \underline{8,} & \text{`` `` } \$\frac{1}{8}. \\ & \$248 & \text{`` `` } \$3\frac{7}{8}. \end{array}$$

Explanation.—64 times $4, or $256, would be the cost at $4 per yard ; and at $\$\frac{1}{8}$ per yard less than $4, the cost must be 64 times $\$\frac{1}{8}$, or $8 less than $256.

RULE.—*Multiply the multiplicand by the integral part of the multiplier plus 1; and diminish the result by such a part of the multiplicand as the fraction lacks of being a whole.*

EXAMPLES FOR PRACTICE.

Multiply	Multiply	Multiply	Multiply
1. $48 by $5\frac{3}{4}$.	4. $250 by $3\frac{4}{5}$.	7. $180 by $7\frac{9}{10}$.	10. $440 by $6\frac{10}{11}$.
2. $75 by $10\frac{2}{3}$.	5. $522 by $4\frac{8}{9}$.	8. $720 by $2\frac{11}{12}$.	11. $288 by $5\frac{7}{8}$.
3. $136 by $6\frac{5}{6}$.	6. $672 by $8\frac{6}{7}$.	9. $342 by $9\frac{5}{6}$.	12. $723 by $9\frac{2}{3}$.

378. To divide when a multiple of the divisor will form a more convenient divisor.

ILLUSTRATIVE EXAMPLE.

Divide $1840 by 25, and $6375 by $7\frac{1}{2}$.

FIRST SOLUTION.

$$\begin{array}{r} \$1840 \\ 4 \\ \hline \$73.60 \end{array}$$

SECOND SOLUTION.

$7\frac{1}{2} + \frac{1}{3}$ of $7\frac{1}{2} = 10$

$\$6375 + \frac{1}{3}$ of $\$6375 = \$8500$

$\$8500 \div 10 = \$850$

EXPLANATION.—Instead of using the inconvenient divisor 25, multiply both divisor and dividend by such a number (4) as will produce the more convenient divisor 100. Then divide the new dividend ($7360) by the new divisor (100), obtaining $73.60, the answer.

In the second solution, since the true divisor ($7\frac{1}{2}$) increased by $\frac{1}{3}$ of itself ($2\frac{1}{2}$) will form a more convenient divisor (10), similarly increase the dividend ($6375) by $\frac{1}{3}$ of itself ($2125), obtaining $8500. Then divide the new dividend ($8500) by the new divisor (10), obtaining $850.

RULE.—I. *Multiply or divide both divisor and dividend by such a number as will simplify the division. Or,*

II. *Add to, or subtract from, the given divisor such a part of itself as will produce a more convenient divisor. Similarly increase or diminish the dividend. Then divide as usual.*

EXAMPLES FOR PRACTICE.

Divide	Divide	Divide
1. $5.80 by 25.	4. $875 by $250.	7. $1527 by 150.
2. $7.50 by $16\frac{2}{3}$.	5. $640 by 125.	8. $918 by 15 (deduct $\frac{1}{3}$).
3. $450 by $12\frac{1}{2}$.	6. $4.35 by $33\frac{1}{3}$.	9. $582 by $7\frac{1}{2}$ (add $\frac{1}{3}$.)

MARKING GOODS.

379. In marking goods, various devices are adopted to prevent the cost and selling price mark from becoming known to outsiders.

Rem. 1.—The device may consist of any word or phrase containing ten different letters or characters, each representing a figure. Thus, take the following word:

b	l	a	c	k	s	m	i	t	h
1	2	3	4	5	6	7	8	9	0

If the selling price is $5.20, and it is required to write the cost ($4.25) by the use of this key-word, the proper mark would be $\frac{\$5.20}{\text{clk.}}$.

Rem. 2.—When both cost and selling price are thus expressed, it is customary to employ two key-words, one for the selling price and the other for the cost price. Thus if "blacksmith" is the key-word for the cost price ($4.25) and "republican" for the selling price ($5.20), the proper mark would be $\frac{\text{ben}}{\text{clk}}$.

Rem. 3.—An extra letter, called a *repeater*, is used to prevent successive repetitions of the same letter or character, and thus afford a clue to the key-word. Hence, taking y as the repeater, and "blacksmith" as the key-word, $5.88 would be written k i y.

Rem. 4.—In trade discounts, the rate of discount on the long price is sometimes indicated by the use of a key-word.

Rem. 5.—The following afford examples of key-words: blackhorse, cash profit, hard moneys, importance, now be sharp, gambolines, Charleston, Cumberland, Hagerstown.

Rem. 6.—Instead of letters, arbitrary characters may be used, as follows:

⊿÷E□⧄⊠○⊖⊗∇◈

EXAMPLES FOR PRACTICE.

Using the key "importance" for the cost price, "blackhorse" for the selling price, and "d" as the repeater, mark the following:

	Cost.	*Gain.*		*Cost.*	*Loss.*
1.	$2.40 per yard	25 %	6.	$2.50 per pair	20%
2.	16¢ " "	12½%	7.	45¢ " "	40%
3.	20¢ " "	30 %	8.	$4.40 " dozen	5%
4.	27¢ " "	33⅓%	9.	20¢ " "	10%
5.	42¢ " "	16⅔%	10.	45¢ " gross	50%

379

RAPID PROCESS OF MARKING GOODS.

380. Retail merchants purchase many articles by the dozen, such as hats, caps, boots, shoes, and notions of various kinds. When goods are thus bought, it is useful to be able to determine instantly the selling price per unit to afford a given per cent. of profit.

381. If the given cost is per dozen, to find the selling price per unit at 20% profit.

ILLUSTRATIVE EXAMPLE.

If shirts are bought at $15 per dozen, at what price should each shirt be sold to afford a gain of 20%?

SOLUTION.

$\frac{1}{10}$ of $15 = $1.50.

EXPLANATION.—At 20% profit, the selling price per doz. is 120% of the cost per dozen; and the selling price per unit must be $\frac{1}{12}$ of 120%, or 10% of the cost per dozen. Hence, 10%, or $\frac{1}{10}$ of $15, must be the selling price per shirt to afford 20% gain.

RULE.—*Divide the cost per dozen by 10. The result will be the selling price per unit at 20% profit.*

EXAMPLES FOR PRACTICE.

1. A merchant bought at auction 6 dozen pairs of boots at $42.50 per dozen pairs. How much must he charge per pair to gain 20%?

2. Bought caps at auction for $10 per dozen. To gain 20% on each cap, what must be my selling price?

3. By selling an article at $1.25 I gain 20%. What was the cost per dozen?

4. A second-hand dealer bought 12 chairs at auction, for which he paid $12.50, and sold them at 20% profit. What did he charge for each chair?

5. By selling shirts at $1.75 each, I gained 20%. What was the cost per dozen?

6. If hats cost $18 per dozen, what must be the selling price of each to yield 20% profit?

7. If chairs are bought at $9.20 per dozen, what must be the selling price per set of 6 to make 20% profit?

382. If the given cost is per dozen, to find the selling price per unit at any % profit.

ILLUSTRATIVE EXAMPLE.

If gloves cost $20 per dozen pairs, what must be the selling price per pair to yield 50% profit?

SOLUTION.

120% of cost = $2.00
30% " " = .50
150% of cost = $2.50

EXPLANATION.—First take $\frac{1}{10}$ of the cost per dozen, obtaining $2 as the selling price per pair at 20% profit, or at 120% of cost. Since the required selling price is 150% of cost, or one-fourth greater than 120%, increase $2 by one-fourth of itself ($.50), obtaining $2.50 as the required selling price.

RULE.—*By Rule,* **381,** *find the selling price per unit at 20% profit; and increase or diminish the result by such a part of itself as the true rate of selling price is greater or less than 120%.*

NOTE.—As a help in marking goods bought by the dozen, we add the following

TABLE.

To make 12½% profit, deduct $\frac{1}{16}$ from the result obtained by Rule, **381.**
" 16⅔% " " $\frac{1}{36}$ " " " " "
" 25% " add $\frac{1}{24}$ to " " " " "
" 30% " " $\frac{1}{12}$ " " " " " "
" 33⅓% " " $\frac{1}{9}$ " " " " " "
" 50% " " $\frac{1}{4}$ " " " " " "

EXAMPLES FOR PRACTICE.

1. If I buy one dozen vests for $28, at what price must I retail them to make 50% profit?

2. At what price must I retail handkerchiefs costing $2.70 per dozen to realize 33⅓% gain?

3. I sold hats costing $8.40 per dozen at 30% profit. At what price were they retailed?

4. If one dozen school-books cost $4.80, at what price must they be retailed to gain 25%?

5. If goods cost $7.20 per dozen, what should be the selling price per unit to yield a profit of 16⅔%?

6. If chairs are bought at $32 per dozen, what must be the selling price per set of 3 to yield 12½% profit?

ANSWERS.

Directions.—The column of bold figures in the margin denotes the page; the bold figures at the beginning of each section, the number of the article; the small, bold figures within the curves, the number of the example; and the figures following the curves, the answer. The heavy dash denotes the beginning of the page whose number is given in the margin.

Art. 17.—(1) $18. (2) $24. (3) $39. (4) $37. (5) $337. (6) **13**
$276. (7) $3122. (8) $3885. (9) $38113. (10) $34638. (11) $26684.
(12) $363361. (13) $338575053.——(14) $67. (15) $61. (16) $70. (17) **14**
$759. (18) $693. (19) $686. (20) $8152. (21) $6852. (22) $20250.
(23) $90942. (24) 9851 lb. (25) 16911 ft. (26) 26008 bbl. (27) 9663 bu.
(28) 10069 yd. (29) $19714. (30) $934461. (31) $3208. (32) $11098.
(33) $7335. (34) 1880.——(35) 40182 mi. (36) $2119. (37) $1833. (38) **15**
$7058. (39) $10202. (40) $6065. (41) $6830. (42) $723. (43) $532918.
(44) 17843 Am.; 7140 Ger.; 6288 Ir.; 1152 Fr.; 1715 It.; 6889 others; 1st
ward, 5723; 2d, 7601; 3d, 7306; 4th, 9428; 5th, 6716; 6th, 4253; total,
41027. (45) 229804 bu. corn; 300493 bu. wheat; 71086 bu. oats; 104171 bu.
barley; 51716 bu. rye; Mon., 99685 bu.; Tues., 88759 bu.; Wed., 132664 bu.;
Thurs., 96195 bu.; Fri., 168496 bu.; Sat., 171471 bu.; total, 757270 bu.

21.—(1) $2944. (2) $6314. (3) $201793. (4) 5652 bu. (5) 826623 ft. **17**
(6) $229567. (7) $4847404. (8) 8891733 lb. (9) $456514. (10) $665876.
(11) $2092211. (12) $1747193. (13) $29117482. (14) $12198952. (15)
$557614403. (16) $10010010. (17) $47882. (18) $38851. (19) $1585862.
(20) $255882. (21) $60152275.——(22) $3647. (23) $13642. (24) $414667. **18**
(25) $2147206. (26) $2622929. (27) $186208. (28) $12515. (29) $11085.
(30) $40520. (31) 62 yr. (32) $86851.

27.—(1) 251250 lb. (2) 149136 bu. (3) 18365530 yr. (4) 126591380 **20**
yd. (5) 1883950872 gal. (6) 4047843891 lb. (7) 29626444512 gal. (8)
$44196133309122. (9) $21664405044656. (10) $7284099602508. (11) $2810700680350. (12) $45963008174718.

29.—(1) 3474198 yd. (2) 38884576 yr. (3) 1407980128 lb. (4) 1759841734 ft. (5) 138603407602 oz. (6) 57955348236 gal. (7) 427875557428 yd. **21**
(8) 124212794445 ft. (9) $648266331713. (10) $234563388288. (11)
$646354836088. (12) $471541405689. (13) $614810407806. (14) $3145058867040. (15) $35223263684736. (16) $28493676328032. (17) 2135066 lb.
(18) 92512 lb. (19) $586. (20) $3042. (21) 2014 trees.

31.—(1) 532008 men. (2) 313776 gal. (3) 5743872 yr. (4) 15312888 **22**
yr. (5) 301746249 lb. (6) $7793785860. (7) $3863921904. (8) $2858156574. (9) $2565441216. (10) $19819170.

33.—(1) 897000 yr. (2) 628500 gal. (3) 137210000 bu. (4) 6386400
ft. (5) 181888000 yd. (6) 1548566400 lb. (7) $5489617875000. (8) $18125000000. (9) $783972000000. (10) $239400000.

24 **37.**—(1) 2261073 gal. (2) 11659477 lb. (3) 35914767 yd. (4) 12318467
bu. (5) 12327348 yr. (6) 1525509 gal. (7) 7712304 oz. (8) 39786714 ft.
(9) $620015080. (10) $348558403. (11) $632572157. (12) $16147820005.
(13) $5325038505. (14) $89235004. (15) $577777778. (16) $88685728643.
(17) 1432 yd. (18) 418 books. (19) 9187 acres.

26 **40.**—(1) 1458252[illegible] times. (2) 31668912[illegible] bu. (3) 5675505 times.
(4) 17102680[illegible] yr. (5) 205087[illegible] times. (6) 2049615[illegible] gal. (7) 896051[illegible]
bu. (8) 327593[illegible] times. (9) 1653250[illegible] miles. (10) 1353433[illegible] gal.
(11) 3762106[illegible] times. (12) $256197[illegible]. (13) $1160826[illegible]. (14)
77977[illegible] times. (15) $37081[illegible]. (16) 1502[illegible] times. (17) 173
trees. (18) $58. (19) $142. (20) 4356 times.

27 **41.**—(1) 127302[illegible] lb. (2) 7196516[illegible] times. (3) 179182369[illegible] yd. (4)
12983831[illegible] times. (5) 1364133804[illegible] times. (6) $6098191. (7) $121398746[illegible].
(8) $6633\frac{48}{125}$ times. (9) 1959[illegible] times. (10) $4323[illegible].

28 **42.**—(1) $91872\frac{683}{1000}$ yd. (2) $517918\frac{36}{100}$ times. (3) $8296\frac{7382}{10000}$ bu. (4)
$435\frac{79886}{1000000}$ yr. (5) $95\frac{6437765}{10000000}$ times. (6) $27836\frac{321}{1000}$ times.

43.—(1) 1795[illegible] lb. (2) 8971[illegible] times. (3) 85[illegible] yd. (4) 700[illegible]
times. (5) 12[illegible] gal. (6) 73[illegible] times. (7) $274[illegible]. (8) $1864[illegible].
(9) 33[illegible] times. (10) $27[illegible]. (11) $1800. (12) 333[illegible] times.

30 **46.**—(1) $7066.00. (2) $11352.00. (3) $522.328. (4) $1215.895. (5)
$635.893.

31 **47.**—(1) $296.37. (2) $2347.35. (3) $98.30. (4) $75.15. (5) $6.96.
(6) $2.89. (7) $267.75. (8) $156.98.

48.—(1) $191401. (2) $1093002.20. (3) $36731.90. (4) $1191360.04.
(5) $7982.25. (6) $12990.86. (7) $2458.869. (8) $8103.005. (9) $1748786.60. (10) $511845.34.

32 **49.**—(1) $325.78. (2) $81.42. (3) $19.42. (4) $493. (5) $292.25.
(6) $77.36—. (7) $1.59+. (8) $18.92+.

33 **50.**—(1) 32718 times. (2) 571 times. (3) 58 times. (4) 9200 times.
(5) 41300 times. (6) 197500 times. (7) 75 times. (8) 67 times. (9) 432
times. (10) 4517800 times.

35 36 **56.**—(1) 2161 pages. (2) 16555 men. (3) 1885. (4) $502.—(5) $3.15.
(6) $8.36. (7) $494.15. (8) $2324.55. (9) $407.50. (10) $2069.55. (11)
37 $42.25. (12) $307.82. (13) $7.68. (14) $2020.—(15) 175 A. (16) 15¢.
(18) 18 da. (19) 36 da. (20) 54 yr. (21) 104400 men. (22) 70 bbl. (23)
85°. (24) 6881 bu. (25) 85¢. (26) 106 lb.

40 **63.**—(1) 3 ft. × 3 × 5. (2) 2 gal. × 2 × 3 × 11. (3) 2 da. × 137. (4) 2 mi. ×
2 × 2 × 7 × 7. (5) 3 men × 5 × 5 × 7. (6) 2 lb. × 2 × 3 × 7 × 11. (7) 3 tons ×
3 × 5 × 11 × 13. (8) 2 yd. × 2 × 2 × 3 × 3 × 3 × 3 × 5. (9) 5 hr. × 5 × 5 × 5 × 7.
(10) 2 oz. × 2 × 2 × 3 × 3 × 11 × 23. (11) 2 ft. × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 ×
19. (12) $2 × 2 × 3 × 3 × 5 × 5 × 11 × 11.

41 **68.**—(1) 504 bu. (2) 540 gal. (3) 315 hr. (4) 504 lb. (5) 120 doz.
(6) 60 yd. (7) 432 yr. (8) $11025. (9) 1260 ft. (10) $1512. (11) $2520.
(12) $1800. (13) $105. (14) 1001 lb. (15) 48 ft.

43 **72.**—(1) 86 or 86 yr. (2) 5 or $5. (3) 19 or 19 lb. (4) 42 or 42
yd. (5) 51 or 51 bu. (6) 19 or 19 bbl. (7) 719 or 719 ft. (8) 2357 or

2357 doz. (9) 60 or 60 tons. (10) 131 or 131 gal. (11) 19 or $19. (12) 43
34 or $34.

75.—(1) $21. (2) 40 times. (3) 35 bbl. (4) 8 times. (5) 20 yd. (6) 44
5 horses. (7) 7 bbl. (8) 12¢. (9) 32 bu. (10) 3 yd.

89.—(1) $\frac{12}{15}$ lb. (2) $\frac{20}{28}$ yd. (3) $\frac{21}{56}$ qt. (4) $\frac{15}{35}$ bu. (5) $\frac{15}{45}$ ft. (6) 48
$\frac{121}{165}$ yr. (7) $\frac{198}{209}$ oz. (8) $\frac{35}{217}$ lb. (9) $\frac{3}{9}$ ft. (10) $\frac{5}{12}$ lb. (11) $\frac{3}{6}$ yd. (12) $\frac{3}{8}$
bu. (13) $\frac{6}{11}$ yr. (14) $\frac{5}{8}$ oz. (15) $\frac{3}{5}$ qt. (16) $\frac{3}{8}$ yd. (17) $\frac{36}{4}$ lb. (18) $\frac{342}{9}$
yr. (19) $\frac{90}{6}$ yd. (20) $\frac{228}{12}$ gal. (21) $\frac{344}{8}$ bu. (22) $\frac{1842}{19}$ oz. (23) $\frac{6165}{45}$ ft.
(24) $\$\frac{40404}{78}$.

90.—(1) $\frac{1}{3}$ gal. (2) $\frac{4}{5}$ yd. (3) $\frac{7}{9}$ yr. (4) $\frac{3}{7}$ ft. (5) $\frac{3}{11}$ bu. (6) $\frac{2}{5}$ tons. 49
(7) $\frac{3}{7}$ oz. (8) $\frac{5}{8}$ bbl. (9) $\frac{2}{5}$ yr. (10) $\frac{7}{11}$ gal. (11) $\frac{8}{11}$ qt. (12) $\frac{6}{7}$ bbl. (13)
$\frac{3}{5}$ yd. (14) $\frac{5}{7}$ gal. (15) $\frac{9}{11}$ bu. (16) $\frac{4}{9}$ lb. (17) $\frac{2}{3}$ bbl. (18) $\frac{3}{7}$ oz. (19) $\frac{8}{11}$
gal. (20) $\frac{7}{8}$ yd.

91.—(1) $\frac{43}{8}$ gal. (2) $\frac{59}{6}$ yd. (3) $\frac{38}{3}$ tons. (4) $\frac{112}{5}$ yr. (5) $\frac{262}{7}$ cd.
(6) $\frac{1685}{12}$ mi. (7) $\frac{2691}{4}$ melons. (8) $\frac{3164}{11}$ bbl. (9) $\frac{8393}{16}$ qt. (10) $\frac{18771}{23}$ ft.
(11) $\frac{17902}{25}$ bu. (12) $\frac{51973}{123}$ yr. (13) $\frac{1248052}{151}$ oz. (14) $\frac{1598038}{315}$ yd. (15)
$\frac{2598692}{419}$ yr. (16) $\$\frac{22846057}{3189}$.

92.—(1) 4 yd. (2) 8 bbl. (3) 23 yr. (4) 28$\frac{5}{9}$ oz. (5) 14 bu. (6) 58$\frac{4}{5}$ 50
gal. (7) 4$\frac{17}{59}$ oz. (8) 19$\frac{21}{23}$ qt. (9) 52$\frac{9}{17}$ yr. (10) 16$\frac{15}{31}$ bbl. (11) 4$\frac{139}{371}$
yd. (12) 85$\frac{1}{23}$ bu. (13) 36$\frac{514}{517}$ gal. (14) 7$\frac{16}{417}$ oz. (15) 24$\frac{41}{113}$ bu. (16)
12$\frac{343}{359}$ bbl.

95.—(1) $\frac{8}{12}$ yd.; $\frac{9}{12}$ yd.; $\frac{10}{12}$ yd. (2) $\frac{12}{24}$ ft.; $\frac{8}{24}$ ft.; $\frac{9}{24}$ ft. (3) $\frac{63}{72}$ bu.; 51
$\frac{66}{72}$ bu.; $\frac{40}{72}$ bu. (4) $\frac{18}{30}$ yr.; $\frac{21}{30}$ yr.; $\frac{16}{30}$ yr. (5) $\frac{27}{36}$ gal.; $\frac{8}{36}$ gal.; $\frac{21}{36}$ gal. (6)
$\frac{70}{105}$ oz.; $\frac{45}{105}$ oz.; $\frac{84}{105}$ oz. (7) $\frac{105}{168}$ ft.; $\frac{72}{168}$ ft.; $\frac{108}{168}$ ft.; $\frac{64}{168}$ ft. (8) $\frac{185}{225}$ bu.;
$\frac{135}{225}$ bu.; $\frac{60}{225}$ bu.; $\frac{81}{225}$ bu. (9) $\frac{196}{504}$ yr.; $\frac{231}{504}$ yr.; $\frac{117}{504}$ yr.; $\frac{64}{504}$ yr. (10) $\frac{105}{210}$
yd.; $\frac{140}{210}$ yd.; $\frac{150}{210}$ yd.; $\frac{42}{210}$ yd. (11) $\frac{168}{252}$ ft.; $\frac{144}{252}$ ft.; $\frac{140}{252}$ ft.; $\frac{99}{252}$ ft. (12)
$\frac{4818}{22176}$ bu.; $\frac{5775}{22176}$ bu.; $\frac{2574}{22176}$ bu.; $\frac{6816}{22176}$ bu.

98.—(1) 1$\frac{13}{24}$ yd. (2) 1$\frac{11}{36}$ gal. (3) 1$\frac{34}{45}$ lb. (4) 1$\frac{17}{72}$ bbl. (5) 2$\frac{1}{12}$ oz. 52
(6) 2$\frac{3}{8}$ bu. (7) 1$\frac{79}{84}$ yr. (8) 2$\frac{53}{280}$ qt. (9) 1$\frac{889}{1890}$ lb. (10) 1$\frac{133}{180}$ yr. (11)
183$\frac{1}{4}$ yd. (12) 448$\frac{49}{72}$ bu. (13) 1052$\frac{23}{32}$ yr. (14) 1029$\frac{98}{175}$ lb. (15) 801$\frac{1}{3}$ oz.
(16) 228$\frac{19}{105}$ yd. (17) 223 yd. (18) 245 yd. (19) 265^{3} yd. (20) 171^{10} 53
lb. (21) 488^{3} lb.

101.—(1) $\frac{1}{12}$ ft. (2) $\frac{41}{63}$ yd. (3) $\frac{1}{6}$ bbl. (4) $\frac{5}{48}$ oz. (5) $\frac{11}{24}$ gal. (6) $\frac{1}{6}$ 54
doz. (7) $\frac{1}{15}$ yr. (8) $\frac{1}{18}$ lb. (9) $\frac{19}{56}$ qt. (10) $\frac{11}{36}$ yd. (11) $\frac{3}{4}$ yr. (12) 3$\frac{13}{15}$
bu. (13) $\frac{11}{12}$ oz. (14) 2$\frac{17}{40}$ ft. (15) 1$\frac{29}{36}$ lb.

102.—(1) $\$303\frac{1}{3}$. (2) $\$313\frac{3}{8}$. (3) $\$223\frac{4}{9}$. (4) $\$404\frac{3}{8}$. (5) $\$291\frac{5}{12}$. 55
(6) $\$262\frac{1}{20}$. (7) $\$432\frac{19}{60}$. (8) $\$87\frac{13}{21}$. (9) $\$57\frac{19}{24}$. (10) $\$225\frac{11}{12}$. (11)
$\$321\frac{3}{14}$. (12) $\$150\frac{11}{18}$. (13) 27^{3} yd. (14) 46^{3} yd. (15) 6^{3} lb.

105.—(1) $\$\frac{2}{3}$. (2) $\$\frac{3}{32}$. (3) $\$\frac{3}{10}$. (4) $\$\frac{1}{2}$. (5) $\$\frac{3}{8}$. (6) $\$\frac{1}{3}$. (7) $\$\frac{1}{6}$. 56
(8) $\$\frac{25}{72}$. (9) $\$\frac{3}{4}$. (10) $\$2\frac{31}{45}$. (11) $3. (12) $\$18\frac{3}{8}$. (13) $\$11\frac{11}{18}$. (14)
$\$\frac{1}{18}$. (15) $\$\frac{1}{6}$. (16) $\$\frac{3}{10}$. (17) $\$516\frac{3}{4}$. (18) $\$98\frac{3}{4}$. (19) $\$416\frac{2}{3}$. (20)
$\$574\frac{2}{3}$. (21) $\$822\frac{2}{3}$. (22) $\$4\frac{1}{3}$. (23) $16. (24) $24. (25) $\$1\frac{1}{4}$. (26) $\$2\frac{1}{3}$.
(27) $\$3\frac{6}{7}$. (28) $\$12\frac{1}{4}$. (29) $\$184\frac{2}{7}$. (30) $42. (31) $52. (32) $20.

106.—(1) $\$2129\frac{5}{8}$. (2) $\$1702\frac{2}{3}$. (3) $424. (4) $343. (5) $\$4335\frac{1}{3}$. 57
(6) $\$1970\frac{3}{8}$. (7) $\$18546\frac{2}{3}$. (8) $\$132173\frac{1}{3}$. (9) $\$27389\frac{1}{4}$. (10) $\$27154\frac{3}{4}$.
(11) $\$54424\frac{4}{7}$. (12) $\$200356\frac{1}{3}$.

58 **107.**—(1) $\$1786\frac{5}{12}$. (2) $\$421\frac{9}{40}$. (3) $\$690\frac{5}{9}$. (4) $\$2076\frac{3}{7}$. (5) $\$25384\frac{7}{18}$.
(6) $\$9354\frac{7}{25}$. (7) $\$8957\frac{5}{13}$. (8) $\$63467\frac{13}{21}$. (9) $\$96628\frac{34}{35}$. (10)
$\$138566\frac{1}{20}$. (11) $\$4855\frac{7}{45}$. (12) $\$46784\frac{27}{35}$.
59 **110.**—(1) $1\frac{1}{4}$ times. (2) $\$\frac{1}{4}$. (3) $1\frac{1}{5}$ times. (4) $\$1\frac{1}{7}$. (5) $33. (6)
$\$5\frac{1}{5}$. (7) $\$\frac{3}{7}$. (8) $\frac{1}{6}$ times. (9) $\frac{1}{8}$ times. (10) $\$2\frac{1}{3}$. (11) $2\frac{2}{7}$ times. (12)
$\$10\frac{1}{2}$. (13) $\frac{1}{21}$ times. (14) $\$\frac{5}{8}$. (15) $605\frac{1}{3}$ times. (16) $\$\frac{6}{7}$. (17) $\frac{1}{24}$ times.
(18) $\$\frac{11}{14}$. (19) $\$1948\frac{1}{3}$. (20) $\frac{1}{18}$ times. (21) $\$\frac{1}{12}$. (22) $\$1\frac{9}{10}$. (23) $\$\frac{3}{34}$.
(24) $\$9\frac{1}{7}$. (25) $\frac{7}{15}$ times. (26) $\frac{3}{35}$ times. (27) $27. (28) $\$\frac{4}{25}$. (29) $\$7\frac{1}{5}$.
(30) $\frac{3}{35}$ times. (31) $\$66\frac{12}{23}$. (32) $107\frac{1}{5}$ times.
60 **111.**—(1) $\$7924\frac{5}{7}$. (2) $21666\frac{2}{7}$ times. (3) $\$103820\frac{1}{12}$. (4) $\$524727\frac{2}{7}$.
(5) $72385\frac{25}{36}$ times. (6) $107074\frac{12}{43}$ times. (7) $\$706353\frac{43}{72}$. (8) $\$167980\frac{41}{210}$.
(9) $\$14943\frac{75}{86}$. (10) $46983\frac{45}{117}$ times. (11) $\$4136\frac{34}{1305}$. (12) $14170\frac{208}{2513}$
times. (13) $\$36295\frac{67}{325}$. (14) $\$47720\frac{743}{1456}$. (15) $158393\frac{421}{509}$ times. (16)
$\$3\frac{1}{32}$. (17) 816 casks.
112.—(1) $\$5\frac{23}{177}$. (2) $\$53\frac{67}{69}$. (3) $17\frac{207}{505}$ times. (4) $\$94\frac{23}{118}$. (5)
$274\frac{454}{513}$ times. (6) $\$144\frac{277}{425}$. (7) $470\frac{119}{404}$ times. (8) $\$58\frac{97}{428}$. (9) $\$54\frac{4225}{6913}$.
(10) $204\frac{465}{1378}$ times. (11) $\$4\frac{1621}{2702}$. (12) $\$157\frac{5029}{5347}$.
63 **115.**—(5) $9\frac{1}{3}$ yd. (6) $\$93\frac{9}{10}$. (7) $7\frac{1}{2}$ lb. (8) $\$\frac{1}{4}$. (9) $4\frac{4}{75}$ mo. (10)
$\frac{405}{448}$ lb. (11) 8 beds. (12) $79\frac{2}{5}$ bu. (13) $\frac{9}{10}$ hr. (14) $8\frac{1}{4}$ da. (15) $974\frac{97}{112}$
mi. (16) $3\frac{139}{153}$ hr. (17) 5¢. (18) $\$\frac{45}{64}$. (19) $5\frac{1}{2}$ lb. (20) $2\frac{1}{17}$ tons. (21)
$12\frac{1}{2}$ da. (22) $2\frac{9}{35}$ da.
65 **116.**—(5) $4200. (6) $\frac{1}{6}$. (7) $\frac{5}{6}$ bbl. (8) $9600. (9) 635 yd. (10)
$1320. (11) $\frac{15}{64}$. (12) $4500. (13) $\frac{3}{5}$. (14) $5180. (15) A $\frac{7}{30}$; B $\frac{7}{15}$; C
$\frac{3}{10}$. (16) $5760. (17) $\frac{6}{35}$.
66 **117.**—(1) $389\frac{3}{7}$ bbl. (2) 6 lots. (3) $773\frac{169}{210}$ sq. rd.—(4) $522\frac{5}{16}$ lb.
(5) $253\frac{7}{16}$ lb. (6) $958\frac{1}{24}$ cu. yd. (7) $2233\frac{1}{3}$ bu. (8) $\$38.99\frac{1}{6}$. (9) 9¢. (10)
$\$40.92\frac{7}{8}$. (11) $\$8.94\frac{27}{35}$. (12) $\$148.57\frac{3}{16}$. (13) $\$24\frac{3}{4}$. (14) $30\frac{19}{180}$ mi.
67 (15) $14\frac{22}{35}$ yd. (16) $\frac{150}{311}$. (17) $21\frac{99}{160}$ gal. (18) 256 acres. (19) A $\frac{1}{4}$; B $\frac{1}{3}$;
C $\frac{5}{12}$. (20) $30720. (21) $\frac{5}{8}$. (22) $8\frac{2}{11}$ da. (23) $\frac{2}{5}$. (24) 232 lb. (25)
270 lb. (26) house $4500; lot $2700. (27) carriage $735; horse $525.
(28) $7\frac{1}{5}$ da. (29) $1\frac{1}{13}$ da. (30) $5100.
72 **129.**—(1) .375 lb. (2) .875 lb. (3) .1875 yr. (4) .4375 bbl. (5)
.34375 yd. (6) .59375 qt. (7) .796875 gal. (8) .578125 bu. (9) .0625 ft.
(10) .046875 yd. (11) .016 lb. (12) .0078125 yr. (13) 9.75 gal. (14)
73 25.3125 mi. (15) 17.078125 yd. (16) 4.07421875 bbl.—(17) .3125 gal.
(18) .109375 bbl. (19) .15625 yr. (20) .265625 lb. (21) .59375 yd. (22)
.390625 bu. (23) 18.1875 rd. (24) 25.078125 T. (25) 19.67–oz. (26)
415.63–lb. (27) 75.583+yr. (28) 91.281+yd. (29) 278.0469–bu. (30)
67.5556–bbl. (31) 8.05469–yr. (32) 15.54545+mi. (33) 46.34288–bu.
(34) 75.19531+lb. (35) 812.931281+gal. (36) 73.418333+yd. (37) 2.9117188–T.
(38) 18.2736667–mi.
74 **130.**—(1) $\frac{1}{4}$ bbl. (2) $\frac{7}{8}$ lb. (3) $\frac{3}{8}$ gal. (4) $\frac{3}{40}$ yd. (5) $\frac{1}{160}$ oz. (6)
$5\frac{1}{800}$ yr. (7) $18\frac{1}{32}$ bu. (8) $5\frac{3}{125}$ ft. (9) $3\frac{4}{3125}$ qt. (10) $7\frac{253}{800}$ yd. (11) $5\frac{11}{32}$
ft. (12) $19\frac{1}{16}$ bbl. (13) $25\frac{1}{6}$ oz. (14) $58\frac{1}{45}$ lb. (15) $\frac{1}{120}$ bu. (16) $\frac{1}{12}$ bbl.
74 **131.**—(1) 389.19798 bu. (2) 1148.9027 lb. (3) 8427.99745 yd.—(4)
75 1795.5031875 yr. (5) 1550.562825 rd. (6) 6864.1822875 mi. (7) 435.84513-

175 cd. (8) 819.886+yr. (9) 1.4039+bu. (10) $271.024—. (11) 4063.034636 A. (12) 5407.06444 mi. 75

132.—(1) 5.6795 lb. (2) 8.8552 yr. (3) 41.91732 ft. (4) 8.6532 yd. (5) 8.6526 bu. (6) 389.9563.oz. (7) 14.37 bbl. (8) .2027 gal. (9) 34.855 rd. (10) 125.804375 A. (11) 10.18563125 mi. (12) 25.5609+da. (13) 2.460+cd. (14) $29.311+. (15) 947.9625 yr. (16) 492.000721 fath. 76

133.—(1) 14864.592 yd. (2) 158.46875 ft. (3) 34.532025 bu. (4) 1.780587 bbl. (5) .91849464 yr. (6) 19.33456 lb. (7) .00037559 T. (8) .000001656 ft. (9) $.00000057645. (10) $.1618125. (11) $4.46162625. (12) $62.919075. (13) 1.348051\frac{1}{3}$. (14) 1.714402\frac{4}{5}$. (15) 32.0101\frac{20}{21}$. (16) $258.73. (17) $582.73. (18) $1644.5865. 77

134.—(1) $8930. (2) 77500 cd. (3) 27000 acres. (4) .0005 times. (5) .3 bu. (6) 46.8 yd. (7) 10000 mi. (8) .0002 times. (9) 12.38 mo. (10) 2350 fath. (11) .024 times. (12) 97.65625 tons. (13) 1000 times. (14) 1.73 times. (15) $620. (16) 21.6 times. (17) 7.5 times. (18) 716200 yd. (19) .0087 times. (20) 4.7578125 times. (21) 297.6825 times. (22) 43540 mo. (23) 7830 rd. (24) 81.4 mi. (25) 316.667—fath. (26) 10.9117—times. (27) 19.780—mi. (28) 216.708+yr. (29) 4.89+times. (30) 2.059—times. (31) $3214.865—. (32) .00016 times. (33) .001+times. (34) 18209.3023+ times. (35) 30.62—pt. (36) .010+ft. (37) .004—yd. (38) 826.667—cd. (39) 5.935—rd. (40) .1317+ fath. (41) .0009—bbl. (42) .86+ times. (43) 2.143—yr. (44) .08—bu. (45) .153+ times. (46) 186.67—gal. (47) 1.854—times. (48) 6.37—yr. (49) 1.8475 times. (50) 3.451298 acres. (51) 41.52385 mi. (52) .0075 bu. (53) $213.519—. (54) .75+times. (55) $.576+. (56) .2668+acres. (57) $.01141—. (58) .016+yr. 78 79

135.—(1) 11999.55 bu. (2) 12654.9 loads. (3) 134.37 lb. (4) 10756.79206 A. (5) $130.62+.—(6) $394.06—. (7) 9.045 bu. (8) 640 bu. (9) $343.4140625. (10) $414.61—. (11) 263.2702 yr. (12) .289. (13) 264.06232 A. (14) 7.18 yd. (15) 393.3675 mi. (16) .317. (17) 133 sheep. (18) 345 A. (19 $400. (20) .35.—(21) $18374. (22) .184. (23) $165. (24) .48. (25) $163.11. (26) Coffee, $80; sugar $54.80. (27) $42880. 80 81

141.—(1) $1327. (2) $40.50. (3) $153. (4) $1343. (5) $147.50. (6) $227.67—. (7) $179.95. (8) $389.25. (9) $3057.10. (10) $419.33+. (11) $780.50. (12) $418.46. (13) $1485. (14) $362.70. (15) $5121.27. (16) $271.88—. (17) $447.15. (18) $421.88—. (19) $265.65. (20) $170.23—. (21) $660. (22) $1177.75. (23) $241.88—. (24) $646.25. (25) $1375. (26) $587.81+. (27) $279. (28) $276. (29) $2708.10. (30) $2272.58. 83

142.—(1) $12.50. (2) $156.25. (3) $16. (4) $48. (5) $139.13—. (6) $83.75. (7) $256.50. (8) $182.50. (9) $28.33+. (10) $685.94—. (11) $148.13—. (12) $3.60. (13) $1.63. (14) $3.10. (15) $4.27—. 84

143.—(1) 2200 yd. (2) 1392 yd. (3) 1602 yd. (4) 57.24 yd. (5) 284 yd. (6) 17.6 yd. (7) 66.4 gal. (8) 1638 gal. (9) 20.85 gal. (10) 496 gal. (11) 48.2 gal. (12) 22.44 gal. (13) 35.52 bu. (14) 258$\frac{1}{3}$ bu. (15) 957$\frac{1}{3}$ bu. (16) 1320 bu. (17) 12.64 bu. (18) 3.624 bu. 85

144.—(1) $47.43+. (2) $45.57. (3) $11.53—. (4) $13.75. (5) $14.22. (6) $68.54+. (7) $137.50—. (8) $68.96+. (9) $20.70. (10) $65.64—., 87

87 (11) \$2.64+. (12) \$11.40−. (13) \$246.73−. (14) \$463.02−. (15)
\$1.14+. (16) \$89.16+. (17) \$11.96−. (18) \$79.25. (19) \$1.49+.
(20) \$2.21−. (21) \$38.26+. (22) \$19.65+. (23) \$5.42+. (24) \$4.73−.
90 **152.**—(1) \$11.13+. (2) \$189.80. (3) \$456.87. (4) \$631.92. (5)
92 \$84. (6) \$146.17. (7) \$77.52. (8) \$152.35. (9) \$358.44. (10) \$19.38.
93 (11) Mar. 3, \$35.97; Mar. 18, \$27; Mar. 23, \$84.15; Mar. 27, \$17.50;
94 monthly statement, \$164.62. (12) \$292.92.
99 **173.**—(1) 5030 m. (2) 72756 far. (3) 30036d. (4) 45696 far. (5)
2014 pt. (6) 415 gi. (7) 64393234 sec. (8) 6240288 oz. (9) 29208 gr.
(10) 503728 oz. (11) 343938 in. (12) 458514108 sq. in. (13) $327\frac{1}{2}$ ft.
(14) 976 cu. ft. (15) $100534\frac{1}{2}$ ft. (16) 2464 cu. ft. (17) 16307 lb. (18)
54953100 sec. (19) 2400 sh. (20) 18160 pwt.
100 **174.**—(1) 983 gal. 2 qt. 1 gi. (2) 22 bu. 1 pk. 4 qt. (3) 15 rm. 4 qu.
22 sh. (4) 80 bu. 3 pk. 5 qt. (5) 7 yr. 92 da. 17 hr. 18 min. (6) 3 sc. 18
yr. (7) £18 1s. 9d. 1 far. (8) \$5.678. (9) 4 T. 16 cwt. 13 lb. 4 oz. (10)
10 lb. 1 oz. 12 pwt. 8 gr. (11) £73 1s. (12) 2 mi. 287 rd. 4 yd. 1 ft. 10 in.
(13) 38 cd. 54 cu. ft. (14) 8 A. 31 P. 29 sq. yd. 1 sq. ft. 36 sq. in. (15)
56891 rd. 3 yd. 2 ft. 6 in. (16) 15 da. 8 hr. 6 min. 13 sec. (17) 21 lb. 11
oz. 13 pwt. (18) 35 cwt. 74 lb. 2 oz. (19) 7 A. 130 P. 8 sq. yd. 4 sq. ft. 72
sq. in. (20) 2450 lb.
101 **175.**—(1) 21 hr. 20 min. (2) 13s. 4d. (3) 1 qt. 1 pt. (4) 1 pk. 6 qt.
(5) 243 da. 8 hr. (6) 6 qt. $1\frac{1}{3}$ pt. (7) 6s. 3d. (8) 133 rd. 1 yd. 2 ft. 6 in.
(9) 4 oz. 16 pwt. (10) 43 P. 19 sq. yd. 2 sq. ft. 36 sq. in. (11) 6 cwt. 25 lb.
(12) 2 ft. $1\frac{5}{7}$ in.

176.—(1) 7s. 6d. (2) 4.4 qt. (3) 164 da. 6 hr. (4) 1 qt. 1 pt. 3 gi.
(5) 1d. 2 far. (6) 51 da. 22 hr. 6 min. 36 sec. (7) 10 oz. 10 pwt. (8) 3
cwt. 18 lb. 5 oz. (9) 60 P. 15 sq. yd. 1 sq. ft. 18 sq. in.
102 **177.**—(1) £.6875. (2) .075 hr. (3) .75 yd. (4) .06 T. (5) .9125 da.
(6) .921875 bu. (7) £.0625. (8) .4375 gal.

178.—(1) £279 6d. (2) 262 lb. 1 oz. 4 pwt. 17 gr. (3) 189 mi. 307
rd. 4 yd. 1 ft. 4 in. (4) 22 gal. 1 pt. (5) £719 14s. (6) 21 T. 17 cwt. 31
lb. 11 oz. (7) 3 pk: 7 qt. 1 pt.
103 **179.**—(1) 20 gal. 2 qt. 1 pt. 3 gi. (2) 5 T. 18 cwt. 41 lb. (3) £2955
17s. 3 far.

180.—(1) 8 yr. 5 mo. 28 da. (2) 27 yr. 9 mo. 4 da. (3) 21 yr. 7 mo.
104 8 da. (4) 12 yr. 11 mo. 29 da. (5) 60 yr. 5 mo. 20 da. (6) 30 yr. 9 mo.
12 da. (7) 7 yr. 9 mo. 17 da. (8) 3 yr. 1 mo. 14 da. (9) 8 yr. 11 mo. 20
da. (10) 79 yr. 8 mo. 24 da.

181.—(1) 94 da. (2) 145 da. (3) 221 da. (4) 188 da. (5) 105 da.
(6) 229 da. (7) 289 da. (8) 269 da. (9) 312 da. (10) 227 da. (11) 174
da. (12) 444 da.
105 **182.**—(1) 141 bu. 1 pk. 4 qt. (2) 46 da. 15 hr. 34 min. 6 sec. (3) 39
lb. 7 oz. 9 pwt. 19 gr. (4) 113 gal. 2 qt. 1 pt. (5) £1647 9s. 4d. (6) 285
rd. 1 ft. 6 in. (7) 1102 yr. 360 da. (8) £1348 13s. 6d. (9) \$90.65+. (10)
\$37. (11) 8183 mi. 40 rd. 4 yd. (12) 92 lb. 8 oz.

183.—(1) 14 lb. 7 oz. 17 pwt. (2) £2 1s. $10\frac{1}{3}$d. (3) 24 bu. 6 qt. 1 pt.

(4) 26 yr. 73 da. 2 hr. 36 min. (5) 9° 53′ 6″. (6) 1 T. 3 cwt. $58\frac{3}{7}$ lb. (7) **105**
8 mi. 106 rd. 3 yd. 2 ft. (8) 7 gal. 2 qt. $0\frac{3}{5}$ pt.▬(10) £2 4s. 2d. $3\frac{141}{385}$ **106**
far. (11) 4 yr. 3 mo. $10\frac{23}{41}$ da. (12) 22 mi. 121 rd. 3 yd. $7\frac{5}{19}$ in. (13)
$81\frac{106}{179}$ cu. ft.

184.—(1) 13 times. (2) 5 times. (3) 28 times. (4) 383940 times.
(5) $909\frac{99}{209}$ times. (6) 8 times.

185.—(1) £710 13s. ½d. (2) $727.23+. (3) 25 forks. (4) $578.75. **107**
(5) $105.60. (6) $124.46. (7) $64.20. (8) $8.82. (9) $200.64. (10) 4
yr. 8 da. (11) $544. (12) 17350 bu. 5 qt. (13) $76.25. (14) $8.13.
(15) $75.87+.▬(16) 53 A. 3 sq. ch. 5½ P. (17) 235 da. (18) $13.72. **108**
(19) 24 da. (20) 84 bbl. and 1 bu. 2 pk. 4 qt. remainder. (21) $150. (22)
2s. 6d. (23) $105.31+. (24) $183.93−. (25) $2661.98−. (26) $9.38.
(27) $383.24−. (28) $217.85+. (29) $2278.23+. (30) $78.94+. (31)
$275.30+. (32) $423.29+. (33) 1763.78+m. (34) 3499.48+fr. (35)
951.75−roubles. (36) 227.61+ gu. (37) 3779.53+ peseta. (38) 301.76+
pounds. (39) £138 5s. 2¾+d. (40) 1512.637+ milreis. (41) 6740.66+
Mex. dollars.

191.—(1) 9.5195275 A. (2) 1064.0625 bu. (3) 95.06961 mi. (4) **110**
1275.9411 gal. (5) 166.4473 lb. (6) 389.80025 A.▬(7) 10.82675 in. (8) **111**
79.5528 M. (9) 677.688 S. (10) 6367.9545 A. (11) 65.4805 Dl. (12)
93.312 G. (13) 65.8988 Hl. (14) 646.0025 Cu. M. (15) $4.83. (16)
$591.13+. (17) $5820.75. (18) $10.90. (19) $2.95+. (20) $124.68+.

196.—(1) 42 A. (2) 112 sq. yd. (3) 61 sq. ft. (4) 373 sq. ft. 78 sq. **113**
in. (5) $23760. (6) $24.84. (7) $150.08−. (8) $16.58−. (10) 12 ft.
(11) 46 ch. (12) 90 rd. (13) 3520 yd. (14) 6 yd. 1 ft. (15) 18 ft.▬(16) **114**
53 rd. (17) 7 yd. 2 ft.

197.—(1) 29 yd. 2 ft. $7\frac{1}{5}$ in. (2) 41 yd. 1 ft. 10½ in. (3) 107 yd. $6\frac{4}{7}$
in. (4) $80.83+.▬(5) $2201\frac{5}{8}$ bricks. (6) 278 flag-stones. (7) 10368 **115**
shingles. (8) 1139 sods. (10) 37 yd. (11) 117 yd. 1 ft. 4 in. (12) $12.

200.—(1) 3024 cu. ft. (2) 60 cu. yd. (3) $76.80. (4) $32.50. (5) **117**
1968¾ lb. (6) $30.94. (7) $22.95. (9) 9 ft. 6 in. (10) 6 ft. (11) 7 ft.
(12) 1 ft. 6 in. (13) 6 ft. 2 in. (14) 8 ft. (15) 8 ft. **118**

204.—(1) 10½ bd. ft. (2) 13⅓ bd. ft. (3) 20 bd. ft. (4) 22½ bd. ft. **119**
(5) 33 bd. ft. (6) 41⅔ bd. ft. (7) 63 bd. ft. (8) 40 bd. ft. (9) 67½ bd. ft.
(10) 37½ bd. ft. (11) $18.56+. (12) $4.20. (13) $22.00−. (14) 24 bd.
ft. (15) $14.69.

205.—(1) $353.94−. (2) $2\frac{52}{63}$ ft. (3) $1923.64−. (4) 9 ft. **120**

206.—(1) $562785\frac{105}{119}$ bricks. (2) $200220\frac{38}{49}$ bricks. (3) $268311\frac{3}{11}$ **122**
bricks.

207.—(1) 133.5925+ bu. (2) 110.055+ bu. (3) 18.8666+ bu. (4) **123**
48.2138 bu. (5) $3783\frac{3}{11}$ gal. (6) 34.9118− bu. (7) 894.9907+ hhd. (8)
167.565+ gal. (10) 3 ft. 7.83+ in. (11) 10 ft. 4.45− in. (12) $.73+.
(13) 6037.318 lb. (14) 4 ft. 5.43+ in. (15) $163.20. (16) 5 ft. (17) 128 **124**
bu. (19) 1028.16 gal. (20) 1175.04 gal. (21) 713836.8 gal. (22) $74.09−.

216.—(1) $56. (2) $15. (3) 48 lb. (4) 353.7 gal. (5) 2388 ft. **127**
(6) $604.62. (7) $1063.47−. (8) $.19−. (9) 316 men. (10) $8918.20.

128 (11) .91 mi.—(12) £130 7s. (13) 3924 ft. (14) 5348 bu. (15) \$690.
(16) \$5377.50. (17) 535 fr. (18) 779 T. (19) \$26250. (20) \$29500.
(21) 493 sheep. (22) 231 girls. (23) \$14062.50. (24) \$822.29. (25)
\$22660. (26) \$1070.60.

129 **217.**—(1) 9 times. (2) 4 times. (3) .10 times. (4) 25%. (5) $33\frac{1}{3}$%.
(6) 52%. (7) 8%. (8) 160%. (9) 20%. (10) $6\frac{2}{3}$%. (11) 25%. (12)
5%. (13) 18%. (14) 34%. (15) 60%. (16) 8%. (17) $33\frac{1}{3}$%. (18)
130 35%. (19) 14%. (20) 20%. (21) 96%. (22) 66%.—(23) $22\frac{2}{9}$%.

218.—(1) 8 bu. (2) 600 yd. (3) \$850. (4) 800 lb. (5) 630 gal.
131 (6) 1250 mi. (7) 500 T. (8) \$7132. (9) 15 fr. (10) \$828. (11) 300 T.
(12) \$8000. (13) 3425 bu. (14) 9000 yd. (15) 80 sheep. (16) 875 trees.
(17) \$5292. (18) \$4322.50. (19) \$1640.40. (20) \$8077.50. (21) \$213.

219.—(1) $16\frac{2}{3}$%. (2) \$115. (3) \$17225. (4) 300 A. (5) \$4836.
132 (6) $4082\frac{2}{3}$ lb. (7) 75%. (8) 468 yd. (9) 13085 Ger.; 15702 Ir.; 2617 Fr.
(10) \$4280. (11) 900 lb. (12) \$699.96. (13) 2150 lb. (14) \$12743.50.
133 (15) A 48%; B 52%. (16) \$16088.63–. (17) 8%.—(18) 40%. (19)
18%. (20) $11\frac{38}{157}$%. (21) 7%. (22) 1700 bbl. (23) $757\frac{1}{7}$%. (24) \$2350.
(25) \$2232. (26) 60% more; $37\frac{1}{2}$% less. (27) \$7716. (28) saltpetre 75%;
sulphur 10%; charcoal 15%. (29) \$351. (30) Oct. 30, \$366.70; Nov. 18,
134 \$374.42; Dec. 24, \$386; Jan. 4, \$405.30.—(31) \$817.50. (32) 40%. (33)
\$3500. (34) \$10.24–. (35) \$19.69–; \$14.06+. (36) 30%, \$24.51; 45%,
\$36.765; 25%, \$20.425. (37) horse \$320; buggy \$250. (38) dress goods
\$30; jewelry \$34.50. (39) wool $863\frac{1}{13}$ lb.; cotton $258\frac{12}{13}$ lb. (40) A \$720.
135 (41) 1st, 2740 bbl.; 2d, 3425 bbl.; 3d, 2329 bbl.—(42) oysters \$92; fish
\$41.40; clams \$33.35. (43) \$34427.43. (44) wife \$8750; son \$9750;
daughter \$6500. (45) 32500 inhab. (46) 30%.

136 **221.**—(1) \$881.46. (2) \$30.06. (3) \$191.10. (4) \$6.90. (5) \$37.50.
(6) \$1965.42. (7) \$35. (8) \$89.42–. (9) \$80.58. (10) \$7.15. (11)
\$4.05. (12) \$1.05. (13) \$5.22. (14) \$5.80. (15) \$7.15. (16) \$47.84.
(17) \$121.04. (18) \$32.48–. (19) \$22.75. (20) \$26.25. (21) \$99.09.
137 (22) \$38.28. (23) \$252. (24) \$1129.50.—(25) 20%. (26) $33\frac{1}{3}$%. (27)
25%. (28) $16\frac{2}{3}$%. (29) $\frac{1}{32}$%. (30) $\frac{5}{8}$%. (31) $2\frac{1}{2}$% gain. (32) $16\frac{2}{3}$%
loss. (33) 10% loss. (34) $33\frac{1}{3}$% gain. (35) $\frac{1}{8}$% loss. (36) \$8976. (37)
\$375. (38) \$825. (39) \$387. (40) \$20700. (41) \$5900. (42) \$325.
(43) \$63.30. (44) \$135.70. (45) \$6894. (46) \$47.80. (47) \$10. (48)
138 \$720. (49) 20%.—(50) \$260. (51) \$1377. (52) 13%. (53) \$5076.
(54) \$53.13. (55) 100%. (56) 135 bu. (57) \$101.04. (58) \$6030. (59)
$16\frac{2}{3}$%. (60) 74¢. (61) 9%. (62) 15%. (63) 12%. (64) \$63. (65)
139 25%.—(66) 20%. (67) \$45. (68) \$1010.60. (69) 40%. (70) \$259.75–.
(71) \$204.70. (72) \$26.25. (73) lost \$240. (74) \$5.70. (75) \$416.
(76) \$4000. (77) 33¢. (78) 30%. (79) 4%. (80) \$311.85. (81) \$155.61.
140 (82) \$800 (83) loss \$12.83–. (84) \$2.53–. (85) gain 40%. (86) \$5.46.
(87) \$53760. (88) loss 4%. (89) \$1111.50. (90) \$5096. (91) 21¢. (92)
\$5600; \$6720.

142 **224.**—(1) \$225.72. (2) \$476.67–. (3) \$350.56–. (4) \$382.11–.
(5) \$294.23–. (6) \$483.59+. (7) \$388.00+. (8) \$337.50. (9) \$312.24–.

(10) $514.60+. (11) $557.46. (12) $577.39+. (13) $1123.02—.——(14) 143
$207.63+.

225.—(1) 14 2/7%. (2) 28%. (3) 23½%. (4) 40 3/10%. (5) 31⅗%. (6)
17 13/300%. (7) 79/100%. (8) 71 38/100%.

226.—(2) $3.20. (3) $14.81+. (4) $6.67—. (5) $41.15+. (6) $30. 144
(8) $500. (9) $138.90+. (10) $272.96—. (11) $16.11—. (12) $19.65—.
(14) 16⅔%. (15) 20%. (16) 33⅓%. (17) 25%. (18) 20%. (19) 18¾%.
(20) 8%.——(21) $52.67. 145

230.—(2) $1.07. (3) $34.76+. (4) $7.75. (5) $2.95. (6) $56.25. 147
(7) $722.25.——(8) $1205.70. (9) $17757.20. (10) $669.75. (11) $17.42. 148
(12) $7634.95. (13) $2512.75. (14) $80.46+. (16) 1⅝%. (17) 8/33%.
(18) 3/16%. (19) 5%. (20) 4%. (21) 2%. (22) ¼%. (23) 3½%. (25)
$2515. (26) $1506. (27) $2440. (28) $3900.——(30) $82. (31) $350. 149
(32) $512. (34) $180. (35) $560. (36) $1600. (38) $151.02+. (39)
$720. (40) $800. (42) $32. (43) $136. (44) $248. (45) $11.04. (46)
5/32%. (47) $536. (48) $530.——(49) 3%. (50) $20.25. (51) $7761.88—. 150
(52) 2½%. (53) $173.20. (54) $322.25. (55) $128.19—. (56) $3008.
(57) ½%. (58) 2%. (59) $9930.27. (60) $591.07.——(61) 26300 lb. (62) 151
77 bbl.; $2.91—. (63) 56 bbl.; 42¢. (64) 3600 bu. (65) $1.50. (66)
$15.90.——(67) $8.75. (68) $3255.69—. (69) $431.09. 152

242.—(1) $2.20+. (2) $30.88—. (3) $2.29+. (4) $9.49—. (5) 157
$1.37—. (6) $44.08—. (7) $121.90+. (8) $70.33—. (9) $1.13+. (10)
$1.74+. (11) $5.66—. (12) $37.35—. (13) $11.94+. (14) $4.66+.
(15) $3.51+. (16) $46.23—. (17) $19.17—. (18) $19.27+. (19)
$2.90—. (20) $9.63—. (21) $11.51+. (22) $380.53—. (23) $13.18—.
(24) $39.92—. (25) $24.62—. (26) $16.00—. (27) $150.07+. (28)
$22.46—. (29) $15.73+. (30) $8.36—. (31) $74.99—. (32) $36.15—.

243.—(1) $5.46+. (2) $1.16+. (3) $1.69—. (4) $6.25+. (5) 159
$17.47—. (6) $10.84—. (7) $.23+. (8) $2.09+. (9) $4.98—. (10)
$6.34+.

244.—(1) $17.13—. (2) $5.06+. (3) $106.52+. (4) $19.40+. (5) 160
$53.24—. (6) $42.13+. (7) $297.34+. (8) $146.26—. (9) $13.51+.
(10) $5.92+. (11) $59.23+. (12) $12.03+. (13) $144. (14) $19.81—.
(15) $15.92—. (16) $37.35—. (17) $27.59—. (18) $81.73—.——(19) 161
$131.25. (20) $390.02+. (21) $54.33+. (22) $169.87+. (23) $20.81+.
(24) $30.27—. (25) $121.90—. (26) $694.72—. (27) $6.23+. (28)
$111.55—. (29) $9.19—. (30) $1.51+. (31) $2.91—. (32) $283.55+.
(33) $34.70+. (34) $62.63+. (35) $226.72+. (36) $12.07+. (37)
$17.34—. (38) $48.37+. (39) $242.25—. (40) $7.00+. (41) $43.57—.
(42) $123.31—. (43) $90.13. (44) $363.98+. (45) $36.08—. (46)
$12.28+. (47) $30.18—. (48) $9.72+. (49) $62.42+. (50) $438.39—.
(51) $7.40+. (52) $188.27—. (53) $2.09—. (54) $8.24+. (55)
$1965.68+. (56) $44.40—. (57) $2.37+. (58) $100.16+.

245.—(1) $3.66+. (2) $3.95. (3) $4.08—. (4) $2.34—. (5) 162
$202.71—. (6) $14.10—. (7) $36.59—. (8) $7.43—. (9) $30.86+.
(10) $16.55—. (11) $7.82—. (12) $295.58+. (13) $13.74—. (14)

162 $7.38+. (15) $14.28+. (16) $5.10−. (17) $1524.75. (18) $9.48+.
163 (19) $4.92+. (20) $24.63+. (21) $20.99+. (22) $25.81+. (23)
$166.65−. (24) $9.65+. (25) $50.03−. (26) $42.71+. (27) $32.65−.
(28) $130.50+. (29) $392.35+. (30) $196.46−. (31) $337.94+. (32)
$14.90+. (33) $4262.77−.

164 **246.**—(1) $.68+. (2) $10.20. (3) $11.35. (4) $56.02+. (5) $7.72+.
(6) $68.96−. (7) $5.83−. (8) $5.58−. (9) $93.16+. (10) $16.65−.

165 **247.**—(1) $239.17−. (2) $21.50. (3) $4.33−. (4) $10.53+. (5)
$13.67−. (6) $194.83+. (7) $3.66+. (8) $10.65. (9) $32.50. (10)
$125.22+. (11) $29.10. (12) $17.50.

248.—(1) $893.99−. (2) $317.82+. (3) $3774.36+. (4) $863.52.
(5) $521.81+. (6) $773.24+. (7) $621.25. (8) $862.07−.

167 **250.**—(1) $.38+. (2) $23.16+. (3) $2.52+. (4) $1.54−. (5)
$5.26−. (6) $9.09−. (7) $62.18+. (8) $12.11+. (9) $19.90−. (10)
$59.99−. (11) $864.54+. (12) $75.07−. (13) $20.94−. (14) $66.84−.
(15) $12.33−. (16) $284.91−. (17) $337.01−. (18) $56.32+.

168 **251.**—(1) 5%. (2) 8%. (3) 4%. (4) 10%. (5) 6%. (6) 9%. (7)
4%. (8) 8%. (9) 9%.

169 **252.**—(1) 3 yr. 5 mo. 18 da. (2) 2 yr. 1 mo. 24 da. (3) 11 mo. 5 da.
(4) 6 da. (5) 18 da. (6) 28 da. (7) 33 yr. 4 mo.; 25 yr.; 20 yr.; 16 yr.
8 mo. (8) 25 yr.; 42 yr. 10 mo. 9 da. (9) Nov. 3, 1888. (10) Aug.
10, 1889.

170 **253.**—(1) $2280. (2) $630. (3) $1500. (4) $595. (5) $350. (6)
$600. (7) $320. (8) $865. (9) $3576.42−. (10) $3956.92+.

171 **254.**—(1) $171. (2) $400. (3) $666. (4) $7331.94+. (5) $8960.
(6) $1200. (7) $4383.55+. (8) $479.80+.

255.—(1) $606.06. (2) $518.80. (3) May 16, 1888. (4) Aug. 10,
172 1889.——(5) $787.79−. (6) $963.19+. (7) $590.57−. (8) 4%. (9)
accurate interest $4.14 less. (10) 5%. (11) 4%. (12) $375.75. (13)
173 $413.42. (14) $326.71−.——(15) $32.49−. (16) $31\frac{1}{13}$%. (17) $1\frac{42}{51}$%.

174 **256.**—(1) $1339.81+. (2) $1053.12+. (3) $594.50+. (4) $1397.18−.
(5) $672.09−. (6) $1339.94−. (7) $1157.66−. (8) $71.71+. (9)
$245.58−. (10) $89.83+. (11) $165.67+. (12) $468.92+. (13)
$3161.69+. (14) $3972.11−.

177 **258.**—(2) $20724.63−. (3) $90305.77+. (4) $23262.27+. (5)
$709.94+. (6) $2581.80−. (7) $6108.95−. (8) $3486.95−. (9)
$4381.74−.

179 **263.**—(1) $832; $2.08. (2) $8220; $100.01. (3) $300; $6.20. (4)
$291; $14.55. (5) $270; $6.30. (6) $259.88−. (7) $580. (8) $2782.50
at 2 mo. (9) $3.60. (10) $242.24+. (11) $26.37−. (12) $3231.37.
(13) $3611.93.

183 **272.**—(1) Apr. 22, 1889. (2) May 3, 1888. (3) Nov. 6, 1889. (4)
Feb. 22, 1888. (5) Nov. 12, 1888. (6) Apr. 21, 1888. (7) June 6, 1889.
(8) Oct. 28, 1888. (9) Aug. 16. 1887. (10) Jan. 3, 1890.

273.—(1) 125 da. (2) 64 da. (3) 94 da. (4) 125 da. (5) 94 da.
(6) 156 da. (7) 63 da. (8) 155 da. (9) 95 da. (10) 126 da.

274.—(1) 138 da. (2) 111 da. (3) 99 da. (4) 97 da. (5) 128 da. **184**
(6) 101 da. (7) 66 da. (8) 149 da. (9) 98 da. (10) 124 da.

280.—(1) Mar. 9, 1889; 36 da.; $9; $1491. (2) Jan. 1, 1890; 54 da.; **186**
$9.98; $940.02. (3) Apr. 11, 1888; 30 da.; $1.63; $323.37. (4) Feb. 11,
1890; 41 da.; $1.37; $148.63. (5) Apr. 8, 1888; 40 da.; $3.33; $496.67.
(6) Mar. 3, 1888; 55 da.; $9.17; $1190.83. (7) June 6, 1889; 52 da.;
$2.38; $272.62. (8) Nov. 16, 1888; 71 da.; $38.66; $2761.34. (9) Nov. 9,
1888; 52 da.; $45.07; $5154.93. (10) $218.98—.——(11) 53 da.; $35.33; **187**
$2964.67. (12) $7.31; $795.24. (13) Jan. 12, 1890; 60 da.; $5.38;
$532.62.——(14) Jan. 11, 1889; 30 da.; $3.26; $486.16. (15) $5.11; **188**
$494.89. (16) $593.25. (17) $34.29; $1802.61.——(18) discounts: $12.47, **189**
$2.22, $8.45; total proceeds, $1728.41. (19) $573.97. (20) Feb. 1, 1889;
56 da.; $12.83; $1487.17. (21) Sept. 17, 1889; 45 da.; $7.02; $1116.38.
(22) Oct. 12, 1889; 102 da.; $21.25; $1228.75. (23) Oct. 8, 1888; 50 da.;
$4.27; $435.23. (24) No. 1—Mar. 3, 1890; 118 da.; $14.75; $985.25.
No. 2—Mar. 3, 1890; 118 da.; $10.33; $689.67. No. 3—Mar. 3, 1890; 118
da.; $8.26; $551.74. No. 4—Mar. 3, 1890; 118 da.; $.93; $62.27.

281.—(1) $680. (2) $372. (3) $400. (4) $240. (5) $540. (6) **190**
$1181.73—. (7) $400. (8) $722.63—.——(9) $761.81. (10) $1020.93—. **191**
(11) $2549.70—.

282.—(1) bank disc't $21.60; true disc't $21.34; difference $.26. (2)
$582.50; $582.55. (3) $574.97; $575.

285.—(1) $493.20. (2) $1317.07. (3) $719.37. (4) $2543. (5) **193**
$2790.03. (6) $1452.99. **194**

286.—(1) $315.13. (2) $520.84. (3) $46.86. (4) $1017.16. (5) **195**
$2790.03. (6) $2906.55. (7) $1495.39. (8) $857.88. (9) $864.89. (10) **196**
$365.15.

295.—(2) $112. (3) $1087.50. (4) $656.25. (5) $876.38—. (6) **200**
$324. (7) $629.63—. (9) $19216.13—. (10) $3258.75. (11) $2681. (12)
$15413.06+. (13) $1553.38—. (14) $2925.38—. (16) $9849.38—. (17)
$2107.38—. (18) $3692.50. (19) $5859.75. (20) $6754.50. (21) $6205.
(23) 3%.——(24) $6\frac{3}{8}$%. (25) $14\frac{1}{4}$%. (26) $12\frac{5}{8}$%. (27) $117\frac{7}{8}$. (28) $8\frac{1}{4}$%. **201**
(29) $32\frac{1}{8}$%. (31) $103\frac{1}{8}$. (32) $23\frac{3}{8}$%. (33) $15\frac{7}{8}$%. (34) $83\frac{3}{4}$. (35) $19\frac{3}{8}$%.
(36) $13\frac{1}{8}$%. (38) $3100. (39) $12500. (40) $17000. (41) $11700. (42)
$6900. (43) $8400.——(44) $7000. (46) $5000. (47) $12000. (48) **202**
$15000. (49) $13800. (50) $3800. (51) $32000. (52) $562.50. (53)
$85\frac{7}{8}$. (54) 150 sh. (55) $3365. (56) $15\frac{5}{8}$%. (57) 74 sh. (58) $157.50.
(59) $196\frac{5}{8}$. (60) $16014.38—. (61) $421.88—. (62) $12800.——(63) **203**
$34\frac{3}{8}$%. (64) $10145.

296.—(2) $75. (3) $70. (4) $150. (5) $160.——(7) $87\frac{3}{8}$. (8) $59\frac{7}{8}$. **204**
(9) $79\frac{7}{8}$. (10) $74\frac{7}{8}$. (12) 5%. (13) 8%. (14) $6\frac{3}{8}$%. (16) $12\frac{1}{2}$%. (17)
$5\frac{1}{8}$%. (18) $3\frac{4}{7}$%. (20) 7%. (21) 5%. (22) 3%. (24) 6%. (25) 12%.
(26) $7\frac{1}{2}$%. (27) 4%.——(28) $149\frac{7}{8}$. (29) $166\frac{3}{8}$. (30) 3%. (31) $4820. **205**
(32) $36000. (33) $52.05. (34) $258. (35) 4%; $50. (36) $370.
(37) $72\frac{8}{11}$. (38) $3200. (39) $2000. (40) $200 gain. (41) 4%.
(42) $300. **206**

208 **302.**—(2) $7236. (3) $8436.25. (4) $3180. (5) $1505.63—. (8)
$3489.50. (9) $1795.80. (10) $2466.67—. (11) $752.81+. (12) $4928.75.
209 (14) $824. (15) $352. (16) $7500. (17) $136. (18) $1200. (21)
$120. (22) $1600. (23) $720. (24) $1200. (25) $12013.96+. (26)
$15387.98—.

212 **308.**—(1) $1649.85. (2) $1699.42—. (3) $3549.92+. (4) $611.96—.
(5) $80.66+. (6) $300.94—. (7) $736.58—. (8) $354.84+. (9) $428.-
63—. (10) $759.09—. (11) $4908.09+. (12) $3646.08—. (13) $194.09.

213 **309.**—(1) £1260. (2) £560. (3) 4161 fr. (4) 1682.37 fr. (5) 3600
gu. (6) 448 gu. (7) 1280 m. (8) 3600 m. (9) £1195 17s. 6¼+d.
(10) 9505.71— fr. (11) 18600.20+ gu. (12) 35697.82+ m. (13) £800.
(14) £740.

215 **315.**—(1) $447.60. (2) $521.55. (3) $1872.50. (4) $406.70. (5)
216 $363.75. (6) $134.25. (7) $418.80. (8) $176.—(9) $1958.75. (10)
$6720. (11) $900. (12) $67.50. (13) $955.50. (14) $182.40. (15)
$466.35. (16) $1409. (17) $3465.40. (18) $6.44—. (19) $385.80. (20)
$87.15. (21) $15.60. (22) $399. (23) duty $469; exchange $955.46+.

217 **317.**—(2) $27. (3) $122.50. (4) $227.70. (5) $103.34+. (6)
218 $27.93. (7) $112.50.—(8) $228.92. (10) 12 m. (11) 9 m. (12) 11 m.
(13) $1.38. (14) 98¢. (15) $1.13. (16) $1.52. (17) 8½ m. on $1 or 85¢
on $100. (18) 32¢ on $100; $26.56. (19) 6 m.; $50.70.

221 **324.**—(1) $40.50. (2) $531.25. (3) Eagle $5294.12—; Manhattan
$7941.18—; Globe $4764.70+. (4) Home $2500; Essex $1500; Howe
222 $2000. (5) $800; $1400. (6) $1333.33+; $5000. (7) $30; $37.50.—(8)
⅞%; 87½¢ on $100. (9) $300. (10) 1¼%. (11) 8%; $400. (12) $12000.
223 (13) $3200. (14) $12000; $1200. (15) $8000. (16) $720.—(17)
$8501.78—. (18) $52006.08—.

225 **330.**—(1) 11⅓¢. (2) $4. (3) 7 mi. (4) $27. (5) $25/32. (6) $5.125.
(7) 2 doz. (8) $13. (9) $6. (10) 198 men. (11) 7 mi. (12) 8 hr.

227 **331.**—(1) $1.17. (2) $1.35. (3) 2175 bu. (4) $120. (5) $16.45.
(6) 2190 bbl. (7) $9.60. (8) 15 sheep. (9) $7.20. (10) $246. (11)
$465. (12) 7⅓ ft. (13) 39 ft. (14) 36 men. (15) 9 men. (16) A $900;
228 B $600.—(17) $9000. (18) $78.91. (19) 25%. (20) 48 da. (21) 12¢.
(22) 144 da.

229 **332.**—(1) $59.80. (2) $63.56. (3) 1430 reams. (4) 9%. (5)
230 $129.60. (6) $306.60.—(7) 23 da. (8) 15 ft. (9) 9 hr. (10) $1368.50.
(11) 14 half-barrels. (12) 13½ da.

232 **336.**—(1) 60¢ on $1; $10388.40. (2) $5299.14. (3) Smith $1800;
Owens $1260. (4) M & Co. $2931; W & Co. $1490; F & Sons $950. (5)
Miles $15579; Thomas $6855.30; Lewis $3937.50. (6) Devon $4588.92;
Heitmueller $4020.66; Patrick $3650.22; Lackland $3003.21; Hopper
$2497.32. (7) J & Co. $1499.20; Henson $540; S & Son $308; Sinclair
$227.20. (8) 82¢ on $1; $5096.30.

234 235 **341.**—(1) 4 mo. (2) 5 mo. 2 da. (3) 3 mo. 9 da.—(4) July 5. (5)
Aug. 20. (6) 7 mo. 8 da. (7) July 19, 1889. (8) 4 mo. (9) 10 mo. (10)
Sept. 30, 1889. (11) Nov. 23, 1889.

342.—(1) July 3, 1889. (2) Mar. 1, 1889. (3) Apr. 4, 1888. (4) 238
Sept. 30, 1889. (5) Feb. 18, 1887.▬(6) Apr. 23, 1889. (7) Apr. 8, 1889. 239
(8) July 6. (9) Aug. 3. (10) May 16, 1888.▬(11) Jan. 19, 1888. (12) 240
May 16, 1889. (13) Aug. 29, 1888. (14) Dec. 20, 1889.

343.—(1) May 20, 1889. (2) Oct. 29, 1888. (3) Dec. 2, 1888. (4) 242
Apr. 17, 1889. (5) June 21, 1889. (6) Oct. 9, 1888.▬(7) Aug. 9, 1888. 243
(8) July 13, 1889. (9) May 18, 1889. (10) July 1, 1888. (11) Mar. 28,
1889. (12) June 3, 1888.

346.—(1) May 2, 1890. (2) Oct. 7, 1889. (3) Feb. 6, 1890. (4) 247
Apr. 12, 1890. (5) May 25, 1889. (6) Apr. 6, 1889.▬(7) Aug. 20, 1890. 248
(8) Apr. 28, 1888. (9) July 16, 1889. (10) Apr. 7, 1890. (11) Aug. 4,
1880.▬(12) June 22, 1886. (13) Aug. 3, 1888. (14) Oct. 21, 1888. (15) 249
Jan. 11, 1889. (16) May 1, 1889.▬(17) Oct. 7, 1888. (18) June 20, 1888. 250
(19) Oct. 12, 1889. (20) Nov. 25, 1888.▬(21) July 18, 1889. (22) Dec. 251
10, 1888. (23) Jan. 5, 1889. (24) July 2, 1887. (25) Sept. 1, 1887. (26)
Aug. 30, 1887. (27) Jan. 18, 1889.

347.—(1) Mar. 16, 1888. (2) Apr. 24, 1889. (3) June 8, 1889. 253

348.—(1) $428.86. (2) $567.84. (3) $516.63.▬(4) $1440.15. (5) 257
$694.43. (6) $1218.95. 258

356.—(1) gain $4024.34. (2) gain $5222.10. (3) gain $7013.13. 262
(4) loss $3998.91. (5) loss $6641.10. (6) loss $1881.25. (7) cap. $2935.
(8) cap. $8430.15. (9) cap. $6129.20. (10) insolv. $989. (11) insolv.
$11870.70. (12) insolv. $3109.50. (13) cap. $13548.31. (14) cap. $15-
505.92. (15) cap. $2434.24. (16) insolv. $2158.76. (17) insolv. $3741.62.
(18) insolv. $255.30. (19) cap. $8409.95. (20) insolv. $3812.70. (21)
$25000. (22) $8980.50. (23) net gain $2825.▬(24) Will $10000; Meyers 263
$6000. (25) Wells $7195.20; Green $4796.80. (26) Insolv. at commenc-
ing—Waite $2000; Foley $2000; Wiley $1000; cap. at closing—Waite
$1593; Foley $1593; Wiley $2593.

357.—(1) Long $2100; Shimer $2400; Dorr $2700. (2) $870; $1020; 264
$1260. (3) Russell $2325.55; Florence $2099.45. (4) Arnold $4500;
Brown $8600. (5) Reid $26647.06; Haines $22852.94.

358.—(1) A $4998.09; B $4266.67; C $3535.24. (2) Wilson $1758.- 266
93; Holmes $1687.50; Farr $1553.57.▬(3) Harvey $1900; Roberts 267
$2225; Williams $875. (4) Merritt $2029.75; Reeves $3970.25.

359.—(1) Cross $15708.43; Davies $11091.57. (2) Horn $19920.36; 270
McLane $13479.64; McLane owes Horn $220.36 int. (3) Herndon 271
$13291.10; Newman $5597.74; Dulin $9211.16; Newman owed interest
to Herndon $19.67, to Dulin $96.88. (4) Gould solv. $17355.13; Hunter
insolv. $1808.87; Nichols solv. $16695.93; Hunter owes interest to Gould
$474.40, to Nichols $515.20.▬(5) Hixon insolv. $1534.01; Merritt solv. 272
$666.19; Pickett insolv. $2032.19. (6) Smith $912.78; Harrison $793.69;
Yeatman $633.53. (7) White $21010; Moore $34580.

360.—(1) 57. (2) 57. (3) 52. (4) 638. (5) 574. (6) 664. (7) 274
6949. (8) 6739. (9) 8664781780. (10) 103685041. (11) 1147421109.
(12) 11599992904.

275 **361.**—(1) $14542.93. (2) $3639.58. (3) $8024.49. (4) $5037.33.

276 **362.**—(1) $572. (2) $792. (3) $396. (4) $583. (5) 275 rd. (6) 198 mi. (7) $176. (8) $891. (9) $297. (10) $495. (11) $374. (12) $682.

363.—(1) $56496. (2) $799018. (3) $5663152. (4) $410091.55. (5) $453952.95. (6) $36033.25. (7) $530895.75. (8) $1043606.30. (9) $167625.15.

277 **364.**—(1) $139510.50. (2) $147804.75. (3) $158233.30. (4) $131011.65. (5) $452339.40. (6) $754508.75. (7) $151524.65. (8) $238939.80. (9) $28434.23.

278 **365.**—(1) $114007.20. (2) $10598.90. (3) $10617.88. (4) $16656.30. (5) $46457.10. (6) $22464. (7) $14677.76. (8) $10111.04. (9) $155129.25. (10) $155322.15. (11) $7473.24. (12) $3216.90.

366.—(1) $525. (2) $756. (3) $384. (4) $810. (5) $5400. (6) $900. (7) $13000. (8) $14700. (9) $7200. (10) $1600. (11) $630. (12) $12600. (13) $1200. (14) $1200. (15) $320. (16) $1200.

279 **367.**—(1) $247715.70. (2) $243540. (3) $60226335. (4) $1087638.75. (5) $5209451.52. (6) $131602.24. (7) $40102686.72. (8) $167186.25. (9) $10149180.80.

280 **368.**—(1) $424575. (2) $84770. (3) $733779.50. (4) $26863.20. (5) $830062.74. (6) $526.32. (7) $981088. (8) $9603. (9) $1007010.

369.—(1) $1296. (2) $1764. (3) $3136. (4) $7225. (5) $5476. (6) $8649. (7) $6724. (8) $4096. (9) $1444. (10) $361. (11) $12769. (12) $14884.

281 **370.**—(1) $3024. (2) $2204. (3) $1281. (4) $2176. (5) $3999. (6) $1776. (7) $2975. (8) $3744. (9) $1581. (10) $4784. (11) $12144. (12) $10125. (13) $11169. (14) $12325. (15) $11264. (16) $11856.

371.—(1) $3074. (2) $5925. (3) $1088. (4) $6885. (5) $4288. (6) $575. (7) $342. (8) $1968. (9) $8556. (10) $4030. (11) $6972. (12) $2860.

282 **372.**—(1) $625. (2) $4225. (3) $2025. (4) $7225. (5) $1225. (6) $5625. (7) $3025. (8) $13225. (9) $9025.

373.—(1) $7216. (2) $5621. (3) $3024. (4) $9024. (5) $621. (6) $2009. (7) $625. (8) $1224. (9) $11021. (10) $221. (11) $9016. (12) $13216.

283 **374.**—(1) $2436. (2) $4875. (3) $1599. (4) $4896. (5) $4851. (6) $4875. (7) $8064. (8) $2491. (9) $8019. (10) $14384. (11) $12036. (12) $9964.

375.—(1) $\$35\frac{1}{16}$. (2) $\$55\frac{1}{4}$. (3) $\$16\frac{1}{64}$. (4) $\$38\frac{1}{16}$. (5) $\$33\frac{1}{4}$. (6) $\$39\frac{1}{3}$. (7) $\$30\frac{1}{4}$. (8) $\$56\frac{1}{4}$. (9) $\$72\frac{1}{4}$.

284 **376.**—(1) $\$90\frac{2}{3}$. (2) $\$112\frac{9}{25}$. (3) $\$160\frac{5}{12}$. (4) $\$339\frac{1}{6}$. (5) $\$.12\frac{2}{3}$. (6) $\$3681\frac{9}{20}$. (7) $\$16.25\frac{3}{32}$. (8) $\$650\frac{6}{25}$. (9) $\$.28\frac{7}{8}$. (10) $\$.72\frac{11}{12}$. (11) $\$.42\frac{15}{64}$. (12) $\$152\frac{5}{81}$.

285 **377.**—(1) $276. (2) $800. (3) $\$929\frac{1}{3}$. (4) $950. (5) $2552. (6) $5952. (7) $1422. (8) $2100. (9) $3363. (10) $3040. (11) $1692. (12) $6989.

378.—(1) \$.232. (2) \$.45. (3) \$36. (4) 3.5 times. (5) \$5.12. (6) \$.1305. (7) \$10.18. (8) \$61.20. (9) \$77.60. **285**

379.—(1) $\frac{aed}{moe}$. (2) $\frac{br}{it}$. (3) $\frac{lh}{me}$. (4) $\frac{ah}{ma}$. (5) $\frac{cs}{om}$. (6) $\frac{led}{mre}$. (7) $\frac{lo}{or}$. (8) $\frac{cbr}{ode}$. (9) $\frac{br}{me}$. (10) $\frac{la}{or}$. **286**

381.—(1) \$4.25. (2) \$1. (3) \$12.50. (4) \$1.25. (5) \$17.50. (6) \$1.80. (7) \$5.52. **287**

382.—(1) \$3.50. (2) 30¢. (3) 91¢. (4) 50¢. (5) 70¢. (6) \$9. **288**